ABSTRACT

Modern railway rolling stock should meet requirements regarding comfort (maximum travel speed with minimum vibrations of wagons, noiselessness of movement, etc.). To eliminate the influence of dynamic loads, rolling stock is equipped with vibration dampers. The objective of the work is to select the parameters of the vibration dampers of rolling stock, depending on its characteristics, to ensure the due indicators of comfort and safety of transportation of passengers and goods by rail. To achieve this objective, applied methods of mathematical modelling were based on numerical programming of operation of dynamic systems. The indicators of vibration dampers are evaluated according to the results of studies of the dynamics of the rolling stock (in particular, of vibration protection rates).

Assessment of dynamic state of the rolling stock implies application of methods of mathematical and physical modelling, which include the development of a physical and mathematical model, a calculation algorithm, and computer programming. The study of the mathematical model by numerical methods makes it possible to carry out a multifactorial experiment using a large number of input parameters (factors) and to select the characteristics of vibration dampers that are optimal for the conditions under consideration.

Keywords: railway, dynamics, rolling stock, vibration damper, parameter estimation, mathematical model, vibration process, amplitude, displacement, speed, acceleration, motion.
INTRODUCTION

To successfully promote products globally, a manufacturer of railway rolling stock should not be guided exclusively by the requirements stated by technical indicators, such as strength, stiffness, stability of rolling stock, since there are also requirements regarding comfort (maximum travel speed with minimal vibrations of wagons, noiselessness of movement, etc.). The latter factors are also important for transportation of different groups of goods.

Currently existing passenger coaches do not always meet the requirements for comfort and safety. It is necessary to develop and create new types and series of coaches that would not only meet modern world standards, but also surpass them.

However, modern scientific and technical literature (e.g., [1, p. 13]) pays less attention to comfort than to the safety of transportation, while these requirements are different in each specific region or a country. Recently, these specifications have started to refer to high speed passenger trains more frequently. In Europe, these requirements are regulated by the European Union Agency for Railways (ERA)¹, in the United States by the Department of Transportation and by the Federal Railroad Administration².

It is known that structural elements of rolling stock perceive weight loads, loads arising in the mode of unsteady motion (traction, braking), inertial loads arising when moving in curved sections of the track. The listed loads are static loads. The dynamic loads arise due to movement of the vehicle along the railway track with geometry and track irregularities.

To eliminate the influence of dynamic loads, modern rolling stock is equipped with vibration dampers. The choice of their parameters depending on the features of rolling stock to ensure the indicators of comfort and safety of movement of passengers and goods by rail is still carried out by trial-and-error method.

The objective of the study is to evaluate the parameters of vibration dampers of rolling stock, depending on the action of dynamic loads with specific indicators. For this, numerical methods of mathematical and physical modelling were applied using MathCad software, followed by an analysis of the results obtained.

RESULTS

Indicators of the Dynamic Properties of the Rolling Stock

Vibration processes resulting from the action of dynamic loads are assessed by indicators of the dynamic properties (DPIs) of rolling stock. DPIs include [2, p. 148]:

- Vibration protection: the degree of protection of equipment of the locomotive and the railway track from vibrations arising during movement.
- Traffic safety: the degree of safety provided during movement.
- Running smoothness: the degree of impact of vibrations of rolling stock on the human body.

These indicators are assessed based on the results of research on the dynamics of rolling stock. Let us dwell in more detail on the main indicators of vibration dampers. These indicators comprise:

- Maximum values of body acceleration values in vertical and horizontal directions, \( \dot{q}_{\text{max}} \).
- Maximum displacement values at the end points of the body \( q_{\text{max}} \), determined by the dimensions and operating conditions of the automatic couplers.
- Maximum values of the dynamic factors in the horizontal \( F_{Dh} \) and vertical directions of the first \( F_{D1} \) and second \( F_{D2} \) spring suspension stages [2, p. 232];
- Safety margin factors of spring deflection value regarding the dynamics \( F_{\text{fin}} \).

The maximum displacement of the ends of the body is determined using the known formula as the amount:

\[
q_{\text{max}} = 0,5(\Delta q_{1-1\text{max}} + \Delta q_{2-2\text{max}}) + \Delta q_{1-1\text{max}} + \Delta q_{2-2\text{max}} \overline{2a_s},
\]

where \( \Delta q_{1-1\text{max}} \) and \( \Delta q_{2-2\text{max}} \) are maximum spring deflections (stroke of the hydraulic damper rod) of the central suspension of the first and second bogies, respectively;

\( a_s, ab \) – half of the base and body length, respectively.


The indicator of horizontal dynamics $F_{1\text{th}}$ is determined by the formula:

$$\Sigma F_y = R_y + R_y - Q - L = -3,125 + 98,25 - 20 - 75 = 0,125 = 0.$$ 

$$F_{1\text{th}} = \frac{Y_{bnY}}{L_a},$$

where $Y_{bnY}$ is dynamic component of the horizontal maximum frame force (frame forces are transverse forces of interaction between the wheel set and the bogie frame of a railway rolling stock unit);

$L_a$ – axial load.

Changes in the dynamics of the first and second stages are shown in Pic. 1.

Pic. 1 shows that it is rational to determine the dependence of the vertical dynamic indicators of the first $F_{D1}$ and the second $F_{D2}$ stages of the spring suspension, depending on speed of vehicle’s movement, by a linear dependence of the type $y = ax + b$, as evidenced by the values of variances, which tend to unity for each obtained function. Moreover, it can be seen from the Pic. 1 that with an increase in speed, the dynamic factor decreases. For example, at a speed of 200 km/h, it decreases by 1,37, which indicates a revision of the parameters of hydraulic dampers for high-speed rolling stock.

Failure to respect smoothness indicators due to the action of vibrations in transient and steady-state modes of movement negatively affects the performance ability of drivers, of service personnel, and the well-being of passengers. These indicators are assessed with various indicators: frequency and amplitude of vibrations.

The authors of numerous studies [3, pp. 1–4; 4, pp. 1–3] note the negative impact of vibrations on the human body, which leads to irreversible consequences associated with changes in its physical and psychological state, which is aggravated over time due to the cumulative nature of the impact of vibrations.

**Methods of Mathematical and Physical Modelling of the Indicators of the Dynamic State of the Rolling Stock**

Rolling stock design widely uses methods of mathematical and physical modelling to determine the indicators of the dynamic state of vehicles. The method of mathematical modelling includes development of a physical and mathematical model, of an algorithm for computing and computer programming. The study of the mathematical model by numerical methods makes it possible to perform a large number of numerical experiments with exhaustive search (variation) of strength, cargo loading, and other parameters of the vehicle, the conditions of its operation, which allows choosing the parameters of the vibration damper design that are optimal for the specific (selected) conditions.

Technical problems associated with vibration processes are based on the generation and solution of multi-mass systems of differential equations (DE), which, as a rule, are solved by numerical methods. The results obtained as a result of the solution make it possible to obtain a mathematical model of the process occurring in the system and to estimate the parameters of the vibration process quantitatively. In our case, the system of differential equations of the system under study is based on the developed design scheme [5, p. 17; 6, p. 125] and is two-mass. The development of the design scheme, based on the research conditions and purposes, considered...
external forces, influencing the dynamic state of the rolling stock under consideration, as a technical system.

When creating a dynamic model, the operational and technical characteristics of the rolling stock were considered as external factors, which in mathematical terms were presented as «harmonic disturbance», expressed as a sine function with an argument corresponding to the rail length \[5, p. 17; 6, p. 125; 7, p. 192\]. Such a choice of input parameters is typical, for example, for the situations of moving along a single uneven track, of entering a curve, in case of impact at the rail joint, collision of vehicles, etc. It has been established \[6, p. 131; 7, p. 214\] that dissipative characteristics of the railway rolling stock, which include free vibrations, damp out in a short time. Proceeding from this, the definition of indicators of dynamic properties will be considered in the mode of steady forced vibrations.

It is known \[8, p. 230\] that disturbances causing forced vibrations in the rolling stock can be divided into:

- kinematic;
- force-based (due to application of external forces);
- parametric.

Kinematic disturbances arise due to geometric irregularities of the track arising in the profile and in the plane, irregularities on the rolling surface of the wheel. Force-based vibrations are excited by the traction moment, periodic forces from the imbalance of the rotating parts of diesel engines, electric motors, compressors, etc. Parametric disturbances are caused by changes in a parameter of the system, for example, in wear parameter.

When analysing vibrations of a vehicle, its natural vibrations can be represented in the form of a second-order differential equation with the missing right-hand side

\[
Mq + Bq + Cq = 0. \tag{1}
\]

Determination of the parameters of the vibration process of the vehicle (natural frequency and amplitude) will make it possible to make a decision on the choice of a vibration damper, which reduces the vibration process itself to a minimum. The problem stated is solved for small deviations of the system from the equilibrium position. In the study of the vibration process obtained as a result of the analysis of the considered model, stability of motion becomes an important indicator. The running stability is reduced to the study of the zero solution of the system according to linearised equations of the perturbed motion of the system, which is a necessary but insufficient condition for ensuring stable operation of the system under consideration. Further analysis of the model, depending on the action of the external load in the form of forced vibrations, makes it possible to determine the indicators of the dynamic properties of the wagon.

**Results of the Practical Phase of the Study**

The study of the model was conducted using MathCad software package platform. The results obtained make it possible to evaluate influence of changes in the suspension parameters on vibrations of the moving parts of the vehicle. Variable parameters belong to vehicle speed, which makes it possible to determine critical speeds and maximum displacements in the suspension system corresponding to each other. Also, by changing suspension parameters, it is possible to outline ways to reduce negative phenomena. This approach to solving the problem is called direct modelling \[7, p. 72; 9, p. 42; 10, p. 54; 11 p. 92\].

The solution of differential equations of movement of the vehicle by numerical methods will make it possible to assess the main processes occurring in the system under various combinations of external influences. For this, nonlinear functions are introduced into the system that do not change the structure of the model. The mathematical model represented by dependence (1) can be sophisticated by introducing the right-hand side by introducing a functional characterising the vibration process, which leads to obtaining new indicators of dynamic movement of the rolling stock.

When conducting research on the dynamic mode (vibrations of the rail vehicle), we introduce the following assumptions:

- The car body and bogies are considered absolutely rigid bodies in comparison with stiffness of springs.
- The track is considered absolutely rigid due to high rigidity of its elements.
- Spring suspension is considered to be inertialess due to smallness of the masses involved in the vibration.
- Forces of internal inelastic resistance in the elements of the track and body and the resistance of the environment are not considered.
### Table 1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Designation and dimension</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross body weight, tare</td>
<td>$m$, kg</td>
<td>40000•17000</td>
</tr>
<tr>
<td>Bogie weight (sprung)</td>
<td>$m$, kg</td>
<td>7500</td>
</tr>
<tr>
<td>Moment of inertia of the body: gross tare</td>
<td>$J_{gy}$, kg•m$^2$</td>
<td>$1,32\times10^6$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$7,2\times10^5$</td>
</tr>
<tr>
<td>Moment of inertia of the bogie</td>
<td>$J_{gy}$, kg•m$^2$</td>
<td>1900</td>
</tr>
<tr>
<td>Body base</td>
<td>$2a_k$, m</td>
<td>18</td>
</tr>
<tr>
<td>Bogie base</td>
<td>$2a_t$, m</td>
<td>2,6</td>
</tr>
<tr>
<td>Vertical rigidity of the first stage of spring suspension</td>
<td>$c_1$, kN/m</td>
<td>120</td>
</tr>
<tr>
<td>Vertical damping of the first stage of suspension</td>
<td>$\beta_1$, kN•s/m</td>
<td>10</td>
</tr>
<tr>
<td>Vertical rigidity of one spring set of the second stage of spring suspension: tare</td>
<td>$c_2$, kN/m</td>
<td>310</td>
</tr>
<tr>
<td></td>
<td></td>
<td>625</td>
</tr>
<tr>
<td>Vertical damping of one spring set of the second stage of spring suspension</td>
<td>$\beta_2$, kN•s/m</td>
<td>80</td>
</tr>
<tr>
<td>Speed of motion</td>
<td>$V$, m/s</td>
<td>45</td>
</tr>
</tbody>
</table>

**Pic. 2. Design scheme of vehicle vibrations in the vertical longitudinal plane [5, p. 93].**
Coloured images in Pics. 3–5 are designated as follow: Displacements: $z_k$ (blue), $z_1$ (dotted line) and $z_2$ (red); angles of rotation: $\phi_k$ (blue), $\phi_1$ (dotted line) and $\phi_2$ (red).
Wheel sets are in constant contact with the rails.

Perturbations from the right and left rails are assumed to be the same (the last assumption allows us to consider the vibrations as flat).

A design scheme has been developed (Pic. 2) to obtain a mathematical model of vibrations in the vertical longitudinal plane for a rolling stock having an axial formula 2–2 with two-tier suspension when passing along irregularities in the track.

The initial data of the design scheme shown in Pic. 2 were based on the operational and technical characteristics of rolling stock shown in Table 1.

Considering the design scheme as a two-mass system, we write down the systems of differential equations of vibrations:

– of the body:

\[ m \ddot{z}_1 + \beta_1 (2z_2 - z_1 - z_2) + c_1 (z_2 - z_1 - z_2) = 0 \]

\[ J_1 \ddot{\phi}_1 + \beta_1 a_1 (a_2 \phi_1 - z_1 + z_2) + c_1 a_1 (2a_2 \phi_1 - z_1 + z_2) = 0; \]

– of the first bogie:

\[ m_1 \ddot{z}_1 - \beta_1 (z_1 - z_1 + a_1 \phi_1) - c_1 (z_2 - z_1 + a_1 \phi_1) + 2 \beta_1 \phi_1 + 2c_1 z_2 = \beta_1 (\eta_1 - \eta_2) + c_1 (\eta_1 - \eta_2); \]

\[ J_1 \ddot{\phi}_1 + 2 \beta_1 a_1 \phi_1 + 2c_1 a_1 \phi_1 = a_1 (\beta_1 (\eta_1 - \eta_2) + c_1 (\eta_1 - \eta_2)); \]

– of the second body:

\[ m_2 \ddot{z}_2 - \beta_1 (z_2 - z_2 - a_2 \phi_2) - c_1 (z_2 - z_2 - a_2 \phi_2) + 2 \beta_1 \phi_2 + 2c_1 z_2 = \beta_1 (\eta_1 - \eta_2) + c_1 (\eta_1 - \eta_2); \]

\[ J_2 \ddot{\phi}_2 + 2 \beta_1 a_2 \phi_2 + 2c_1 a_2 \phi_2 = a_1 (\beta_1 (\eta_1 - \eta_2) + c_1 (\eta_1 - \eta_2)). \]

The equations were solved in MathCad Prime 4.0 package using the Runge–Kutta numerical method (the computation step in MathCad software package is selected automatically). The study of the properties of the dynamic system was performed by changing the resistance parameter of dampers of the first stage of spring suspension in the range of values from 1000 to 500000 N•s/m, the values of the amplitude of vibrations of the system and the period being recorded.

A graphic interpretation of the computation results is shown in Pics. 3–5.

The analysis of the results has shown that the vibration period of the body and the bogies with any changes in the resistance parameter of the damper remains unchanged: for bogies it is of 0.57 s, and for the body it is of 0.50 s (Pic. 6).

At \( b_2 = 80000 \) N•s/m the parameters of resistance of axle box dampers \( b_1 \) are within the range from 80000 to 150000 N•s/m, so we can recommend them as rational parameters.

Hydraulic vibration dampers with the specified parameters used for the rolling stock help to reduce wear and damage to running gears, improve ride comfort and traffic safety, as well as to reduce repair and maintenance costs.

CONCLUSIONS

Safety of movement and operation of rolling stock is due to a set of dynamic quality indicators: vibration protection, traffic safety and smoothness.

It was found that it is rational to determine the indicator of the vertical dynamics of the first \( F_{D1} \) and the second \( F_{D2} \) spring suspension stages depending on the speed of vehicle movement by a linear dependence of the type \( y = ax + b \), as
evidenced by the values of variances that tend to unity for each function obtained. As the speed increases, the dynamic factor decreases. For example, at a speed of 200 km/h, it decreases by 1,37, which indicates the need to revise the parameters of hydraulic dampers for high-speed rolling stock.

The disturbing factor acting on the considered dynamic system is deemed to be presented by a model which is a periodically changing function, for example, a sinusoid, the argument of which changes with a period corresponding to the rail length (multi-support beam). It has been established that the main characteristic of hydraulic vibration dampers is the dependence of the change in the resistance force of the hydraulic damper on the piston movement speed.

The selected design schemes of the vibration process in the vertical longitudinal plane during vehicle movement allow estimating the basic dynamic properties of the system under the accepted assumptions that do not contradict the theory of elasticity and dynamics of solids and do not significantly affect the calculation results.

Analysis of the results of the study of the dynamic model has shown that the vibration period for the first and second wheel sets remains unchanged for any changes in the viscosity parameter of the hydraulic dampers:
• For the first wheel set it is equal to 0,57 s, and for the second it is equal to 0,50 s.
• The vibration amplitude of the first wheel set is minimal at \( b_1 = 50000 \) and \( b_2 = 80000 \text{ N} \cdot \text{m/s} \) (with a further changes in ratio, this value begins to increase).

The use of hydraulic vibration dampers with the characteristics identified during the study for the rolling stock, besides satisfying the requirements for comfort, improvement of the running smoothness and traffic safety when transporting passengers and goods, helps to reduce wear and damage to the running gears of cars and, as a result, to diminish their repair and maintenance costs.

REFERENCES


