# CALCULATION OF TRACTION CURRENT HARMONICS' INFLUENCE ON TRACK CIRCUIT

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## ABSTRACT

Track circuits, as well as other elements of railway automation, are in the zone of influence of electric rolling stock and, in particular, of traction current. In terms of electromagnetic compatibility this is an area of risk that requires specific tools of study and control. Mathematical description has been developed to calculate the influence of harmonics of traction current, flowing in the contact wire of adjacent track of double-track sections, and of its harmonic components, on track circuits. Equivalent circuit and system of differential equations are shown, which are designed to solve problems to ensure operational reliability of a railway line.

_Keywords: railway, electromagnetic compatibility, mathematical description, track circuit, traction current harmonics, contact wire, adjacent track._

**Background.** *The problem of electromagnetic compatibility of electric rolling stock and railway automation devices including track circuits is quite acute in Russia and abroad [1–7].*

*When calculating the influence of traction current harmonics on operation of the track circuit, it is usually represented as a single-wire power interconnected lines with resistance track lines $z_1$ and $z_2$. These lines are subject to induction influence of traction currents flowing in the contact wire both of considered and adjacent tracks [8].*

*The value of traction current in rails at any point of the section between traction substation and electric locomotive is the sum of two components, the first of which does not depend on coordinates of the controlled point and is the induced current. The second term is determined by the coordinate point and decreases as the distance from the electric locomotive grows [9].*

*Given the inductive influence of traction currents, flowing in the contact wire of considered and adjacent track, the amount of current in rails and voltage to ground at any point between the electric locomotive and the traction substation can be found via solution of differential equations according to [10, 11]. In drawing up the equivalent circuit for differential equations the impact of high-voltage power line is further considered [12–14].*

*In operation conditions train situation in the sections between traction substations is determined by the schedule of movement of trains, by regulations on equipment maintenance by personnel, and by those technological «windows» (time intervals) when the motion of electric rolling stock on one track is not performed. In such a situation disturbing influence of traction current, flowing in the contact wire of an adjacent track, may appear which is expressed as a false occupation of track circuit when there is no train.*

**Objective.** *The objective of the authors is to consider a method of calculation of traction current harmonics' influence on track circuit.*

**Methods.** *The authors use general scientific methods, engineering and algebraic calculations, tools of electrical engineering.*

**Results.** *Hence there is a need to assess the degree of general influence of traction current in the contact wire of an adjacent track without taking into account processes in the contact wire of a considered track with no trains on it (as shown in Pic. 1).*

*Designations demonstrated in the circuit:*

*$z_1$, $z_2$ are specific resistances of single rails;*

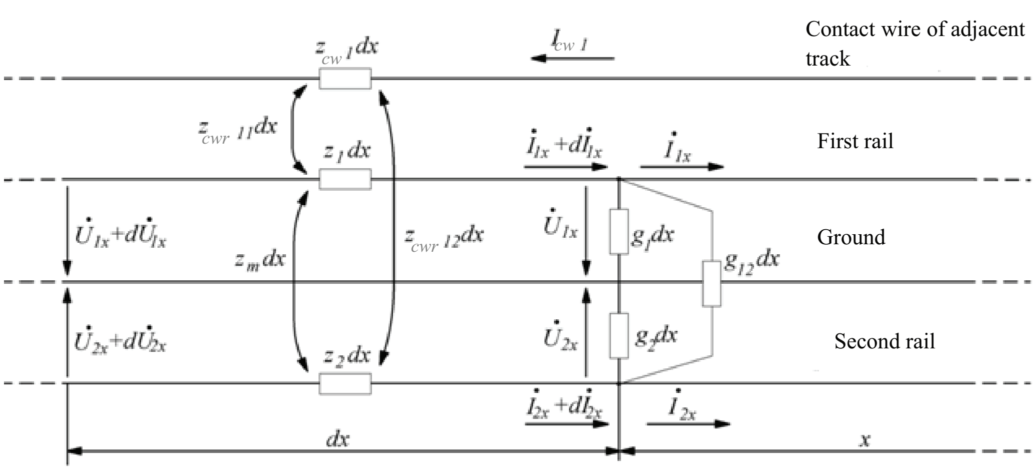
*$z_m$ is resistance of mutual inductance of rails;*

*$z_{cwr11}$, $z_{cwr12}$ are resistances of mutual inductance of contact wire and each rail;*

*$z_{cw1}$ is specific resistance of contact wire;*

*$g_1$, $g_2$ are specific earthing conductivities of rails;*

*$g_{12}$ is specific conductivity of the upper layer of ballast and sleepers;*




***Pic. 1. Equivalent circuit of element dx of a rail line, taking into account the influence of traction current in the contact wire of an adjacent track.***


*x is a distance from the end of the rail line, where the load is connected;*

$\dot{I}_{1x},\ \dot{I}_{2x}$ *are respectively currents in the first and second rail lines with the positive direction from the supply end to the load;*

$\dot{U}_{1x},\ \dot{U}_{2x}$ *are voltages of first and second rail lines with respect to ground with positive direction from rails to the ground;*

$\dot{I}_{cw1}$ *is current flowing in the contact wire of an adjacent track with positive direction from traction substation.*

*Differential equations for voltages and currents of the rail line have a form:*

$$\frac{d\dot{U}_{1x}}{dx} = z_1\dot{I}_{1x} + z_m\dot{I}_{2x} + z_{cwr11}\dot{I}_{cw1}; \qquad (1)$$

$$\frac{d\dot{I}_{1x}}{dx} = (g_1 + g_{12})\dot{U}_{1x} - g_{12}\dot{U}_{2x}; \qquad (2)$$

$$\frac{d\dot{U}_{2x}}{dx} = z_2\dot{I}_{2x} + z_m\dot{I}_{1x} + z_{cwr12}\dot{I}_{cw1}; \qquad (3)$$

$$\frac{d\dot{I}_{2x}}{dx} = (g_2 + g_{12})\dot{U}_{2x} - g_{12}\dot{U}_{1x}. \qquad (4)$$

*Conductivity "contact wire of an adjacent track–ground" is taken equal to zero, i.e.,* $\frac{d\dot{I}_{cw1}}{dx} = 0$.

*As a result of solving a system of differential equations (1)–(4) we get:*

$$\dot{U}_{1x} = P(A_1 sh\gamma_1 x + A_2 ch\gamma_1 x) + Q(A_3 sh\gamma_2 x + A_4 ch\gamma_2 x); \qquad (5)$$

$$\dot{U}_{2x} = \frac{M\gamma_1 + Pg_{12}}{g_2 + g_{12}}(A_1 sh\gamma_1 x + A_2 ch\gamma_1 x) + \frac{N\gamma_2 + Qg_{12}}{g_2 + g_{12}}(A_3 sh\gamma_2 x + A_4 ch\gamma_2 x); \qquad (6)$$

$$\dot{I}_{1x} = A_1 ch\gamma_1 x + A_2 sh\gamma_1 x + A_3 ch\gamma_2 x + A_4 sh\gamma_2 x + \frac{I_{cw1}(z_2 z_{cwr11} - z_m z_{cwr12})}{z_m^2 - z_1 z_2}; \qquad (7)$$

$$\dot{I}_{2x} = M(A_1 ch\gamma_1 x + A_2 sh\gamma_1 x) + N(A_3 ch\gamma_2 x + A_4 sh\gamma_2 x) - \frac{I_{cw1}(z_m z_{cwr11} - z_1 z_{cwr12})}{z_m^2 - z_1 z_2}, \qquad (8)$$

*where*

$$P = \gamma_1\frac{g_2 + g_{12}(1+M)}{g_1 g_2 + g_1 g_{12} + g_2 g_{12}};\quad Q = \gamma_2\frac{g_2 + g_{12}(1+N)}{g_1 g_2 + g_1 g_{12} + g_2 g_{12}};$$

$$M = \frac{\gamma_1^2 - (z_1 g_1 + z_1 g_{12} - z_m g_{12})}{z_m g_1 + z_m g_{12} - z_2 g_{12}};$$

$$N = \frac{\gamma_2^2 - (z_1 g_1 + z_1 g_{12} - z_m g_{12})}{z_m g_1 + z_m g_{12} - z_2 g_{12}};$$

$\gamma_1$ *is coefficient of propagation of earth wave path in two-wire rail line:*

$$\gamma_1 = \sqrt{\frac{1}{2}a + \sqrt{\frac{1}{4}a^2 - b}}; \qquad (9)$$

$\gamma_2$ *is coefficient of propagation of phase wave path in two-wire rail line:*

$$\gamma_2 = \sqrt{\frac{1}{2}a - \sqrt{\frac{1}{4}a^2 - b}}; \qquad (10)$$

*where* $a = z_1(g_1+g_{12}) + z_2(g_1+g_{12}) - 2z_m g_{12}$;
$b = (z_m^2 - z_1 z_2)(g_1 g_2 + g_1 g_{12} + g_2 g_{12})$.

*These expressions are used for mathematical description of track circuit operation, taking into account the influence of traction current flowing in the contact wire of the adjacent track of double-track sections, and its harmonic components.*

*For rail lines with parameters* $z_1 = z_2 = z_p$ *solution of differential equations (1)–(4) will look like:*

$$\dot{U}_{1x} = P(A_1 sh\gamma_1 x + A_2 ch\gamma_1 x) + Q(A_3 sh\gamma_2 x + A_4 ch\gamma_2 x); \qquad (11)$$

$$\dot{U}_{2x} = \frac{M\gamma_1 + Pg_{12}}{g_2 + g_{12}}(A_1 sh\gamma_1 x + A_2 ch\gamma_1 x) + \frac{N\gamma_2 + Qg_{12}}{g_2 + g_{12}}(A_3 sh\gamma_2 x + A_4 ch\gamma_2 x); \qquad (12)$$

$$\dot{I}_{1x} = A_1 ch\gamma_1 x + A_2 sh\gamma_1 x + A_3 ch\gamma_2 x + A_4 sh\gamma_2 x + \frac{I_{cw1}(z_p z_{cwr11} - z_m z_{cwr12})}{z_m^2 - z_p^2}; \qquad (13)$$

$$\dot{I}_{2x} = M(A_1 ch\gamma_1 x + A_2 sh\gamma_1 x) + N(A_3 ch\gamma_2 x + A_4 sh\gamma_2 x) - \frac{I_{cw1}(z_m z_{cwr11} - z_p z_{cwr12})}{z_m^2 - z_p^2}. \qquad (14)$$

*In case of good insulation of rail lines conductivity of one rail relative to ground $g_1$ and conductivity between rails $g_{12}$ are equal to zero. The conductivity of another rail line $g_{on}$ is determined by conductivity of contact network supports. This situation is typical for the winter season. Due to the fact that contact network supports must be connected to rail line at equal distances from each other, the conductivity of these supports can be regarded as distributed, i.e. $g_2 = g_{on}$.*

*Given $z_1 = z_2 = z_p$ equations (1)–(4) will take a form:*

$$\frac{d\dot{U}_{1x}}{dx} = z_p \dot{I}_{1x} + z_m \dot{I}_{2x} + z_{cwr11} \dot{I}_{cw1}\ ; \qquad (15)$$

$$\frac{d\dot{I}_{1x}}{dx} = 0\ ; \qquad (16)$$

$$\frac{d\dot{U}_{2x}}{dx} = z_p \dot{I}_{2x} + z_m \dot{I}_{1x} + z_{cwr12} \dot{I}_{cw1}\ ; \qquad (17)$$

$$\frac{d\dot{I}_{2x}}{dx} = g_{on} \dot{U}_{2x}\ . \qquad (18)$$

*Solution of equations (15)–(18) is:*

$$\dot{U}_{1x} = x\left[A_1\left(z_m - \frac{z_p^2}{z_m}\right) - I_{cw1}\left(\frac{z_p z_{cwr12}}{z_m} - z_{cwr11}\right)\right] + \frac{z_m}{\gamma_{on}}\left(A_2 sh\gamma_{on} x + A_3 ch\gamma_{on} x\right) + A_4; \qquad (19)$$

$$\dot{U}_{2x} = \frac{\gamma_{on}}{g_{on}}(A_2 sh\gamma_{on} x + A_3 ch\gamma_{on} x)\ ; \qquad (20)$$

$$\dot{I}_{1x} = \frac{-z_p A_1 - I_{cw1} z_{cwr12}}{z_m}\ ; \qquad (21)$$

$$\dot{I}_{2x} = A_1 + A_2 ch\gamma_{on} x + A_3 sh\gamma_{on} x\ , \qquad (22)$$

*where $\gamma_{on}$ is coefficient of propagation of wave of two-wire rail line with good insulation of rails, $\gamma_{оп} = \sqrt{g_{оп} z_p}$.*

**Conclusion.** *From equations (16), (21) it is obvious that if $g_1 = g_{12} = 0$, then the current flowing in the first rail is constant. This result fully meets the given conditions of the task. It confirms the validity of the proposed mathematical apparatus for calculation of traction current harmonics influence on track circuits.*

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