MODELS OF INTERACTION OF WHEEL AND RAIL AT HIGH SPEED

Loktev, Alexei A., *Moscow State University of Railway Engineering (MIIT), Moscow, Russia.*
Vinogradov, Valentin V., *Moscow State University of Railway Engineering (MIIT), Moscow, Russia.*
Buchkin, Vitaly A., *Moscow State University of Railway Engineering (MIIT), Moscow, Russia.*

ABSTRACT

The authors evaluate the process of dynamic contact «wheel-rail» through analysis of several simulation models of interaction, allowing to take into account different rheological properties of contacting bodies. The research apparatus comprises equations describing the motion of wheel and rail in the vertical direction, which are then solved using initial conditions and numerical iterative scheme, assuming that at small time intervals individual values behave linearly. The proposed algorithm allows to determine that at medium and high speeds of vehicle movement the elastic-plastic dependence of Alexandrov-Kadomtsev is most appropriate to describe the nature of interaction between the wheel and the the rail, as well as to find individual dynamic values.

Keywords: railway, wheel, rail, dynamic effects, viscoelastic contact force, elastic-plastic interaction, mechanical characteristics, interaction patterns, dynamic bending, local buckling.

Background. *During the construction of high-speed rail lines it is necessary to take into account changes in the classical model of contact of wheel set and rail. On the one hand, high speed does not allow plastic deformation in the vicinity of wheel and rail contact area [1–4] to fully develop, on the other hand, grown speed increases dynamic component of final stresses and efforts [5–7], and the ratio of classical Hertz theory [8, 9] for contact between two bodies does not provide a sufficient approximation of the results of experiments [10], either does not allow to describe all the processes occurring in the wheel and the rail.*

Requirements of engineering practice in the transport sector force researchers to improve rheological models of contacting bodies, to assess in more detail the nature of their interaction and a wide range of effects accompanying them [11, 12].

Objective. *The objective of the authors is to consider models of wheel and rail interaction at high speeds.*

Methods. *The authors use general scientific methods, comparative analysis, graph construction, mathematical apparatus.*

Results. *The study of the process of dynamic contact loading «wheel-rail» (Pic. 1) offers to consider several contact models of interaction and different rheological properties of interacting bodies:*

1. Quasi-static model of Hertz [2, 7]:

$$P = ka^{3/2}. \tag{1}$$

2. Linear elastic model [7, 8]:

$$P = E_1(\alpha - w). \tag{2}$$

3. Viscoelastic model [9]:

$$P(t) = E_1(\alpha - w) - \frac{E_1}{\tau_1}\int_0^t (\dot{\alpha} - \dot{w}) e^{-\frac{t-t'}{\tau_1}} dt'. \tag{3}$$

4. Elastoplastic model of Kilchevsky [6]:

$$\alpha = \begin{cases} bP^{2/3}, & dP/dt > 0,\ P < P_b, \\ bP^{2/3} + Pd, & dP/dt > 0,\ P > P_b, \\ bP^{2/3} + P_{\max}d, & dP/dt < 0,\ P_{\max} > P_b. \end{cases} \tag{4}$$

5. Elastoplastic model of Alexandrov-Kadomtsev [6]:

$$\alpha = \begin{cases} bP^{2/3}, & dP/dt > 0,\ P_{\max} < P_1, \\ (1+\beta)c_1 + (1-\beta)Pd, & dP/dt > 0,\ P_{\max} > P_1, \\ b_f P^{2/3} + \alpha_p(P_{\max}), & dP/dt < 0,\ P_{\max} > P_1. \end{cases} \tag{5}$$

6. A model in which the interaction force is divided into stages of loading and unloading [11]. For loading stage one can use (1), for unloading stage:

$$P = P_m\left[\frac{(\alpha - \alpha_0)}{(\alpha_m - \alpha_0)}\right]^q,\ \alpha_0 = \begin{cases} \beta(\alpha_m - \alpha_{cr}); & \alpha_m > \alpha_{cr}, \\ 0\ ; & \alpha_m \le \alpha_{cr}. \end{cases} \tag{6}$$

7. Viscoelastic model with fractional derivatives of Riemann-Liouville [12]:

$$P + \tau^\gamma D^\gamma P = E_1 \tau^\gamma D^\gamma(\alpha - w),$$

$$D^\gamma P = \frac{d}{dt}\int_0^t \frac{P(t-t')}{\Gamma(1-\gamma)t'^\gamma}dt'. \tag{7}$$

In the expressions (1)-(7) α is a local buckling of materials of the rail and the wheel; k – coefficient depending on the geometry of contacting bodies and elastic characteristics of materials; E_1 – modulus of elasticity of the interaction region of the wheel set and the rail; w – movement of the lower edge of the rail; $\lambda = 5.7$,

$$b = \left(\left(9\pi^2(k_1+k)^2\right)/16R\right)^{1/3},\ k_1 = \left(1-\sigma_1^2\right)/E_1,$$

$$k = \left(1-\sigma^2\right)/E,\ \chi = \pi k_{pl}\lambda,\ P_1 = \chi^3\left(3R(k_1+k)/4\right)^2,$$

$$d = 1/2\chi R,\ b_f = R_f^{-1/3}\left(3(k_1+k)/4\right)^{2/3},\ R_p^{-1} = R^{-1} - R_f^{-1},$$

$$R_f = \left(4/3(k_1+k)\right)P_{\max}^{1/2}\chi^{-3/2},$$

$$\alpha_p(P_{\max}) = (1-\beta)P_{\max}\left(2\chi R_p\right)^{-1},\ \beta = 0.33,$$

$$c_1 = 3\chi^{1/2}(k_1+k)/8,\ k_{pl}$$

is the smallest of plastic constants of interacting bodies; σ_1, E_1 – Poisson's ratio and the modulus of elasticity for the wheelset [2, 8]; $\tau_1 = \eta_1/E_1$, τ_1 – relaxation time in the case of a viscoelastic model; η_1 – viscous resistance coefficient; γ ($0 < \gamma \le 1$) – fractionality parameter; P_m – maximum contact force before unloading stage; α_m – maximum buckling of the material in the interaction zone; α_0 – current value of buckling; q and β – experimentally determined constants; α_{cr} – critical buckling.

In the simulation of the dynamic interaction between the wheel and the rail, taking into account homogeneous and isotropic materials of two bodies and smoothness of contacting surfaces equations of motion of the wheel and the rail after the start of the interaction takes a form

$$m\ddot{y} = -P(t),\ EI\frac{\partial^4 w}{\partial x^4} + \rho F \ddot{w} = P(t)\delta(x - \xi), \tag{8}$$

where m is the mass of the wheel; y – displacement of the wheel; w(x, t) – deflection of the rail; E – Young's *modulus; I – moment of inertia of the rail section with respect to the midline; F – cross-sectional area of the rail; ρ – density of its material; δ (x-ξ) – delta of Dirac function; x – coordinate measured along the axis of the rail, the dot above the value means a partial derivative with respect to time.*

Equations (8) and (9) are integrated with account of initial conditions

$$\dot{w}\big|_{t=0}=0, \quad \dot{\alpha}\big|_{t=0}=V_0 \qquad (9)$$

and of dependence between interaction force and local bearing (1)-(6).

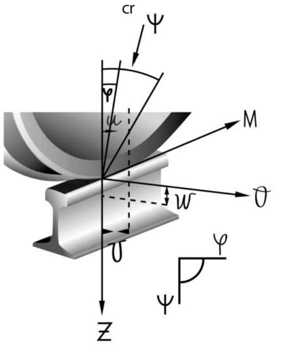
After integration of the defining relation we obtain a functional equation for determining the contact force:

$$V_0 t-\frac{1}{m}\int_0^t\int_0^{t_1}P(t_2)\,dt_2\,dt_1=k'P^{2/3}+$$
$$+\sum_n A_n\int_o^t P(\tau)\sin\omega_n(t-\tau)\,d\tau, \qquad (10)$$

where coefficients A_n depend on a type of eigenfunctions and eigenvalues in the task.

Solution of integral-differential equation of the type (10) for relationships (1) – (7) is found numerically by computer, based on the assumption that at each sufficiently small interval $(n-1)\tau \le t \le n\tau$ unknown quantities vary linearly $\dot{P}(n\tau)=(P_n-P_{n-1})/\tau$. Such computing iterative scheme is most often used in the presentation of unknown quantities in the form of expansions in series of special functions [6–10]. This is connected to the fact that the final determining equation contains the summation based on two indices.

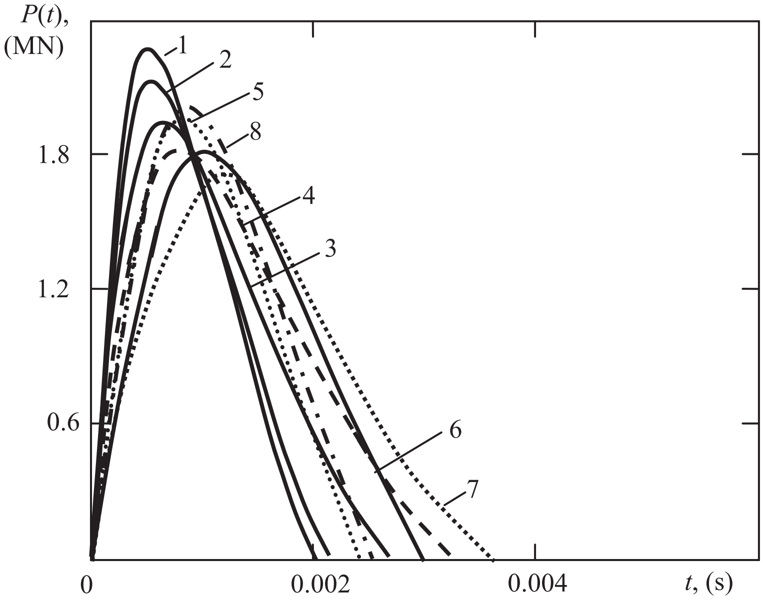
Pic. 2, 3 show dependences of the contact force on time. In Pic. 2 curves 1, 2, 3, 4, 5, 6, 7 are constructed using the relations (1), (2), (3), (4), (5), (6) and (7) respectively, and the curve 8 is taken from [2], the speed of the vehicle V_0 = 80 km / h. In Pic. 3 curves 1,2 correspond to V_0 = 200 km / h, curves 3 and 4 – V_0 = 150 km / h, curves 5 and 6 – V_0 = 100 km / h, curves 1, 3 and 5 were calculated using the equations (5), and curves 2, 4, 6 – by means of relations (3).



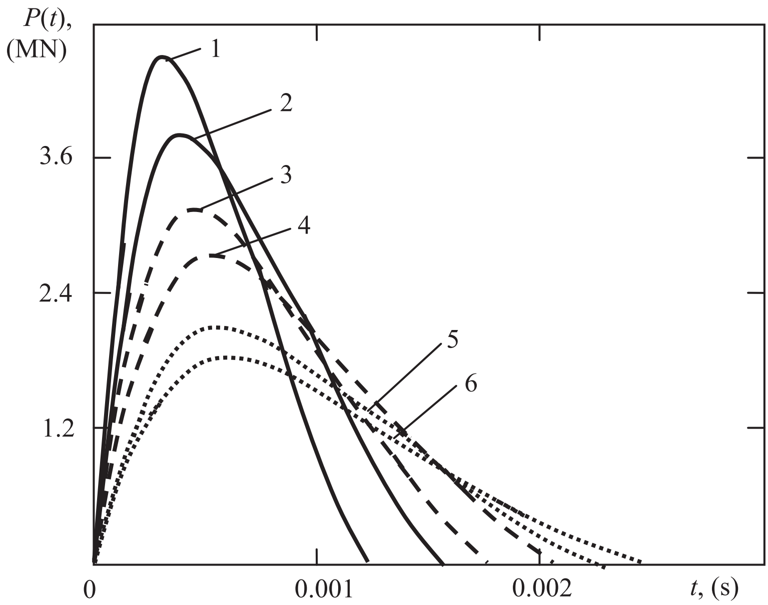
Pic. 1. The model of contact interaction of wheel and rail.

Pic. 2 shows that models (3) and (5) give the best approach to the experimental results, and that at speeds of ≈80 km / h rheological properties of the wheel and the rail have a significant influence on the maximum value of the contact force and the time of interaction. Pic. 3 compares the results of the use of visco-elastic (3) and elastic-plastic dependences (5). With an increase in the initial speed of interaction of wheel and rail (vehicle speed) the difference between values of maximum contact forces obtained using relations (3) and (5), and the interaction time are growing.

Conclusion. *In course of the study graph dependences of contact force on time for various models of contact interaction of wheel and rail were produced, which showed that even at vehicle speeds of 80 km / h it is necessary to take into account viscoelastic and elastic-plastic properties of materials. Modification of existing relations for contact force and local buckling allowed to make the obtained graph dependences closer to the results of the experiment. The proposed algorithm makes it possible to select a mathematical model of contact*



Pic. 2. Dependence of the contact force on time for different models of contact interaction of wheel-rail.



Pic. 3. Dependence of contact force on time for different speeds of vehicle movement.

that most accurately describes the processes in the interacting bodies, and to get values of the maximum force of interadction and contact time, close to the experimental data.

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Information about the authors:
Loktev, Alexei A. – D.Sc. (Physics and Mathematics), professor, head of department, dean of the faculty of the Russian Open Transport Academy of Moscow State University of Railway Engineering (MIIT), Moscow, Russia, aaloktev@yandex.ru.
Vinogradov, Valentin V. – D.Sc. (Eng.), professor, first vice-rector – vice-rector for education of Moscow State University of Railway Engineering (MIIT), Moscow, Russia, +7(495) 684–21–10.
Buchkin, Vitaly A. – D.Sc. (Eng.), professor, head of the department of designing and construction of railways of Moscow State University of Railway Engineering (MIIT), Moscow, Russia, vp@vpm770.ru.

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