DETERMINING PARAMETERS OF DYNAMIC IMPACT IN THE PLATE OF BALLASTLESS TRACK

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ABSTRACT

A mathematical model of wave surface propagation in the flat element, which is a carrier plate of ballastless track under dynamic action of wheel set, is demonstrated. An algorithm of accounting different rheological properties of interacting bodies, based on the analytical method of representation of unknown quantities in the form of expansions in terms of spatial coordinates and time, initial and boundary conditions, is offered. The method of constructing a wave pattern in the plate is based on the principle of superposition of two separate tasks: contact problem of initial dynamic load application and wave plate deformation problem over time, including through the dissemination of elastic waves with finite speeds. In the article the wave problem is solved in an analytical form for displacement functions.

Keywords: *dynamic impact, plate Uflyand-Mindlin-Reissner, elastic waves, orthotropic properties, spherical functions, ballastless railway track.*

Background. *Investigation of the behavior of structures of ballastless track under dynamic load stimulates the search for mathematical models that can maximally facilitate an understanding of the phenomena. In the study of wave processes in bodies such as the carrier plate, it is proposed to use models of an ideal elastic medium [1, 2]. If it is taken into account [3, 4] that the objects have all the properties of such a medium, the differential equations of wave propagation in bodies with density* ρ *can be obtained from the general equations of equilibrium of a rectangular prism with edges dx, dy, dz, selected near an arbitrary point, equating to zero forces and inertial terms [5, 6].*

Objective. *The objective of the authors is to demonstrate a way to determine parameters of dynamic impact in the plate of ballastless track.*

Methods. *The authors use general scientific and engineering methods, mathematical apparatus, analysis.*

Results. *Differential wave equations for displacement in elastic media will generally take a form:*

$$(\chi+G)\frac{\partial\Delta}{\partial x}+G\nabla^2u=\rho\frac{\partial^2u}{\partial t^2},$$

$$(\chi+G)\frac{\partial\Delta}{\partial y}+G\nabla^2v=\rho\frac{\partial^2v}{\partial t^2},$$

$$(\chi+G)\frac{\partial\Delta}{\partial z}+G\nabla^2w=\rho\frac{\partial^2w}{\partial t^2}. \qquad (1)$$

Here: Δ *is volume deformation;* ∇^2 *– second order Laplace operator;* $\chi=\nu E/(1+\nu)(1-2\nu)$; *E and G are elasticity and shear modulus, respectively;* ν *– Poisson's ratio; u, v, w – displacement of elements of an elastic medium along the coordinate axes 0x, 0y, 0z.*

The dynamic behavior of the plate in this case is described by wave equations such as Uflyand–Mindlin–Reissner equation having a hyperbolic structure and taking into account shear deformations and rotational inertia of cross sections [7–10]. Finding dynamic and kinematic parameters of deformation of bearing reinforced plate of ballastless track assumes the solutions of a defining system of equations, written in dimensionless form using substitutions:

$$\tau=t\sqrt{c_1}/h,\ c_1=E_r/(1-\sigma_r\sigma_\theta)\rho,$$
$$w=w/h,\ u=u/h,\ v=v/h,\ r=r/h, \qquad (2)$$

where w(r, θ), u(r, θ) and v(r, θ) are displacements along the normal and tangents in directions r, θ respectively; φ(r, θ) and ψ(r, θ) are functions of rotation angles in two tangential directions; E_r, E_θ *and* σ_r, σ_θ *– Young module and Poisson's ratio for directions r, θ; h – thickness of reinforced concrete plate; t – time passed since the start of dynamic impact of a vehicle.*

When using the substitutions (2) the system of equations that define the dynamic behavior of the carrier plate, can be written as

$$\frac{\partial^2\varphi}{\partial r^2}+\frac{1}{r}\frac{\partial\varphi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\varphi}{\partial\theta^2}-\frac{1}{r^2}\frac{c_2}{c_1}\varphi+\frac{c_2\sigma_r+c_3}{c_1r}\frac{\partial^2\psi}{\partial r\partial\theta}-$$
$$-\frac{c_2+c_3}{c_1r^2}\frac{\partial\varphi}{\partial\theta}+\frac{12c_4}{c_1}\left(\frac{\partial w}{\partial r}-\varphi\right)=-\frac{\partial^2\varphi}{\partial\tau^2},$$

$$\frac{c_4}{c_1}\left(\frac{\partial^2w}{\partial r^2}-\frac{\partial\varphi}{\partial r}\right)+\frac{c_4}{c_1}\left(\frac{\partial w}{r\partial r}-\frac{\varphi}{r}\right)+$$
$$+\frac{c_4}{c_1}\left(\frac{\partial^2w}{r^2\partial\theta^2}-\frac{\partial\psi}{r\partial\theta}\right)=\frac{\partial^2w}{\partial\tau^2}+q_1, \qquad (3)$$

$$\left(\frac{\partial^2u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right)+\frac{c_3}{c_1r^2}\frac{\partial^2u}{\partial\theta^2}-\frac{c_2}{c_1}\frac{u}{r^2}+$$
$$+\frac{c_2\sigma_r+c_3}{c_1r}\frac{\partial^2v}{\partial r\partial\theta}-\frac{c_2+c_3}{c_1r^2}\frac{\partial v}{\partial\theta}=\frac{\partial^2u}{\partial\tau^2},$$

$$\frac{c_2}{c_1r^2}\frac{\partial^2v}{\partial\theta^2}+\frac{c_3}{c_1}\left(\frac{\partial^2v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}-\frac{v}{r^2}\right)+$$
$$+\frac{\sigma_\theta+c_3}{c_1r}\frac{\partial^2u}{\partial r\partial\theta}+\frac{c_2+c_3}{c_1r^2}\frac{\partial u}{\partial\theta}=\frac{\partial^2v}{\partial\tau^2},$$

$$\frac{c_3}{c_1}\left(\frac{\partial^2\psi}{\partial r^2}+\frac{1}{r}\frac{\partial\psi}{\partial r}-\frac{\psi}{r^2}\right)+\frac{c_2}{c_1r^2}\frac{\partial^2\psi}{\partial\theta^2}+\frac{\sigma_\theta+c_3}{c_1r}\frac{\partial^2\varphi}{\partial r\partial\theta}+$$
$$+\frac{c_2+c_3}{c_1r^2}\frac{\partial\varphi}{\partial\theta}+\frac{12c_5}{c_1}\left(\frac{1}{r}\frac{\partial w}{\partial\theta}-\psi\right)=-\frac{\partial^2\psi}{\partial\tau^2},$$

where

$$c\ =E_\theta/(1-\sigma\ \sigma_\theta)\rho,\quad c_3=G_{r\theta}/\rho,$$

$$c_4=KG_{rz}/\rho,\ c_5=KG_{\theta z}/\rho,\quad q_1=qh/\rho c_1\ ;$$

ρ *– density of plate material and θ – external load* *(here from dynamic impact);* $E_r\sigma_r = E_\theta\sigma_\theta$, $K = 5/6$,

D_r, D_θ, C_r, C_θ –stiffness of bending and tension-compression for directions r, θ; G_{rz}, $G_{\theta z}$ – shear modulus in planes indicated in lowercase index.

Parameters c_1, c_2, c_3, c_4, c_5, included in the equation (3) represent the speed squares of edges of elastic waves, the indices of the values indicate the type of wave surface: 1, 2 determine quasi-longitudinal wave surface s of tension-compression, extending along the directions r and θ (up and down the rails respectively), the index of 3 – a quasi-transverse wave shear surface in the plane rθ, indices 4, 5 – quasi-transverse wave shear surfaces in directions normal to the planes rz, θz. The wave surface speeds are directly dependent on mechanical anisotropic properties of plate material in the corresponding directions. We may assume that the set of values of these speeds determines the elastic properties of the bearing concrete plate [9, 11].

The system of equations (3) may be solved in Laplace space. All types of available in (3) linear and angular displacements, as well as an external load q(t, r, θ) (function associated with the contact force P(t)) should be written in the form of expansions in power series [10, 11] on spherical functions using Legendre polynomials [5]. This representation of unknown values allows to link the unknown functions and their derivatives of various orders up to the unknown coefficients, which will not increase the number of unknown functions after differentiation

$$\bar{x} = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} x_{2n+m} P_{2n+1}\left(\cos\frac{\pi r}{2R}\right)\cos(m\theta), \qquad (4)$$

$$\bar{q}_1 = \frac{P(p)}{\pi R_c^2}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}(4n+3)P_{2n+1}\left(\cos\frac{\pi r_1}{2R}\right)P_{2n+1}\left(\cos\frac{\pi r}{2R}\right)\cos(m\theta). \qquad (5)$$

In these expressions R is characteristic size of carrier plate of the base; r_1 – coordinate of place of dynamic effects of wheel on the upper structure of railway track; x – value that can be set to basic movements ϕ, ψ, w, u, v. These values are in the image space, which is confirmed by the bar above. P(p) – dependence of interaction forces on the parameter of space images.

To determine the coefficients of expansions (4), (5) we use their representation in the form of Laurent series near the considered point of the plate [6, 7]:

$$x_{2n+m} = x_{2n+m}^0\varepsilon^0 + x_{2n+m}^1\varepsilon^1 + x_{2n+m}^2\varepsilon^2 + x_{2n+m}^3\varepsilon^3, \qquad (6)$$

where $\varepsilon = p^{-2}$, p – parameter of Laplace space.

Substituting the series (4), (5) with account of (6) in the system (3) and equating the coefficients with the same degrees ε, we obtain systems of linear algebraic equations, from which we find the coefficients of correlations (6):

$$\psi_{2n+m}^3 = P(p)P_s,$$

$$\varphi_{2n+m}^3 = P(p)\left(\frac{c_1}{c_4}\frac{R_1\cos\alpha_1}{A_1^3} - P_s\right),$$

$$\psi_{2n+m}^2 = P(p)P_s\left(1 + D_1\cdot\frac{B_1^3}{A_1^3} - D_2\right) - \frac{P(p)R_1\cos\alpha_1}{A_1^3}\frac{c_1}{c_4}D_1,$$

$$\varphi_{2n+m}^2 = P(p)\frac{B_1^3}{A_1^3}P_s\left[-2 - \frac{D_3}{A_1^3} + D_2\right] +$$

$$\frac{P(p)R_1\cos\alpha_1}{A_1^3}\frac{c_1}{c_4}\left(1 + \frac{D_3}{A_1^3}\right), \qquad (7)$$

$$\psi_{2n+m}^i = \left[P(p)\frac{c_1}{c_4}\left(A_1^3 - A_2^3R_1\cos\alpha_1\right) - \varphi_{2n+m}^{i+1}\left(A_1^3A_2^2 - A_2^3A_1^2\right) - \psi_{2n+m}^{i+1}\left(A_1^3B_2^2 - A_2^3B_1^2\right) - \psi_{2n+m}^{i+2}B_2^1\right]\Big/\left(B_2^3A_1^3 - B_1^3A_2^3\right),$$

$$\varphi_{2n+m}^i = \frac{P(p)R_1\cos\alpha_1}{A_1^3}\frac{c_1}{c_4} - \frac{B_1^3}{A_1^3\left(B_2^3A_1^3 - B_1^3A_2^3\right)}\left[\frac{c_1}{c_4}\frac{P(p)}{A_1^3}\left(A_1^3 - A_2^3R_1\cos\alpha_1\right) - \varphi_{2n+m}^{i+1}\left(A_1^3A_2^2 - A_2^3A_1^2 + \frac{A_1^2}{A_1^3}\right) - \psi_{2n+m}^{i+1}\left(A_1^3B_2^2 - A_2^3B_1^2 + \frac{B_1^2}{A_1^3}\right) - \psi_{2n+m}^{i+2}A_1^3B_2^1\right],$$

$$w_{2n+m}^3 = -C_1^3\psi_{2n+m}^3,$$

$$w_{2n+m}^i = C_1^2\psi_{2n+m}^{i+1} - C_1^3\psi_{2n+m}^i,$$

$$v_{2n+m}^3 = \frac{\cos\alpha_1 P(p)S_p\left(T_2^3\cos\alpha_2 - T_1^3\sin\alpha_2\right)}{T_1^3S_2^3 - S_1^3T_2^3},$$

$$u_{2n+m}^3 = \cos\alpha_1\sin\alpha_2\frac{P(p)S_p}{S_2^3} - \frac{v_{2n+m}^3T_2^3}{S_2^3},$$

$$u_{2n+m}^i = \frac{u_{2n+m}^{i+1}\cos(m\theta) + \cos\alpha_1\cos\alpha_2P(p)S_p}{S_2^3} - \frac{v_{2n+m}^iT_2^3}{S_2^3},$$

$$v_{2n+m}^i = \frac{\left(u_{2n+m}^{i+1}\cos(m\theta) + \cos\alpha_1\cos\alpha_2P(p)S_p\right)S_2^3 - \left(v_{2n+m}^{i+1}\cos(m\theta) + \cos\alpha_1\sin\alpha_2P(p)S_p\right)S_1^3}{T_1^3S_2^3 - S_1^3T_2^3}.$$

Here i = 0, 1, 2, the index after comma denotes the partial derivative with respect to the specified coordinate.

After inverse Laplace transform movement can be written in the space of originals as a function of time, two coordinates and interaction forces on the plate (8), where Dirac (τ-τ_1) is a delta function of initial time interval:

$$x(\varphi,\theta,\tau) = \frac{(1-\sigma_\theta\sigma_r)h}{\pi R_c^2E_r}\int_0^\tau\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}(4n+3)P(\tau_1)\times$$

$$\times P_{2n+1}\left(\cos\left(\frac{\pi r_1}{2R}\right)\right)P_{2n+1}\left(\cos\left(\frac{\pi r}{2R}\right)\right)\cos(m\theta)\times$$

$$\times\left[x_{2n+m}^0 Dirac(\tau-\tau_1) + x_{2n+m}^1(\tau-\tau_1) + \frac{x_{2n+m}^2(\tau-\tau_1)^3}{6} + \frac{x_{2n+m}^3(\tau-\tau_1)^5}{120}\right]d\tau_1. \qquad (8)$$

Relation (8) connects displacements of plate points with the power of dynamic influence from it on the wheel set. Contact force is determined by displacements of points of the flat element in a place of interaction application and gets its numerical expression in a functional equation [3, 7]

$$y(t) = V_0t - \frac{1}{m}\int_0^t P(t_1)(t-t_1)dt_1, \qquad (9)$$

where $y(t) = \alpha(t) + w(t)$ *is total displacement of*

wheels of the carriage; m – reduced mass of the wheel; t – time counted from the start of interaction of bodies in the selected point; t_1 – integration variable.

Time dependences α(t) and P(t) are determined by solving the contact problem [3, 8, 9], the results of which are combined with the results of the wave problem. The solution of equation (9) is found numerically by computer, based on the assumption that on each sufficiently small interval $(n-1)\tau \le t \le n\tau$ *unknown quantities vary linearly.*

Conclusion. *These are approaches and the logic of finding the dynamic effects parameters in the plate of ballastless track, if we use the proposed mathematical models. The presence in the calculation of an ideal elastic medium can more accurately determine the unknown quantities in view of the wave pattern in the flat member. And it is certainly an advantage of the demonstrated method.*

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