AVERAGE WEIGHT CALCULATION OF A FREIGHT TRAIN

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ABSTRACT

Average weight of freight trains is an important indicator of the use of the existing tonnage rating, power of traction means and defining, in terms of given volume of traffic, size of freight trains movement on sections of railway lines. This indicator plays a leading role in technical and economic calculations, but methods, which are used, provide a basis to seek corrective options. The author offers his interpretation of the well-known formula that is used for selecting optimal tonnage rating of freight trains (characterized by full-length and full-weight) and bases his calculations on certain assumptions. With the help of a modified formula difference of the calculated average weight of about 2.3% is achieved.

ENGLISH SUMMARY

Background. *Professor K. K. Tihonov [1] proposed a formula that allows establishing the dependence of the average weight of the train from tonnage rating, length standard of station receiving-departure tracks, weight structure of freight traffic on gage section.*

The basis of this dependence is a histogram of the distribution of train linear loads (Pic. 1), which was calculated on the basis of statistical data on the actual weights and lengths of trains.

In Pic.1 P^* – average train linear load; P_H – train linear load, defined by tonnage rating Q_H and useful length of station receiving-departure tracks ($l_{ст}$ – a). See formula (1).

Average weight of a train with a tonnage rating Q_H is calculated by the formula (2), where α_i – frequency of i- category of a histogram;

P_i – average value of train linear load in i-category of a histogram;

k – number of histogram`s category, the right border of which coincides with $P_{H.}$

Frequency of i-category of a histogram is defined as a ratio (3), where

m_i – number of statistical samples, which got in i- category of a histogram of train linear loads distribution;

$m_{общ}$ – total statistical sample.

In deriving formula (2) [1, p.68] trains in accordance with the rules of formation are either full-weight (formed by tonnage rating) or full-length (formed in accordance with the length of station receiving-departure tracks). Under these conditions, average gross train weight will be (4), where A- given daily volume of transportation work; $n_{пс}$, $n_{пв}$ – average daily traffic size of full-length and full-weight trains, respectively.

In his subsequent calculations professor K. K. Tihonov assumed that the number of full-length and full- weight trains does not depend on what was the original tonnage rating at which statistics on weights and lengths of trains was collected, and histogram of train linear loads distribution was built corresponding to this initial tonnage rating.

And here the question arises, how will the formula (2) change, if this assumption is abandoned?

Objective. *The author aims at showing his interpretation of calculation formula of freight trains weight.*

Methods. *The author uses mathematical methods and comparative method.*

Results. *Transportation work, defined in terms of existing tonnage rating $Q_н^ф$ and corresponding train linear load $P_н^ф$, will be (Pic. 2a) formula (5), where the first summand defines transportation work of full-length trains, and the second- of full-weight trains.*

If in the calculation tonnage rating is assumed to be less than the current one ($Q_н < Q_н^ф$), then (Pic. 2a) the number of full- length trains will be (6) and the number of full- weight trains will be (7).

In the formula (7) the first summand defines the number of full- weight trains, which under current tonnage rating $Q_н^ф$ were full- length trains. The second summand is the number of full-weight trains under tonnage rating $Q_н$, which were also full- weight trains, but under tonnage rating $Q_н^ф$.

Thus, when $Q_н \le Q_н^ф$:

$$Q^*_{бр} = \frac{A}{n_{пс} + n_{пв}} = \frac{\sum_{i=1}^{k_ф} P_i m_i (l_{ст} - a) + Q_н^ф \sum_{i=k_ф+1}^{n} m_i}{\sum_{i=1}^{k} m_i + \sum_{i=k+1}^{k_ф} \frac{P_i (l_{ст} - a) m_i}{Q_н} + \sum_{i=k_ф+1}^{n} \frac{Q_н^ф}{Q_н} m_i}.$$

Dividing the numerator and denominator by $m_{общ}$ the formula (8) will be obtained.

In case, when tonnage rating is more than the current one – $Q_н \ge Q_н^ф$ (Pic. 2b), the number of full-length trains:

$$n_{пс} = \sum_{i=1}^{k_ф} m_i + \sum_{i=k_ф+1}^{k} \frac{Q_н^ф m_i}{P_i (l_{ст} - a)}.$$

And the number of full-weight trains:

$$n_{пв} = \sum_{i=k+1}^{n} \frac{Q_н^ф m_i}{Q_н}.$$

Average gross train weight in such cases will be equal to:

$$Q^*_{бр} = \frac{A}{n_{пс} + n_{пв}} = \frac{\sum_{i=1}^{k_ф} P_i m_i (l_{ст} - a) + Q_н^ф \sum_{i=k_ф+1}^{n} m_i}{\sum_{i=1}^{k_ф} m_i + \sum_{i=k_ф+1}^{k} \frac{Q_н^ф m_i}{P_i (l_{ст} - a)} + \sum_{i=k+1}^{n} \frac{Q_н^ф}{Q_н} m_i}.$$

Dividing the numerator and denominator by $m_{общ}$, the formula (9) will be obtained.

To compare the results of calculation by formula of professor K. K. Tihonov (2) and equations (8) and (9) the results of calculations presented in [1, p.73] are used. Comparative data are shown in Table 1.

The formula of professor K. K. Tihonov was used in calculating the average train weight in case of changes in the length of station receiving-departure tracks.

First in formula (2) it is necessary to change the value ($l_{ст}$ – a) and train linear load $P_н = \frac{Q_н}{(l_{ст} - a)}$. However, this is not enough, since histogram of train linear loads distribution was built at the existing length of station receiving-departure tracks $l_{ст}^ф$ and current tonnage rating $Q_н^ф$. Therefore, calculation of transportation work, the number of full-weight and full-length trains after elongation of tracks will be different, and therefore the formula to calculate average gross weight of a train after elongation of station tracks will differ.

Then train linear load, corresponding to tonnage rating and new length standard of station receiving-

Pic. 1. Histogram of train linear load distribution.

departure tracks after their extension is denoted via $P_н$:

$$P_н = \frac{Q_н}{(l_{cm} - a)},$$

and via $P_н^ф$ – train linear load, corresponding to the current tonnage rating and length standard of station tracks:

$$P_н^ф = \frac{Q_н^ф}{(l_{cm}^ф - a)}.$$

Transportation work that was done under the current tonnage rating and length standard of station receiving-departure tracks will be (10).

Formula (10) differs from the formula (5) only in that the transportation work is determined by the existing length standard of station receiving-departure tracks, i. e. prior to their elongation.

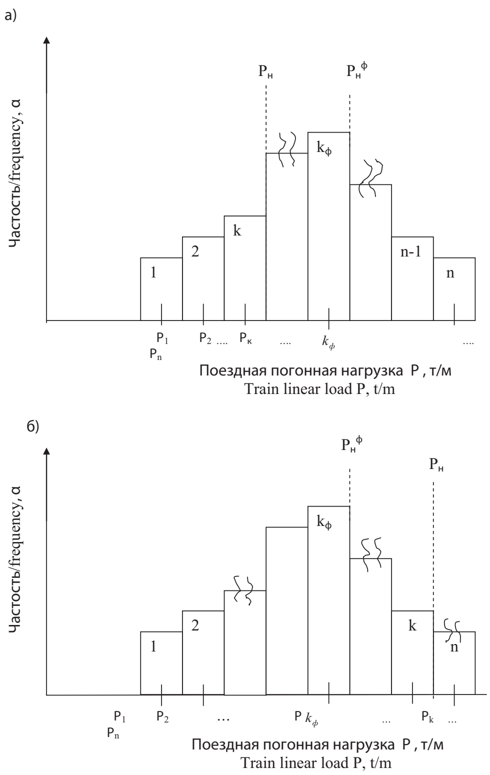
If $P_н \le P_н^ф$, the number of full-length trains after elongation of station tracks will be (Pic. 2a) defined by formula (11).

The number of full-length trains (Pic. 2a) with the tonnage rating $Q_н$ will be defined by formula (12).

The first summand defined the number of full-weight trains, which prior to elongation of station tracks were full- length trains. The second summand determines the number of full-length trains with new tonnage rating $Q_н$.

Substituting parameters obtained in (10–12) in (4) and dividing the numerator and denominator by the total $m_{общ}$, if $P_н \le P_н^ф$, the formula (13) will be obtained.

And if $P_н \ge P_н^ф$ (Pic. 2b) the number of full- length trains will be (14), where the first summand defines the



Pic. 2. Histogram of train linear loads under different values of tonnage ratings.

Table 1

Comparison of calculation results of average train weight

Весовая норма $Q_н$ /*Tonnage rating, $Q_н$*	Средний вес поезда по формуле (2) / *Average train weight under formula (2)*	Средний вес поезда по формулам (8), (9) при исходной весовой норме $Q_н^ф$/*Average train weight under formulas (8), (9) with initial tonnage rating $Q_н^ф$*					
		3000	3500	4000	4500	5500	6500
6500	4783	4518	4568	4613	4657	4741	4783
6000	4740	4490	4539	4580	4624	4703	4740
5500	4626	4410	4456	4495	4535	4599	4626
5000	4466	4288	4331	4366	4401	4447	4466
4500	4218	4093	4128	4157	4180	4210	4218
4000	3860	3780	3807	3826	3840	3856	3860
3500	3429	3386	3402	3414	3421	3428	3429
3000	2976	2959	2967	2971	2973	2975	2976
2500	2496	2496	2496	2496	2496	2496	2496

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number of full-length trains after elongation of station tracks and the second one- the number of the full- length trains, which were full- weight trains prior to elongation.

The number of full- weight trains under tonnage rating $Q_н$ will be determined by formula (15).

Substituting (10), (14), (15) in the formula (4), under $P_н \geq P_н^ф$ the formula (16) will be obtained.

Calculation results for average weight of a train by the formula (2) and formulas (13), (16) are presented in Table 2 under $l_{ст}^ф – a = 800$ m, $l_{ст} – a = 1000$ m.

Histogram of train linear load and values of train average weight by the formula (2) are taken from the work [1, c.73].

Conclusion. *The results obtained in Table 1 lead to the following conclusions:*

1. Calculation of the average weight of a train by the formula of professor K. K. Tihonov (2) and equations (8) and (9) is the same in the case if the initial tonnage rating, at which statistics on weights and lengths of freight trains was collected, corresponded to maximum train linear load (in Table 1 it is the initial tonnage rating $Q_н^ф$=6500 tons). Full matching also takes place at the very low value of tonnage rating – at the level of the first category of a histogram of train linear loads distribution (in the table – tonnage rating is 2,500 tons). In all other cases, the results of the calculations do not match.

2. The discrepancy in the calculation of average train weight by the formula of professor K. K. Tihonov and formulas (8) and (9) is largely dependent on the nature of the distribution of train linear loads and the current (initial) tonnage rating. For example, when the calculated tonnage rating is 5500 tons weight and the current tonnage rating is 3000 tons, weight of the train according to the formula of professor is 4624 tons, and according to the proposed formula is 4410 tons. The difference in calculation of the average train weight is 214 tons or 4.8%.

3. From formula (8) as a special case can be obtained version of the formula (2). It is enough to take $Q_н^ф = P_{max}(l_{ст} – a)$. Then $k_ф = n$, and the formula (8) is transformed into formula (2).

The data in Table 2 lead to the following conclusions:

1. Length standard of station receiving-departure tracks, which existed prior to elongation, and tonnage rating, which was in force prior to elongation affect the calculated gross train weight, which will be obtained after the elongation of station tracks. For example (see Table 2), when tonnage rating is 5000 tons and $l_{ст}$=1050 m, the average weight of a train by the formula (2) is 4466 tons. By the proposed formula and tonnage rating prior to elongation of tracks $Q_н^ф$=3200 tons, the average weight of a train after elongation is 4366 tons. Difference of the calculated average weight of the train is 100 tons, i. e. 2.3%.

2. Difference in calculating the average gross weight of the train by the formula (2) and formulas (13), (16) is absent only when train linear load calculated for the tonnage rating prior to elongation of tracks is maximum. If after the elongation of tracks, tonnage rating and corresponding train linear load are within 2.0–3.5 t / m, the difference in calculating the average weight of the train is low. For example, for a train when $Q_н^ф$=3000–3500 t (Table 2), this difference does not exceed 15 t, i. e., 0.4%.

Keywords: freight train, average weight, calculation formula, optimal weight standards, theoretical basis of calculations.

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