MANAGEMENT MODEL FOR CONTAINER COMPANY WITHIN LOGISTICS CHAIN

Lievin, Sergey B. *– Ph.D. (Tech), D. Sc. candidate at Moscow State University of Railway Engineering (MIIT), Moscow, Russia.*

ABSTRACT

The article substantiates a model of management of container handling and forwarding company within a logistic chain consisting of four links. Weighted mean value of mathematical expectation of the arrival of goods to finite logistics link and of mathematical expectation of delivery time constitute an objective function of the model. The expenses of a company are equated to managerial impact.

ENGLISH SUMMARY

Background. *Containerization of freightage made a significant contribution to optimization of traffic flows, to reduction of transportation costs, facilitated international and domestic trade. Standardization of transported units, which contributed to productivity of transport sector, was the main factor of wide containerization. The standardization permitted to avoid handling of different packages and kinds of goods during transport operations, to optimize distribution of goods within loading volumes of transport vehicles, to facilitate the provisions for goods' safety, to overcome illegal acts from the part of carriers' agents and stevedore companies.*

The standardization also permitted to reduce importance of some features that had been previously guidelines for optimization of logistics activities of transportation company. For instance relevant dimensional characteristics of every unit of cargo (a container) are limited by one of two possible types of containers that can be considered.

But all the characteristics that are usual for all other modes of transport have maintained their important role (e. g. departure point, delivery point, safety and transit time within logistics chain).

In most cases the volume of payments received by modern logistics company depends on the level of conformity of its activities with the requirements of its customers concerning the above mentioned characteristics of transportation. Consequently, the capacity of container company to meet the requirements depends on technology of container handling and carriage, as well as to rationally organize the management process.

Objectives. *The objective of research was to offer a model of logistics chain which is relevant and can be subject to numerical analysis.*

Methods *consist in development of mathematical model based on objective function, which is a weighted mean value of mathematical expectation of the arrival of goods to finite logistics link and of delivery time.*

Results.

1. <u>Notion of managerial impact</u>. Even without touching the issues of goods routing, it is to be supposed that the logistics chain of a container enterprise consists of a set of links, each of them being characterized by specific impact on container flows that transit such links of logistics chain. The initial and finite links of chains (source of origin and sink) are of particular importance as they represent an interface with incoming flow of containers (presumably from forwarder) and exit stream (presumably to finite customer).

The following lettering is proposed: index $i = 0,1,..., N$ responds to (consecutive) element of logistics chain from source $i = 0$ through sink $i = N$. At a given moment a container emerges at the source, then it transits logistics chain, being under managerial impact in the chain's element number $i = 1, ..., N$ (see Pic. 1).

It is to be noted that the source of logistics chain is equal to a forwarder who does not belong to container company, and therefore there is no managerial impact e_0 that might correspond to the source. It explains the used numeration of the elements of logistics chain from 0. The source is an element 0 of logistics chain but it does not make a part of container company, and managerial impacts $e_1,..., e_N$ correspond to chain's links $i = 1,..., N$, which are managed by container company.

Without losing commonness we can identify managerial impacts e_i with corresponding costs (expenses) of a container company. Therefore the measuring units of managerial impact are in our case Russian rubles.

It is important that if in natural division of logistics chain of a container company into links (e. g. loading in port-railway transshipment complex, conveyance from one rail station to another) we can find some kinds of managerial impact (not only delaying of departure time of a container at cargo station till train collecting, but also additional measures of guarding the containers), then a proposed theoretical model can take such situation into consideration, to incorporate it by dividing relevant chain's link into several links (concerning respectively delaying of departure of a container and taking supplementary security measures).

Relevant managerial impacts will change during a container's transiting through logistics chain, so that characteristics of cargo will also stochastically change, and the study will deal with probability model of interaction between goods' characteristics and managerial impacts.

Let's study a model containing two characteristics: safe conditions of goods $x_i \in \{0,1\}$, where value 1 is equal to safe goods and value 0 is equal to unsafe (lost) goods, and the time t_i of arrival of the goods at the finite point of the link i. Probabilistic distribution of those characteristics results from managerial impacts at all previous stages of logistics chain.

In order to further reduce the dimensions of the model it is possible to suppose that real impact will be caused by current link and by the link that immediately precedes it. This assumption corresponds to reality as the decisions at the moment of arrival of a container to current link of a chain are taken on the basis of the time of arrival and safety conditions of the container coming from previous link.

It is possible to assume as physical limitations the reduction (increasing) of relevant time series $x_i \geq x_{i+1}$ (the damaged goods can't be restored to original conditions during transportation) and $t_i \leq t_{i+1}$ (for all $i = 0,1,..., N–1$).

So, taking into account initial probabilistic distribution of the value of arrival of the goods at the source t_0, and assumption that goods are in appropriate conditions ($x_0=1$), further destiny of a container in logistics chain is determined by managerial impacts of links e_i and by the characteristics of relationship between those impacts and relevant transformations of probabilistic distributions of the vector of goods' characteristic's (x_i, t_i).

Rational organization of management process of a container company will be determined in that case by the choice of a plan $F_i(\cdot,\cdot,\cdot)$ of managerial actions e_i, depending on impact values of a previous link e_{i-1} and on container conditions in it (x_{i-1}, t_{i-1}):

$$e_i = F_i(e_{i-1}, x_{i-1}, t_{i-1}).$$

The choice of a plan of managerial impacts should be realized with the objective of maximizing of a determined objective function, related to probabilistic distribution of finite conditions of a container, deducting total direct costs of a container enterprise:

$$G(cdf(x_N, t_N - t_0)) - E\sum_{i=1}^{N} e_i$$

where cdf is a probabilistic distribution of relevant random element, and G is an objective function.

Objective function can be formulated as weighted mean value of mathematical expectations of safe conditions of cargo and of delivery time, where ratios of weight of each criterion can be determined through a level of importance of respective factor. Determining of importance level should consider for penalties which are imposed on a container company in case of loss (damage) of cargo or of delay in delivery.

Alternative objective function can be expressed by probability of simultaneous provision of safety of cargo and of in-time delivery. The proposed model is sufficiently flexible to be calculated for different objective functions.

Finally, different combinations of objective functions can be used, as well as attempts can be taken to reduce variability of results (regarding cargo safety and delivery time) by taking into account ratios of variation etc.

2. Mathematical description of the model. Let it be assumed that weighted mean value of mathematical expectations of safe conditions of cargo and of mathematical expectation of delivery time (the least with negative contribution) was taken for an objective function:

$$G(cdf(x_N, t_N - t_0)) = \alpha Ex_N - \beta E(t_N - t_0).$$

Let 4 links be taken as example of logistics chain (N = 3): source of the flow originated at forwarder i = 0; cargo rail station, receiving the cargo for further delivery i = 1; railway section i = 2, and cargo station at destination (it is also the sink for containers' stream) i = 3. The relevant chart is shown in Pic.2.

In order to comprehensively describe the model it is necessary to determine the character of influence of managerial impacts e_1, e_2 and e_3 on distribution of relevant random elements, describing cargo conditions in container at the finite point of respective link of logistics chain: $(x_i, t_i)^3_{i=1}$.

Managerial impact at the station of departure refers to anchoring of a container to a certain train, to waiting of the train, to loading. It is possible to suppose that the impact will be associated with increase in expectation time and with probability of cargo damaging, but also with better outlook for safety and in-time delivery of cargo at the next link of logistics chain (which is direct carriage of a container in a train). Simple form of such managerial impact can be expressed as follows:

$$P_0(x_i = 1) = x_i exp^{-\lambda_1 e_1} x_0,$$

$$t_1 = t_0 + (\gamma_1 e_1 + \alpha_1)\varepsilon_1,$$

where ε_1 is a random value, distributed exponentially with parameter μ_1.

The P_i within those expressions designates conditional probability of relevant event under the condition that all the information of time periods (including i^{th} period) is known and received. In other words, $P_i(A) = P(A \mid x_i, \varepsilon_i, x_{i-1}, \varepsilon_{i-1}, K)$. The abridged designation of conditional mathematical expectations below is expressed in the same manner like $E_i z = E_i(z \mid x_i, \varepsilon_i, x_{i-1}, \varepsilon_{i-1}, K)$.

Parameters x_1, λ_1, γ_1, α_1 and μ_1 should be selected empirically (on the basis of statistics data on the activities of the given link within logistics chain), so that specified influence of managerial impact e_1 approximates in best possible manner the real situation.

Under the described mode the result of managerial impact within the second link of logistics chain can be formalized as follows:

$$P_1(x_2 = 1) = x_2(1 - exp^{-\lambda_2 e_2})x_1,$$

$$t_2 = t_1 + (\gamma_2 e_2 - \delta_2 e_1 + \alpha_2)\varepsilon_2,$$

where ε_2 is a random value, distributed exponentially with parameter μ_2.

As previously described, parameters x_2, λ_2, γ_2, α_2 and μ_2 should be evaluated on the basis of collected statistical data. It is also assumed that random values ε_1, ε_2 and realization of the risks of cargo damage (described above by relevant probabilities) are independent on each other.

The Pic. 3 illustrates an example of a chart of dependency of probability of cargo safety within the second link of logistics chain

$$P_1(x_2 = 1) = x_2(1 - exp^{-\lambda_2 e_2})x_1$$

on the value of managerial impact e_2 within the same link, based on assumption that $x_2=\lambda_2=1$, and the cargo was safe at previous stage ($x_1=1$).

The Pic.4 illustrates the chart of dependency of time t_2 of transition of cargo from link 2 (railway section) to link 3 (arrival station) of logistics chain on the values of managerial impacts e_1 and e_2 in link 1 (departure station) and link 2 (railway section) with the assumption that $\gamma_2 = \delta_2 = \varepsilon_2 = 1$, $t_1=10$, $\alpha_2=40$.

Finally, the managerial impact at arrival station will concern only the issues of cargo safety:

$$P_2(x_3 = 1) = x_3(1 - exp^{-\lambda_3 e_3})x_2,$$

$$t_3 = t_2 + \alpha_3.$$

3. Solution of the problem. It is more convenient to begin solving of the problem starting from last time periods (using so called backward induction), as most parts of fuzziness were realized at previous stages of the model and, as the model assumes, has already been known to the subject of managerial decision-making. According to the above, the decision of optimizing agent on the last managerial impact will be equivalent to solving of the following problem:

$$\max_{e_3} \alpha E_2 x_3 - \beta E_2(t_3 - t_0) - e_1 - e_2 - e_3$$

under the condition that

$$P_2(x_3 = 1) = x_3(1 - exp^{-\lambda_3 e_3})x_2,$$

$$t_3 = t_2 + \alpha_3,$$

$$e_3 \geq 0.$$

It is to note that $t_3 = t_2 + \alpha_3$ within that expression is a fixed value and does not depend on managerial *impact e_3, whose value we are searching for.*

Besides, it is possible to see that $E_2 x_3 = 1 \cdot P_2(x_3 = 1) = x_3(1 - exp^{-\lambda_3 e_3})x_2$. *So the optimization problem is reduced to*

$$\max_{e_3 \geq 0} \alpha x_3 \left(1 - exp^{-\lambda_3 e_3}\right) x_2 - e_3,$$

and its solution is found on the basis of conditions of the first order:

$$e_3 = \frac{1}{\lambda_3} ln \alpha x_3 \lambda_3,$$
$$if\ \alpha x_3 \lambda_3 > 1$$
$$and\ x_2 = 1,$$

$$e_3 = 0,$$
$$if\ \alpha x_3 \lambda_3$$
$$x_2 \leq 1.$$

In other words, optimum plan $F_3(\cdot,\cdot,\cdot)$ of managerial impact e_3 within the third link of logistics chain is expressed by:

$$e_3 = F_3(e_2, x_2, t_2) = \frac{1}{\lambda_3} ln \alpha x_3 \lambda_3,$$
$$if\ \alpha x_3 \lambda_3 > 1$$
$$and\ x_2 = 1,$$

$$e_3 = F_3(e_2, x_2, t_2) = 0,$$
$$if\ \alpha x_3 \lambda_3$$
$$x_2 \leq 1.$$

It should be noted that optimum plan does not depend on the value of managerial impact within previous link of logistics chain e_2, neither on arrival time t_2 at the finite destination point, but on the safe condition of cargo at the end of previous link. Informally, the result means that it is more important to deploy efforts (managerial impact e_3) to ensure further safety of cargo in the case of safe cargo (x_2 = 1), than in the case of unsafe (damaged, lost) cargo (x_2=0).

The problem of finding of optimum level of managerial impact within the second link of logistics chain is formulated similarly to the procedure used for the last link:

$$\max_{e_2} \alpha E_1 x_3 - \beta E_1(t_2 + \alpha_3 - t_0) - e_1 - e_2 - E_1 e_3$$

under appropriate conditions. It should be noted that

$$E_1 x_3 = E_1(E_2(x_3)) = E_1\left(x_3\left(1 - exp^{-\lambda_3 e_3}\right)x_2\right).$$

Taking into account the optimum level of managerial impact e_3, found previously:

$$E_1 x_3 = x_3\left(1 - \frac{1}{\alpha x_3 \lambda_3}\right) P_1(x_2 = 1),$$
$$if\ \alpha x_3 \lambda_3 > 1$$

and

$$E_1 x_3 = 0,$$
$$if\ \ \alpha x_3 \lambda_3 \leq 1.$$

Besides

$$E_1 e_3 = \frac{1}{\lambda_3} \ln(\alpha x_3 \lambda_3) P_1(x_2 = 1) = \frac{1}{\lambda_3} \ln(\alpha x_3 \lambda_3) x_2 \left(1 - exp^{-\lambda_2 e_2}\right) x_1,$$
$$if\ \alpha x_3 \lambda_3 > 1,$$

and

$$E_1 e_3 = 0,$$
$$if\ \alpha x_3 \lambda_3 \leq 1.$$

Consequently, if $\alpha x_3 \lambda_3 > 1$ then the problem of finding of optimum e_2 is reduced to

$$\max_{e_2} \left(\alpha x_3 - \frac{1}{\lambda_3} - \frac{1}{\lambda_3} \ln(\alpha x_3 \lambda_3)\right) \bullet$$
$$\bullet x_2\left(1 - exp^{-\lambda_2 e_2}\right) x_1 - \beta E_1(t_2 - t_1) - e_2$$

under determined conditions. While solving the problem of variable t_2, we have

$$\max_{e_2 \geq 1} \left(\alpha x_3 - \frac{1}{\lambda_3} - \frac{1}{\lambda_3} \ln(\alpha x_3 \lambda_3)\right) x_2 \left(1 - exp^{-\lambda_2 e_2}\right) x_1 -$$
$$- \frac{\beta}{\mu_3}(\gamma_2 e_2 - \delta_2 e_1 + \alpha_2) - e_2,$$

what is equivalent to

$$\max_{e_2 \geq 0} - \left(\alpha x_3 - \frac{1}{\lambda_3} - \frac{1}{\lambda_3} \ln(\alpha x_3 \lambda_3)\right) \bullet$$
$$\bullet x_2 exp^{-\lambda_2 e_2} x_1 - \left(\frac{\beta}{\mu_3}\gamma_2 + 1\right) e_2.$$

Similarly, in case of $\alpha x_3 \lambda_3 \leq 1$ optimum managerial impact e_2 is found by solving the problem

$$\max_{e_2 \geq 0} - \frac{\beta}{\mu_3}(\gamma_2 e_2 - \delta_2 e_1 + \alpha_2) - e_2,$$

so $e_2 = 0$.

Therefore, if $\alpha x_3 \lambda_3 > 1$, then the optimum plan of managerial impact within the second link of logistics chain is as follows:

$$e_2 = F_2(e_1, x_1, t_1) = \frac{1}{\lambda_2} \ln \frac{\mu_3 \lambda_2 \chi_2 (\alpha \chi_3 - (1 + \ln(\alpha \chi_3 \lambda_3))/\lambda_3)}{\beta \gamma_2 + \mu_3},$$

$$if\ \frac{\mu_3 \lambda_2 \chi_2 (\alpha \chi_3 - (1 + \ln(\alpha \chi_3 \lambda_3))/\lambda_3)}{\beta \gamma_2 + \mu_3} > 1\ and\ x_1 = 1;$$

$e_2 = F_2(e_1, x_1, t_1) = 0$, if

$$\frac{\mu_3 \lambda_2 \chi_2 (\alpha \chi_3 - (1 + \ln(\alpha \chi_3 \lambda_3))/\lambda_3)}{\beta \gamma_2 + \mu_3} x_1 \leq 1.$$

At the same time

$e_2 = F_2(e_1, x_1, t_1) = 0$,

if $\alpha x_3 \lambda_3 \leq 1$.

The last step to be taken in order to completely solve the problem, is to find optimum plan of managerial impact within the first link of logistics chain, taking into account the found optimum plans of managerial impact within all further links.

It is possible to express objective function to find e_1 as:

$$\max_{e_1} \alpha E_0 x_3 - \beta E_0(t_3 - t_0) - e_1 - E_0 e_2 - E_0 e_3.$$

Then we have

$$E_0x_3 = E_0\left(E_1\left(E_2\left(x_3\right)\right)\right) = E_0\left(E_1\left(\chi_3\left(1-exp^{-\lambda_3 e_3}\right)x_2\right)\right).$$

In other words,

$$E_0x_3 = \chi_3\left(1-\frac{1}{\alpha\chi_3\lambda_3}\right)E_0\left(P_1\left(x_2=1\right)\right)=$$

$$=\chi_3\left(1-\frac{1}{\alpha\chi_3\lambda_3}\right)E_0\left(\left(1-exp^{-\lambda_2 e_2}\right)x_1\right),$$

if $\alpha\chi_3\lambda_3>1$

and

$E_0x_3=0$, if $\alpha\chi_3\lambda_3\leq 1$

Consequently,

$$E_0x_3=\chi_3\left(1-\frac{1}{\alpha\chi_3\lambda_3}\right)\bullet$$

$$\bullet\left(1-\frac{\beta\gamma_2+\mu_3}{\mu_3\lambda_2\chi_2(\alpha\chi_3-(1+\ln\left(\alpha\chi_3\lambda_3\right)/\lambda_3)}\right)P_0(x_1=1)=$$

$$=\chi_1\chi_3\left(1-\frac{1}{\alpha\chi_3\lambda_3}\right)\bullet$$

$$\bullet\left(1-\frac{\beta\gamma_2+\mu_3}{\mu_3\lambda_2\chi_2(\alpha\chi_3-(1+\ln\left(\alpha\chi_3\lambda_3\right)/\lambda_3)}\right)exp^{-\lambda_1 1}x_0,$$

if

$$\frac{\mu_3\lambda_2x_2\left(\alpha x_3-\left(1+\ln\left(\alpha x_3\lambda_3\right)\right)/\lambda_3\right)}{\beta\gamma_2+\mu_3}>1,$$

and

$$E_0x_3=0,$$

if

$$\frac{\mu_3\lambda_2x_2\left(\alpha x_3-\left(1+\ln\left(\alpha x_3\lambda_3\right)\right)/\lambda_3\right)}{\beta\gamma_2+\mu_3}x_1\leq 1$$

or

$$\alpha x_3\lambda_3\leq 1.$$

Further on

$$E_0\left(t_3-t_0\right)=E_0\left(t_2+\alpha_3-t_0\right)=E_0\left(t_2-t_1\right)+E_0\left(t_1-t_0\right)+\alpha_3.$$

We have

$$E_0\left(t_1-t_0\right)=E_0\left(\left(\gamma_1e_1+\alpha_1\right)\varepsilon_1\right)=\frac{\gamma_1e_1+\alpha_1}{\mu_1}\ \text{and}$$

$$E_0\left(t_2-t_1\right)=E_0\left(\left(\gamma_2e_2-\delta_2e_1+\alpha_2\right)\varepsilon_2\right)=$$

$$=E_0\left(E_1(\left(\gamma_2e_2-\delta_2e_1+\alpha_2\right)\varepsilon_2\right)=$$

$$=\frac{1}{\mu_2}E_0\left(\gamma_2e_2-\delta_2e_1+\alpha_2\right).$$

In other words, if

$$\frac{\mu_3\lambda_2x_2\left(\alpha x_3-\left(1+\ln\left(\alpha x_3\lambda_3\right)\right)/\lambda_3\right)}{\beta\gamma_2+\mu_3}x_1>1,$$

$$E_0\left(t_2-t_1\right)=\frac{\alpha_2-\delta_2e_1}{\mu_2}+\frac{\gamma_2}{\lambda_2\mu_2}\bullet$$

$$\bullet\ln\frac{\mu_3\lambda_2x_2\left(\alpha x_3-\left(1+\ln\left(\alpha x_3\lambda_3\right)\right)/\lambda_3\right)}{\beta\gamma_2+\mu_3}P_0(x_1=1)=$$

$$=\frac{\alpha_2-\delta_2e_1}{\mu_2}+\frac{\gamma_2x_1}{\lambda_2\mu_2}\bullet$$

$$\bullet\ln\frac{\mu_3\lambda_2x_2\left(\alpha x_3-\left(1+\ln\left(\alpha x_3\lambda_3\right)\right)/\lambda_3\right)}{\beta\gamma_2+\mu_3}\exp^{-\lambda_1e_1}x_0.$$

At the same time if

$$\frac{\mu_3\lambda_2x_2\left(\alpha x_3-\left(1+\ln\left(\alpha x_3\lambda_3\right)\right)/\lambda_3\right)}{\beta\gamma_2+\mu_3}x_1\leq 1,$$

or

$$\alpha x_3\lambda_3\leq 1,$$

$$E_0\left(t_2-t_1\right)=\frac{\alpha_2-\delta_2e_1}{\mu_2}.$$

Besides in case

$$\frac{\mu_3\lambda_2x_2\left(\alpha x_3-\left(1+\ln\left(\alpha x_3\lambda_3\right)\right)/\lambda_3\right)}{\beta\gamma_2+\mu_3}>1$$

we have

$$E_0e_2=\frac{1}{\lambda_2}\ln\frac{\mu_3\lambda_2x_2\left(\alpha x_3-\left(1+\ln\left(\alpha x_3\lambda_3\right)\right)/\lambda_3\right)}{\beta\gamma_2+\mu_3}P_0(x_1=1)=$$

$$=\frac{x_1}{\lambda_2}\ln\frac{\mu_3\lambda_2x_2\left(\alpha x_3-\left(1+\ln\left(\alpha x_3\lambda_3\right)\right)/\lambda_3\right)}{\beta\gamma_2+\mu_3}\exp^{-\lambda_1e_1}x_0.$$

On the contrary in case

$$\frac{\mu_3\lambda_2x_2\left(\alpha x_3-\left(1+\ln\left(\alpha x_3\lambda_3\right)\right)/\lambda_3\right)}{\beta\gamma_2+\mu_3}\leq 1$$

or

$$\alpha x_3\lambda_3\leq 1$$

we have $E_0e_2=0$.

Finally we have

$$E_0e_3=\frac{1}{\lambda_3}\ln\left(\alpha x_3\lambda_3\right)P_0\left(x_2=1\right)=$$

$$=\frac{x_2}{\lambda_3}\ln\left(\alpha x_3\lambda_3\right)E_0\left(\left(1-\exp^{-\lambda_2e_2}\right)x_1\right)=$$

$$=\frac{x_2}{\lambda_3}\ln\left(\alpha x_3\lambda_3\right)\bullet$$

$$\bullet\left(1-\frac{\beta\gamma_2+\mu_3}{\mu_3\lambda_2x_2\left(\alpha x_3-\left(1+\ln\left(\alpha x_3\lambda_3\right)\right)/\lambda_3\right)}\right)\bullet$$

$$\bullet P_0\left(x_1=1\right)=$$

$$=\frac{x_1x_2}{\lambda_3}\ln\left(\alpha x_3\lambda_3\right)\bullet$$

$$\bullet\left(1-\frac{\beta\gamma_2+\mu_3}{\mu_3\lambda_2x_2\left(\alpha x_3-\left(1+\ln\left(\alpha x_3\lambda_3\right)\right)/\lambda_3\right)}\right)\bullet$$

$$\bullet\exp^{-\lambda_1e_1}x_0$$

in case if

$$\frac{\mu_3\lambda_2x_2\left(\alpha x_3-\left(1+\ln\left(\alpha x_3\lambda_3\right)\right)/\lambda_3\right)}{\beta\gamma_2+\mu_3}>1.$$

But in the case if

$$\frac{\mu_3\lambda_2 x_2\left(\alpha x_3-\left(1+\ln\left(\alpha x_3\lambda_3\right)\right)/\lambda_3\right)}{\beta\gamma_2+\mu_3}\le 1$$

or

$$\alpha x_3\lambda_3\le 1$$

we obtain $E_0 e_3 = 0$.

By reducing all the obtained expressions in unique objective function in order to find optimum level of managerial impact e_i, *the following problem to find* e_1 *is formulated:*

$$\max_{e_1\ge 0} - A\exp^{-\lambda_1 e_1} - Be_1,$$

where

$$B=1-\beta\left(\frac{\delta_2}{\mu_2}-\frac{\gamma_1}{\mu_1}\right),$$

at the same in the case

$$\frac{\mu_3\lambda_2\chi_2(\alpha\chi_3-(1+\ln\left(\alpha\chi_3\lambda_3\right))/\lambda_3)}{\beta\gamma_2+\mu_3}>1$$

$$A=-\alpha\chi_1\chi_3\left(1-\frac{1}{\alpha\chi_3\lambda_3}\right)\bullet$$

$$\bullet\left(1-\frac{\beta\gamma_2+\mu_3}{\mu_3\lambda_2\chi_2(\alpha\chi_3-(1+\ln\left(\alpha\chi_3\lambda_3\right)/\lambda_3)}\right)x_0-$$

$$-\beta\frac{\gamma_2\chi_1}{\lambda_2\mu_2}\ln\frac{\mu_3\lambda_2\chi_2\left(\alpha\chi_3-(1+\ln\left(\alpha\chi_3\lambda_3\right))/\lambda_3)\right)}{\beta\gamma_2+\mu_3}x_0+$$

$$+\frac{\chi_1}{\lambda_2}\ln\frac{\mu_3\lambda_2\chi_2\left(\alpha\chi_3-(1+\ln\left(\alpha\chi_3\lambda_3\right))/\lambda_3)\right)}{\beta\gamma_2+\mu_3}x_0+$$

$$+\frac{\chi_1\chi_2}{\lambda_3}\ln(\alpha\chi_3\lambda_3)\left(1-\frac{\beta\gamma_2+\mu_3}{\mu_3\lambda_2\chi_2(\alpha\chi_3-(1+\ln\left(\alpha\chi_3\lambda_3\right)/\lambda_3)}\right)x_0,$$

and in the case

$$\frac{\mu_3\lambda_2\chi_2(\alpha\chi_3-(1+\ln\left(\alpha\chi_3\lambda_3\right))/\lambda_3)}{\beta\gamma_2+\mu_3}x_0\le 1$$

or $\alpha\chi_3\lambda_3\le 1$

$$A = 0.$$

By solving the reduced problem we can see that optimum level of managerial impact within the first link of logistics chain in cast if $A\lambda_1 / B > 1$ *and* $B > 0$ *is equal to*

$$e_1=\frac{1}{\lambda_1}ln\frac{A\lambda_1}{B};$$

and if $A\lambda_1 \le B$ *and* $B \ge 0$ *it is equal to*

$$e_1 = 0.$$

Finally, there is no practicable solution (the model is not specified correctly) if $B<0$ *or if simultaneously* $B = 0$ *and* $A>0$*, but that case can't emerge in real conditions.*

Conclusions. *The proposed model considers logistics chain as an aggregate of the elements (links) that are consecutively transited by a container during its delivery. The container is subject to managerial impacts that influence characteristics of cargo safety and delivery time which are considered for by the model. Managerial impacts are identified with relevant costs.*

Cargo features (cargo safety and delivery time) are changing, depending on managerial impacts within each link with consideration for external factors that are determined by probabilistic distribution, so on the whole the model is a probabilistic one.

The managerial impacts are selected in order to obtain the maximum value of objective function. The proposed model can consider different objective functions, but the described example uses weighted mean value of mathematical expectation of cargo safety and of delivery time as objective function. The article has studied a variant of logistics chain consisting of 4 links (forwarder, departure station, section of railway, destination station), where the problem of finding of optimum managerial impact is solved with exactitude, and explicit formulae of supposed plan are formulated.

Key words: containerization of cargo flows, logistics chain, mathematical model, costs for container company, probability of cargo safety.

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Координаты автора (contact information): Лёвин С.Б. (Lievin S.B.) – info@a-trans.ru.

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