SPATIAL OSCILLATION OF A RAIL FLAT-CAR

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ABSTRACT

The article describes design diagrams of four-axle rail flat car with two anti-symmetrically located heavy cargos with elastic dissipative elements (supports), whose common center of mass coincides with the center of mass of a flat car. The authors pose a problem of safety of a flat car's movement, caused by dislocation of the center of mass of cargo, its weight, speed of movement, certain parameters of elastic and dissipative supports, with regard to stability of the wheel against possible derailment and risk of transversal dumping of a car in rail track's curves due to transversal horizontal forces. It is the reason to study, first of all by theoretically, the dynamics of a car, to determine admissible limits of transversal and longitudinal dislocation of the center of mass of cargo with regard to the axis of symmetry of a car, and to find rational parameters of elastic and dissipative supports for cargo that ensure safety of traffic.

The developed design diagrams are based on the mechanical system «flat car – cargo» that includes 13 solid bodies (car frame, two cargos, four side frames, two bars over springs, four wheelsets of two bogies). The authors studied bouncing, rocking, rolling, side drifting, wobbling of a car and cargo, as well as bouncing, rocking, rolling of side frames of bogies and wobbling of bogies.

The authors propose algebraic expressions to determine deformation of spring sets and supports for cargo, deformation of the track with determined vertical and horizontal irregularities, reaction of the rail to the wheels' impact (if there are quenching forces of viscous friction and unilateral relation of wheels and rails), and vertical and horizontal forces influencing spring sets and elastic dissipative supports for cargo.

In order to develop differential equations the researchers used d'Alambert's principle. They developed a system of 23 differential equations, describing spatial oscillation of a flat car.

ENGLISH SUMMARY

Background. *Many of heavy-weight goods transported by flat cars have own springing or vertical and transversal horizontal elastic and dissipative supports based on a car frame, serving to reduce dynamic forces and to increase safety of delivered goods (cars, road and construction machines, car-mounted hoisting cranes).*

When cargo is located anti-symmetrically, the loading capacity of a car is used in a more efficient way and fewer cars are needed. But when cargo is located anti-symmetrically and with regard to some features of elastic and dissipative supports it is possible to come across a risk of oscillation amplitudes that can have negative impact on safety of a car's movement resulting in reduced stability against derailment and transversal dumping of a car when moving in curves of rail track.

Void specifications of loading [1] foresee rules of anti-symmetric location of two rigid heavy-weight cargos without bearing on a car's floor through elastic and dissipative supports, while their common center of mass should coincide with the center of mass of a car or be very close to it. But those standards can't be used for the cargo with elastic and dissipative elements linking it to the frame of a flat car.

There are studies dedicated to oscillation of freight cars, particularly for conditions of anti-symmetric location of heavy-weight cargo with dislocated center of its mass transversally or longitudinally (for one of those conditions taken separately) regarding the car. The studies [2,3] concern cargo rigidly bearing on the car's floor, and studies [4,5] regard cargo with elastic and dissipative elements.

Objectives*. In order to determine more specific parameters it is necessary to theoretically study dynamics of a flat car with anti-symmetrically located cargo, whose centers of mass are dislocated longitudinally or transversally along the car to different sides of its symmetry axis, and for conditions when common center of mass of the cargos of similar weight could coincide with the center of mass of a flat car or be dislocated during loading.*

Methods*. The researchers develop mathematical model (geometric design diagram, force design diagram, mechanical system of solid bodies, system of 23 differential equations based on d'Alambert's principle).*

Results*. In order to study spatial oscillation of a flat car with two anti-symmetrically located cargos, having similar weight and elastic and dissipative elements (supports) and whose common center of mass coincides with the center of mass of a flat car if the distance from centers of mass of respective cargos to the center of mass of a flat car is the same, the authors developed geometric design diagram (pic. 1) and force design diagrams (pic. 2, 3, 4 for respectively side, top and end view).*

The authors have studied a mechanical system «flat car – cargo» that includes 13 solid bodies (car frame, two cargos, four side frames, two bars over springs, four wheelsets of two bogies). The centers of mass of a platform, and of two cargos are located in points O, O1 and O2, the centers of mass of each cargo are dislocated from the center of mass of a flat car for respective distances X1 and X2 longitudinally, and for respective distances У1 and У2 transversally.

The generalized taken coordinates are as follow: q1 – bouncing of the frame of a flat car, q3 and q5 – bouncing of respectively first and second cargo, q2 – rocking of the frame of a flat car, q4 and q6 – rocking of respectively first and second cargo, q7 – rolling of the frame of a flat car, q8 and q9 – rolling of respectively first and second cargo, q10 – wobbling of the frame of a flat car, q11 and q12 – wobbling of of respectively first and second cargo, q13 и q15– bouncing of side frames of bogies, q14 and q16 – rocking of side frames of bogies, q17 and q18 – wobbling of bogies, q19 and q20 – side drifting of respectively first and second cargo, q21 – side drifting of the frame of a flat car, q22 and q23– rolling of side frames of bogies.

Relative vertical deformations of springing sets are equal to:

$\Delta_{кв}1 = q1 - q2L_в - q7\,B + q22\,B - q13,$

$\Delta_{кв}2 = q1 - q2L_в + q7\,B - q22\,B - q13,$

$\Delta_{кв}3 = q1 + q2L_в - q7\,B + q22\,B - q15,$

$\Delta_{кв}4 = q1 + q2L_в + q7\,B - q22\,B - q15.$

Relative horizontal deformation of springing sets are equal to:

$\Delta кг1 = q21 - q10\,Lв - (\eta г1,2 + \eta г3,4)$,
$\Delta кг2 = - q21 + q10\,Lв + (\eta г1,2 + \eta г3,4)$,
$\Delta кг3 = q21 + q10\,Lв - (\eta г5,6 + \eta г7,8)$,
$\Delta кг4 = - q21 - q10\,Lв + (\eta г5,6 + \eta г7,8)$,
where $\eta гi$ is an amplitude of horizontal irregularity of a track ($\eta = \eta 0\ Sin\ \omega t$).

Relative vertical deformations of elastic supports of cargo are determined by equations:

$\Delta гв1 = q3 - q1 + q4\,l1x - q2\,(X1 - l1x) - q8\,l1y + q7\,(Y1 + l1y)$,
$\Delta гв2 = q3 - q1 + q4\,l1x - q2\,(X1 + l1x) + q8l1y + q7\,(Y1 - l1y)$.
$\Delta гв3 = q3 - q1 - q4\,l1x - q2\,(X1 - l1x) - q8l1y + q7\,(Y1 + l1y)$,
$\Delta гв4 = q3 - q1 - q4l1x - q2\,(X1 - l1x) + q8l1y + q7\,(Y1 - l1y)$,
$\Delta гв5 = q5 - q1 + q6l2x + q2\,(X2 - l2x) - q9l2y - q7\,(Y2 - l2y)$,
$\Delta гв6 = q5 - q1 + q6l2x + q2\,(X2 - l2x) + q9l2y - q7\,(Y2 + l2y)$,
$\Delta гв7 = q5 - q1 - q6l2x + q2\,(X2 + l2x) - q9l2y - q7\,(Y2 - l2y)$,
$\Delta гв8 = q5 - q1 - q6l2x + q2\,(X2 + l2x) + q9l2y - q7\,(Y2 + l2y)$.

Relative horizontal deformations of elastic supports of cargo are determined by equations:

$\Delta гг1 = q19 - q21 + q11l1x - q10\,(X1 + l1x) + q8l1z$,
$\Delta гг2 = - q19 + q21 - q11l1x + q10\,(X1 + l1x) - q8l1z$,
$\Delta гг3 = q19 - q21 - q11l1x - q10\,(X1 - l1x) + q8l1z$,
$\Delta гг4 = - q19 + q21 + q11l1x + q10\,(X1 - l1x) - q8l1z$,
$\Delta гг5 = q20 - q21 + q12l2x - q10\,(X2 - l2x) - q9l2z$,
$\Delta гг6 = - q20 + q21 - q12l2x + q10\,(X2 - l2x) + q9l2z$,
$\Delta гг7 = q20 - q21 - q12l2x - q10\,(X2 + l2x) - q9l2z$,
$\Delta гг8 = - q20 + q21 + q12l2x + q10\,(X2 + l2x) + q9l2z$,
where l1z, l2z are heights of centers of mass of respectively first and second cargo over the center plate bearing of a flat car.

Vertical deformations of the track are determined by equations:

$\Delta R1 = q13 + q22\,S - \eta 1$, $\Delta R2 = q13 - q22\,S - \eta 2$, $\Delta R3 = q13 + q22S - \eta 3$,
$\Delta R4 = q13 - q22\,S - \eta 4$, $\Delta 5 = q15 + q23 - \eta 5$, $\Delta R6 = q15 - q23\,S - \eta 6$,
$\Delta R7 = q15 + q23\,S - \eta 7$, $\Delta R8 = q15 + q23\,S - \eta 8$.

Vertical forces, influencing elastic and dissipative supports of transported goods, are equal to: $Rzi = Czi\,\Delta гвi + \beta гi$, $(I = 1–8)$,
where Czi is vertical rigidity of i^th elastic and dissipative element of cargo,

$\Delta гвi$ are relative vertical dislocations of i^th elastic and dissipative element of cargo,

$\beta вi$ is an equivalent ratio of viscous friction of i^th elastic and dissipative element of cargo in vertical plane.

Transversal horizontal forces in elastic and dissipative elements between the cargo and the frame of a flat car are equal to:

$Rym = Cym\,\Delta гг\,m + \beta гm$, $(m = 1–8)$,
where Cym is a transversal horizontal rigidity of m^th elastic and dissipative element of cargo,

$\Delta ггm$ are relative transversal dislocations of elastic and dissipative element of cargo,

$\beta гm$ is an equivalent ration of viscous friction m^th elastic and dissipative element of cargo in transversal plane.

Vertical forces influencing springing sets of a flat car are equal to:

$Rтвi = Cz\,\Delta твi + \beta твi$, $(I = 1–4)$,
where Cz is a rigidity of i^th springing set in vertical plane,

$\Delta твi$ are relative vertical dislocations (deflection) of i^th springing set of a flat car,

$\beta твi$ is an equivalent viscous friction of i^th springing set in vertical plane.

Transversal horizontal forces influencing springing sets of a flat car are equal to:

$Rтгm = Cтгm\,\Delta тгm + \beta тгm\ тгm$, $(m = 1–4)$,
where Стгm is a transversal horizontal rigidity of m^th springing set of a flat car,

$\Delta тгm$ are relative transversal dislocations of springing set,

$\beta тгm$ is an equivalent viscous friction of m^th springing set in horizontal transversal plane.

The moment of friction forces in center plates which emerge during bogies' turning in horizontal plane are determined by expression:

$Mтр\ 1,2 = \mu п$,
where d is a diameter of central plate,

$\mu п$ is a ratio of friction between central plate of a flat car and bearing of a central plate of a bogie.

Vertical reactions of springing sets are determined by expression:

$Pi = Cтв\,(f - \Delta\ твi) + Fтвi\ Sign$, $(I = 1–4)$, $(f + \Delta твi) > 0$,

$Pi = 0\ (f + \Delta твi) < 0$,
where $\Delta твi$ are vertical deformations of springing sets of bogies,

f is static deflection of a springing set of bogies,

Cтв is a vertical rigidity of a springing set,

Fтвi is a force of a dry friction.

In order to determine horizontal transversal reaction of springing sets it is necessary to replace Cтв in above expression by Cтг which is horizontal transversal rigidity of a springing set.

Vertical (Riв) and horizontal (Riг) reactions of a rail to wheels' impact on the rail in case of presence of quenching forces of viscous friction in the track and of unilateral link between the rail and the wheel are equal to:

$Riв,\ г = C\,(x)\,[f\,(x) + \Delta i] + Fп\ Sign\ \Delta i$, $(i = 1–8)$, при $[(f\,(x) + \Delta i] \geq 0$ и

$Ri = 0\ [f\,(x) + \Delta i] < 0$,
where C (x) is a vertical rigidity of the track under the wheels of a flat car,

f (x) is a static deflection of the track under the wheels,

Δi is a dynamic deformation of the track in vertical plane,

Fп is a force of dry friction equivalent to static deformation of the track under the wheels.

The value of the forces of viscous friction between the rail and the wheel is determined by expression, developed in the study [6]:

$Ti = Sig$, $(i = 1–8)$,
where μ is a ratio of friction between the wheel and the rail.

The value of horizontal projections of the rails is equal to $Ni = Ri\ tg\ \alpha i$,
where αi is an angle of inclination of full reaction force of the track from the vertical line, determined by linear interpolation on the basis of the table of trajectory of the point of contact between wheel and rail, which depends on a value of side dislocation of a wheel regarding a rail head [6].

The following system of 23 differential equations, based on the mathematical expressions above, describes spatial oscillations of a flat car with two anti-symmetrically located heavy-weight cargos with elastic and dissipative elements, linking them to a flat car:

$M - (Rz1 + Rz2 + Rz3 + Rz4) - (R + R + R + R) + P1 + P2 + P3 + P4 - Mg = 0,$

$Jx + (P3 + P4 - P1 - P2)\,L_в + (R1z1 + R1z2)(x2 - l2x) + (R1z3 + R1z4)(x2 + l2x) -$

$-(Rz1 + Rz2)(x1 + l1x) - (Rz3 + Rz4)(x1 - l1x) = 0,$

$m1 = Rz1 + Rz2 + Rz3 + Rz4 - m1\,g = 0,$

$J1x + (Rz1 + Rz2)\,l1x - (Rz3 + Rz4)\,l1x = 0,$

$m2 + R1z1 + R1z2 + R1z3 + R14 - m2\,g = 0,$

$J2x + (R1z1 + R1z2)\,l2x - (R1z3 + R1z4)\,l2x = 0,$

$Jz - (P2 + P4)\,B + (p1 + P3)\,B - (Rz1 + Rz3)(У1 + l1y) + (Rz2 + Rz4)(У1 - l1y) - -(R1z1 + R1z3)(У2 - l2y) + (R1z2 + R1z4)(У2 + l2y) = 0,$

$J1z - (Rz2 + Rz4)\,l1y + (Rz1 + Rz3)\,l1y + (Ry1 + Ry3)\,l1z - (Ry2 + Ry4)\,l1z + +Q_в1h_в1 = 0,$

$J2z - (R1z2 + R1z4)\,l2y + (R1z1 + R1z3)\,l2y + (R1y1 + R1y3)\,l2z - (R1y2 + +R1y4)\,l2z + Q_в2h_в2 = 0,$

$Jy - (Ry1 - Ry2)(x1 + l1x) - (Ry3 - Ry4)(x1 - l1x) + (R1y4 - R1y3)(x2 + l2x) +$

$+ (R1y2 - R1y1)(x2 - l2x) - (N2 - N1)(L_в + l_т) - (N4 - N3)(L_в - l_т) + (N6 - N5)\,x$

$x\,(L_в - l_т) + (N8 - N7)(L_в + l_т) = 0,$

$J1y + (Ry1 + Ry4)\,l1x - (Ry2 + Ry3)\,l1x = 0,$

$J2y + (R1y1 + R1y4)\,l2x - (R1y2 + R1y3)\,l2x = 0,$

$m_т + R1 + R2 + R3 + R4 - P1 - P2 - m_т\,g = 0,$

$J_{тх} + (R3 + R4)\,l_т - (R1 + R2)\,l_т - (-)\,l_{рк} - (-)\,l_{рк} = 0,$

$m_т + R5 + R6 + R7 + R8 - P3 - P4 - m_т\,g = 0,$

$J_{тч} + (R7 + R8)\,l_т - (R5 + R6)\,l_т - (-)\,l_{рк} - (-)\,l_{рк} = 0,$

$J_{ту} + (T1 + T3)\,S - (T2 - T4)\,S + (N2 + N3)\,l_т + M_{тр}1\,Sign = 0,$

$J_{ту} + (T5 + T7)\,S + (N6 + N7)\,l_т - (N5 + N3)\,l_т + M_{тр}2Sign = 0,$

$m1 + Ry1 + Ry3 - Ry2 - Ry4 - Q_в1 = 0,$

$m2 + + - - - Q_в2 = 0,$

$M - Ry1 - Ry3 + Ry2 + Ry4 - R1y1 - R1y3 + R1y2 + R1y4 + (N1 + N3 + N5 + N7) -$

$-(N2 + N4 + N6 + N8) - Q_в = 0,$

$J_{zт} + (P4 - P3)\,B + (R5 + R7)\,S - (R6 + R8)\,S - (N1 + N3 - N5 - N7) = 0,$

$J_{zт} + (P2 - P1)\,S + (R1 + R3)\,S - (R2 - R4)\,S - (N2 + N4 - N1 - N3) = 0,$

where qi are generalized coordinates,

M is a mass of the sprung elements of a flat car (frame, bars over the springs of the bogies), m1, m2 is a mass respectively of the first and the second cargo,

mт is a mass of not sprung elements of the bogies,

Jx, Jy, Jz are the moments of inertia of sprung elements of a flat car regarding respectively axis X, У and Z,

J1x, J1y, J1z are the moments of inertia of the first cargo regarding axis X, У, Z,

J2x, J2y, J2z are the moments of inertia of the second cargo regarding respectively axis X, У, Z,

X1, X2, Y1, Y2 are the distances between the centers of mass of respectively first and second cargos,

2l1x, 2l1y, 2l1z, 2l2x, 2l2y, 2l2z are geometric dimensions of respectively first and second cargos,

2Lв is the length of a base of a flat car,

2lт is a length of a base of bogie,

2B is a distance between centers of springing sets of bogies,

2S is a distance between the circles of driving of the wheels of a wheelset,

Qв is a force of the wind that has an impact on the frame of a flat car,

Qв1, Qв2 are forces of the wind, that has an impact in the side surface of cargos,

Ri are reactions influencing the wheels from the part of the track,

Ti are longitudinal forces when wheels are slipping along the rails.

Conclusions. *The developed design diagrams and differential equations (mathematical model), describing spatial oscillations of a flat car with two anti-symmetrically located heavy-weight cargos with elastic and dissipative elements of a link to the frame of a flat car, allow studying dynamic features of four-axle rail car, as assessing safety concerning risks of derailment and transversal dumping in the curves, as well as determining of limits of acceptable transversal and longitudinal dislocations of the centers of mass of the cargo regarding symmetry axis of a car and depending on its weight and traffic speed.*

The solution of the described system of differential equations can be achieved, for instance, by well-known and efficient iteration-difference method of numerical integration with automatic choice of integration step, developed by professor V. Hussainov, using modern personal computers.

Key words: *transport, flat-car, anti-symmetrically located heavy-weight cargo, spatial oscillation, design diagrams, differential equation, mathematical model.*

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