



# Model of Vehicle Motion Along a Curved Trajectory Taking into Account Rolling Friction



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## ABSTRACT

This article is devoted to the problem of adequate mathematical description of the process of vehicle movement along a curved trajectory considering the impact of the correlation of the dynamic and kinematic parameters of this process. The importance of considering the impact of forces of resistance to vehicle movement in the direction of rolling, when moving along a curved trajectory, on the minimum permissible radius of curvature at a given speed or on the maximum possible speed of the vehicle for a given radius of curvature of its trajectory is emphasised.

At present, when a vehicle moves along a curved trajectory, the minimum radius of curvature at a given speed or the maximum permissible speed for a given radius of curvature of the trajectory are determined without taking into account the resistance forces to movement in the direction of rolling. This leads to an unjustified overestimation of the maximum permissible speed of the vehicle when moving along a curved trajectory with a given radius of curvature, which, in

turn, when creating software for unmanned vehicles, can lead to an increase in accidents.

The article proposes an innovative solution to this problem by taking into account the influence of rolling friction and resistance moments in the elements of the vehicle transmission on the magnitude of the centripetal force acting on the vehicle when moving along a curved trajectory.

The objective of the study is to develop a mathematical model of vehicle movement along a curved trajectory taking into account the influence of rolling friction and resistance forces in the vehicle transmission on the maximum possible vehicle speed at a given radius of curvature of the trajectory and on the minimum possible radius of curvature of the trajectory at a given vehicle speed.

It has been established that the force of resistance to vehicle movement in the rolling direction reduces the magnitude of the centripetal force ensuring vehicle movement along a curved trajectory of a certain curvature by an amount equal to the force of resistance to movement in the rolling direction of the vehicle.

**Keywords:** rolling friction, rotational resistance force, minimum radius of curvature, maximum permissible speed, centripetal force, coefficient of sliding friction or adhesion of the wheel to the road surface.

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## BACKGROUND

The issue of vehicle movement along a curved trajectory has been the subject of many studies, both theoretical and experimental. The approaches and methods used to solve this problem are quite diverse. Also, the depth of study of various details of the wheel-ground interaction process varies in different studies.

In most sources, vehicle movement along a curved trajectory is considered as the movement of a material point along a circle under the action of a single centripetal force caused by sliding friction or wheel adhesion to the road surface:

$$F_c = F_{tr} = fN = fmg,$$

where  $f$  – coefficient of sliding friction or adhesion of a wheel to the road surface;

$N$  – force of normal pressure of road surface on wheel, N;

$g$  – acceleration of gravity,  $m/s^2$ ;

$m$  – vehicle weight, kg.

The minimum allowable turning radius is determined from the equation

$$R_{min} = \frac{v^2}{fg}, \quad (1)$$

where  $v$  – speed of a car, m/s.

This model describes the movement of a wheel along a circle of radius  $R$ , provided that the coefficient of rolling friction, or more precisely, the coefficient of resistance to the movement of the wheel in the direction of its rolling is zero. In fact, this is not the case. The coefficient of resistance to the movement of the wheel in the direction of its rolling is always different from zero, and the greater is it, the greater is the minimum turning radius at a given vehicle speed. If the coefficient of resistance to the movement of the wheel in the direction of its rolling and the coefficient of sliding friction are equal, regardless of the angle of rotation of the wheel, it will continue to move in a straight line. This happens on a slippery road due to a decrease in the coefficient of sliding friction, which is reflected in formula (1). But what will happen with an increase in the coefficient of rolling friction or with an increase in the moment of resistance to wheel rotation is not reflected in dependence (1).

Quite a few works are devoted to the study of the laws of movement of a vehicle along a curved trajectory. In the work [1], «the reaction forces of the soil are considered as functions of the coordinates of the instantaneous sliding of the contact patch, which allows us to take into account the mode of movement of the wheel

(leading, driven or braking), the elastic properties of the pneumatic tire and the deformation of the soil, introduce different sizes, the shape of the contact and the magnitude of the normal load on the wheels». In the same work, it is stated that «the components of the friction force are related to each other by the ratio  $P = \sqrt{P_x^2 + P_y^2}$ .

Therefore, if the lateral component reaches the adhesion limit  $P_{ymax} = \varphi N$ , then the longitudinal component must be equal to zero  $P_x = 0$ , which contradicts the very possibility of movement. Consequently, the maximum value of the lateral force is always less than the adhesion limit  $P_y < \varphi N$  [1].

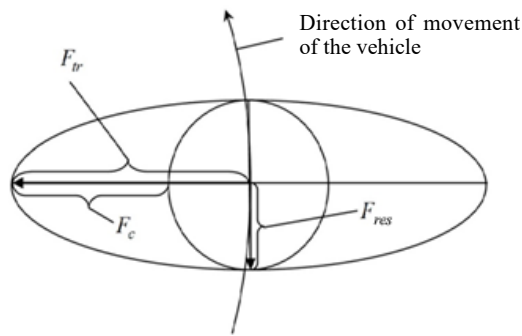
The paper [2] studies the process of curvilinear motion of a single wheel on the ground. The article presents new dependencies and calculation schemes «for determining the force factors and track depth depending on the position of the wheel turning pole». It is very interesting that the authors assume that «the soil is a continuous homogeneous plastic medium with a horizontal surface». However, it is worth noting that the authors then make the assumption that «the pressure on the side surface of the wheel from the soil at a given contact point is directly proportional to the soil deformation» [2].

That is, for plastic soil the authors applied Hooke's law, which, as is known, applies only to absolutely elastic bodies. This would be valid for the flanges of train wheels and steel rails, but for plastic soil Hooke's law is unacceptable. Plastic soil has nonlinear properties and does not obey Hooke's law. Therefore, it is necessary to take these features into account and apply appropriate models to analyse the process of curvilinear movement of a wheel on plastic soil.

There are quite a lot of pseudo-scientific descriptions of the process of curvilinear motion of a vehicle in open sources. One of these sources presents an interesting approach to explaining the process. According to this approach, «each car has mass and inertia; when entering a turn, its weight, one might say, «continues to move» in a straight line, that is, the centre of mass of the car tends to go beyond the turning circle with a force determined by its acceleration (i.e. speed). The vector of this force is directed perpendicular to the turning radius»<sup>1</sup>.

Or «during the movement of a car along a circle, two forces arise: centripetal, which keeps

<sup>1</sup> How does a car turn? [Electronic resource]: <https://avtonauka.ru/electronika/kak-avtomobil-povorachaet.html>. Last accessed 27.03.2024.



Pic. 1. Diagram of forces acting on a vehicle when moving along a curved trajectory [performed by the authors].

the car on the circle, and centrifugal, which tends to throw it away from the center of rotation»<sup>2</sup>.

In the work [3], the authors developed a mathematical model of the movement of a car along a circle. Systems of equations were compiled that describe the movement of a car on a flat horizontal surface. The process of entering a turn was also modelled and a comparative analysis of the obtained calculated dependencies with experimental data was carried out.

A large number of studies are devoted to the development of mathematical models and the assessment of the degree of controllability and stability of a car when driving through a bend, determining critical driving modes and maximum speeds of controlled movement for a given radius of curvature of the trajectory of the vehicle [4–10].

However, there is no precise quantitative estimate of the influence of the coefficient of resistance to the movement of the wheel in the direction of its rolling on the value of the minimum permissible turning radius of the vehicle at a given speed or of the inverse problem. This is an open problem and an interesting area for research.

The objective of this work is to develop a mathematical model of vehicle motion along a curved trajectory taking into account the influence of rolling friction and resistance forces in the vehicle transmission on the maximum possible vehicle speed at a given radius of curvature of the trajectory and on the minimum possible radius of curvature of the trajectory at a given vehicle speed.

The relevance of this study is due to the need to create software for unmanned vehicles

that are becoming increasingly widespread [11–15].

In turn, an integral part of the software are decision-making algorithms based on mathematical models that adequately reflect various nuances of the vehicle movement process and convert signals from sensors into commands for vehicle systems and controls [16–19].

## METHODS AND ASSUMPTIONS

Let us consider the motion of a one-wheeled vehicle in order to consider it a material point and reasonably apply the laws of theoretical mechanics. The transition to two-wheeled and four-wheeled vehicles, taking into account the length of their wheelbase, the location of the centre of gravity, the type of drive, the influence of the driving force (torque), etc., will be made in further studies.

Let us consider the rectilinear motion of a wheel in the absence of a driving force.

The wheel is acted upon from the road surface by: the force of rolling friction

$$F_k = \frac{\lambda}{r} N$$

and the force of resistance to rotation on the part of the transmission (friction in shaft bearings, gear pairs, etc.)

$$F_{\text{tran}} = \frac{M_{tr}}{r},$$

where  $\lambda$  – coefficient of rolling friction, m;

$r$  – wheel's radius, m;

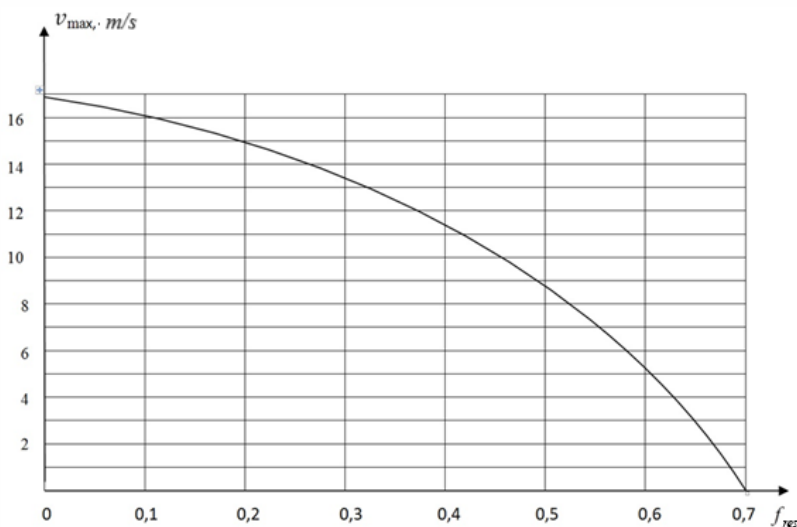
$N = -mg$  – the force of normal pressure of the road surface on the wheel;

$M_{tr}$  – total friction moment in the transmission elements transmitted to the wheel.

Thus, during rectilinear movement of a vehicle, each of its wheels is subject to a force of resistance to movement equal to the sum of

<sup>2</sup> Car turns. [Electronic resource]: <https://ustroistvo-avtomobilya.ru/bez-rubriki/povoroty-avtomobilya/>. Last accessed 27.03.2024.





Pic. 2. Dependence of the maximum permissible speed of a vehicle along a curved trajectory on the coefficient of resistance to movement [performed by the authors].

the rolling friction force of the wheel and the friction forces in the transmission elements of the wheel:

$$F_{res} = F_k + F_{tran} = \frac{\lambda}{r} N + \frac{M_{tr}}{r}.$$

Then the coefficient of resistance to the movement of the vehicle wheel in the direction of its rotation will be equal to:

$$f_{res} = \frac{\lambda}{r} + \frac{M_{tr}}{rN}.$$

**RESULTS**

Taking into account the coefficient of resistance to the movement of the vehicle wheel in the direction of its rotation, the authors obtained the dependence of the minimum permissible turning radius of the vehicle on its speed:

$$R_{min} = \frac{v^2}{(f - f_{res})g} = \frac{v^2}{\left[ f - \left( \frac{\lambda}{r} + \frac{M_{tr}}{rN} \right) \right] g}. \tag{2}$$

Graphically, the denominator of the right side of formula (2), in the general case, is a friction ellipse (Pic. 1).

In the case, where  $f_{res} = 0$ , we get a dependence (1).

In the case, where  $f = f_{res}$  the denominator in (2) is zero, we get an uncontrolled vehicle, i.e. there is no minimum turning radius.

Maximum speed at a given turning radius:

$$v_{max} = \sqrt{Rg \left[ f - \left( \frac{\lambda}{r} + \frac{M_{tr}}{rN} \right) \right]}. \tag{3}$$

The authors have established that the centripetal force that ensures the movement of

a vehicle along a circle of radius  $R$  is equal to the arithmetic difference between the force of sliding friction of the wheel and the force of resistance to the movement of the wheel in the direction of its rotation:

$$F_c = F_{tr} - F_{res} = fN - \frac{\lambda}{r} N - \frac{M_{tr}}{r}. \tag{4}$$

In Pic. 1, the force  $F_c$  is equal to the difference in the lengths of the semi-axes of the friction ellipse.

Pic. 2 shows the dependence obtained by the authors of the maximum permissible speed of a vehicle  $v_{max}$  along a curved trajectory of radius  $R = 40\text{ m}$  on the coefficient of resistance to movement  $f_{res}$  with a coefficient of sliding friction or adhesion of a wheel to a dry asphalt surface  $f = 0,7^3$ .

**CONCLUSIONS**

The analysis of the obtained results shows that the magnitude of the force of resistance to the movement of the wheel in the direction of its rotation has a significant effect on the magnitude of the maximum permissible speed at a given radius of curvature of the trajectory of the vehicle.

The greater is the magnitude of the force of resistance to the movement of the wheel in the direction of its rotation, the smaller is the magnitude of the centripetal force that ensures the

<sup>3</sup> Coefficients of static and sliding friction: reference information. [Electronic resource]; [https://alexfl.pro/inform/inform\\_stali5.html](https://alexfl.pro/inform/inform_stali5.html). Last accessed 03.06.2024.

movement of the vehicle along a curved trajectory, and the lower is the permissible speed of the vehicle at a given radius of curvature of the trajectory of its movement.

The greater is the magnitude of the force of resistance to the movement of the wheel in the direction of its rotation, the greater is the minimum permissible radius of curvature of this trajectory at a given speed of the vehicle.

## REFERENCES

1. Noskov, N. K., Troyanovskaya, I. P., Titov, S. A. Mathematical model of force interactions wheel ground when the machine turns. *Bulletin of the South Ural State University series Mechanical Engineering Industry*, 2017, Vol. 17, Iss. 3, pp. 5–15. DOI: 10.14529/engin170301.
2. Shukhman, S. B., Soloviev, V. I. Mathematical model of curvilinear motion of a wheel on the ground [Matematicheskaya model krivolineinogo dvizheniya koleasa po gruntu]. *News of higher educational institutions. Transport and power engineering*, 2012, Iss. 8, pp. 24–31. EDN: PBJFUJ.
3. Fedotov, A. I., Tikhov-Tinnikov, D. A., Bykov, A. V., Lysenko, A. V. Mathematical model of vehicle circular movement. *Proceedings of ISTU*, 2017, Vol. 21, Iss. 2, pp. 199–207. DOI: 10.21285/1814-3520-2017-2-199-207.
4. Tarasik, V. P., Puzanova, O. V. Methodology for assessing the controllability and stability of a car when driving on a bend [Metodika otsenki upravlyaemosti i ustoychivosti avtomobilya pri dvizhenii na virazhe]. *Bulletin of Belarusian-Russian University. Mechanical Engineering*, 2019, Iss. 3 (64), pp. 93–103. DOI: 10.53078/20778481\_2019\_3\_93.
5. Tarasik, V. P. Methodology for Simulation Modelling of the Test Mode for Controllability and Stability of a Vehicle When Entering a Turn [Metodika imitatsionnogo modelirovaniya rezhima ispytaniya na upravlyaemost i ustoychivost avtomobilya pri vkhode v povorot]. *Bulletin of Belarusian-Russian University. Mechanical Engineering*, 2019, Iss. 2 (63), pp. 44–53. DOI: 10.53078/20778481\_2019\_2\_44.
6. Lyubimov, I. I., Builov, Yu. A. Study of the relationship between vehicle wheel slip and suspension stiffness [Issledovanie svyazi uvoda koleasa avtomobilya s zhestkostyu podveski]. *Vestnik SSTU*, 2013, Vol. 2, Iss. 1 (70), pp. 192–195. EDN: REAWZV
7. Baulina, E. E., Dementyev, Yu. V., Iturralde, P., Kislov, A. A. On the issue of studying the stability and controllability of a hybrid vehicle with a drive type that changes during movement [K voprosu issledovaniya ustoychivosti i upravlyaemosti gibridnogo avtomobilya s izmenyaemym v protsesse dvizheniya tipom privoda]. *Bulletin of the MSTU «MAMI»*, 2012, Vol. 1, Iss. 2 (14), pp. 29–38. EDN: PUWPTD.
8. Butin, D. A., Tumasov, A. V., Vashurin, A. S., Anikin, A. A., Palutin, Yu. I. Study of rolling modes of pneumatic tires during controllability and stability tests [Issledovanie rezhimov kacheniya pnevmaticheskoi shiny pri ispytaniyakh upravlyaemosti i ustoychivosti]. *Proceedings of NSTU named after R. E. Alekseev, Nizhny Novgorod*, 2018, Iss. 3 (122), pp. 107–110. DOI: 10.46960/1816-210X\_2018\_3\_107.
9. Kravets, V. N., Musarskiy, R. A. Study of wheel slip during vehicle turning [Issledovanie skolzheniya koleasa pri povorote avtomobilya]. *News of higher educational institutions. Transport and power engineering*, 2014, Iss. 6 (651), pp. 35–38. EDN: SEZQFF.
10. Dyachuk, M. V., Petrenko, D. I. Simulation of car steerability. *Visnyk of the Pridneprovsk state academy of civil engineering and architecture*, 2009, Iss. 12 (141), pp. 29–37.
11. Chikrin, D. E. Logic and structure of the construction of the control system of unmanned vehicles [Logika i struktura postroeniya sistemy upravleniya bespilotnykh transportnykh sredstv]. *Bulletin of Samara Scientific Center of the Russian Academy of Sciences. Computer Science, Computer Engineering and Control*, 2021, Vol. 23, Iss. 4, pp. 96–102. [Electronic resource]: [http://www.ssc.smr.ru/media/journals/izvestia/2021/2021\\_4\\_96\\_102.pdf](http://www.ssc.smr.ru/media/journals/izvestia/2021/2021_4_96_102.pdf). Last accessed 27.05.2024.
12. Zezyulin, D. V., Tyugin, D. Yu., Tumasov, A. V., Groshev, A. M. [et al]. Development of a system for unmanned control of the movement of a vehicle with an electric drive [Razrabotka sistemy bespilotnogo upravleniya dvizheniem transportnogo sredstva s elektroprivodom]. *Proceedings of NSTU named after R. E. Alekseev, Nizhny Novgorod*, 2018, Iss. 1 (120), pp. 165–174. DOI: 10.46960/1816-210X\_2018\_1\_165.
13. Kuznetsova, M. V., Veremeenko, E. G. Prospects for the implementation of unmanned control of motor transportation [Perspektivy vnedreniya bespilotnogo upravleniya avtomobilnymi perevozkami]. *Young researcher of the Don – electronic journal of DSTU*, 2018, Iss. 5 (14), pp. 67–72. EDN: YHKGYP.
14. Yuzaeva, A. G., Kukartsev, V. V. Unmanned vehicles: dangers and development prospects [Bespilotnie avtomobili: opasnosti i perspektivy razvitiya]. *Actual problems of aviation and cosmonautics*, 2016, Vol. 2, Iss. 12, pp. 120–122. EDN: XRYBJP.
15. Dikmen, M., Burns, C. M. Autonomous Driving in the Real World: Experiences with Tesla Autopilot and Summon. Automotive UI 16: *Proceedings of the 8th International Conference on Automotive User Interfaces and Interactive Vehicular Applications*. [New York]: [Association for Computing Machinery], 2016, pp. 225–228. DOI: 10.1145/3003715.3005465.
16. Herrmann, A., Brenner, W., Stadler, R. Autonomous Driving: How the Driverless Revolution Will Change the World. Bingley: Emerald Publishing, 2018, xiii, 445 p. ISBN: 978-1-78714-834-5.
17. Leong, W. L., Martinel, N., Huang, S., Micheloni, C., Foresti, G. L., Teo, R. S. H. An Intelligent Auto-Organizing Aerial Robotic Sensor Network System for Urban Surveillance. *Journal of Intelligent and Robotic Systems: Theory and Applications*, 2021, Vol. 102, Iss. 2, Art. 33. DOI: 10.1007/s10846-021-01398-y.
18. Akbari, Y., Almaadeed, N., Almaadeed, S., Elharrouss, O. Applications, databases and open computer vision research from drone videos and images: a survey. *Artificial Intelligence Review*, 2021, Vol. 54, Iss. 5, pp. 3887–3938. DOI: 10.1007/s10462-020-09943-1.
19. Litman, T. Autonomous Vehicle Implementation Predictions Implications for Transport Planning. 2021. [Electronic resource]: <https://vtpi.org/avip.pdf>. Last accessed 24.05.2024. ●

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