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Methodology for Determining the Rational Order of Using Shunting Locomotives at a Passenger Station

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ABSTRACT

Railway transport is one of the most important modes of passenger transportation in the Russian Federation. In 2023, it accounted for more than 35 % of all travellers. At the same time, long-distance transportation accounts for almost 37 % of the total volume of transportation by rail. The analysis of the dynamics of changes in passenger turnover and transportation volumes suggests that the market has not only recovered from the unprecedented decline of 2020 but continues to grow steadily. The growth in passenger transportation volumes entails an increase in the volume of work at passenger and passenger technical stations. The load on station complexes increases (especially during periods of mass transportation). Carriers are constantly working on new transportation products that are distinguished by speed and level of comfort. For example, today, JSC FPC alone offers its customers about 90 combinations of offers that differ not only in the category of train, type of car, but also in the range of services offered during the travel. In this case, obviously, the key requirement, regardless of the class, is the high quality of preparation of trains for the trip and the coordinated work of all passenger departments.

Currently, most studies of the passenger infrastructure of railway transport are devoted to the study of its individual objects, for example, to the design and operation features of transfer hubs, the technology of passenger and passenger technical stations. At the same time, little attention is paid to the integrated functioning of the «passenger station – passenger technical station – transport interchange hub as of a single system. The article proposes a mathematical formalisation of the process of rational sequence of servicing passenger trains using linear programming methods, which allows solving a number of operational problems and conducting research to assess the level of influence of various factors on the performance of passenger and passenger technical stations.

Keywords: *transport interchange hub, passenger station, passenger technical station, occupancy of receiving and departure tracks, load of shunting locomotive, regularity of station operation.*

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INTRODUCTION

The functioning of the complex «passenger station – passenger technical station – transport interchange hub (hereinafter – TIH)» as a single system is based, first, on the technology of servicing passenger trains [1–5]. The procedure for servicing passenger trains at stations is fundamentally different from that of cargo trains.

Thus, with a certain load of «station service devices» and uneven arrival of cargo trains for servicing, a queue appears, depending on the «capacity» of the arrangements that ensure the transition of cargo cars from one service system to the next one. The time spent by cargo cars (cargo trains) at the station, unlike passenger cars, depends on the «capacity» of the service arrangements, the volume of car flows and is often not tied to specific schedule threads.

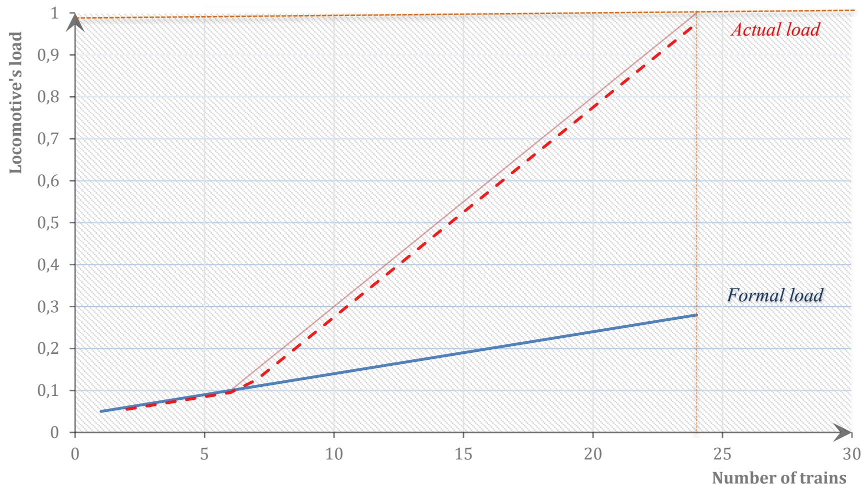
In passenger traffic, based on the turnover schedule, trains are tied to certain threads of the train schedule for departure and their servicing is subordinated to the main goal – departure of a ready train with passengers at a fixed time, according to the schedule. Under these conditions, the load of servicing arrangements (shunting locomotives, receiving and departure tracks, equipment tracks, etc.) should not be formally determined as a share of the time of the work directly performed in relation to the time under consideration (day or the value of the intensive period of work). The preparation and delivery of trains for departure according to the schedule will assume certain intervals between the end of operations with one train and the beginning of work with another. If these intervals are of such a duration that they cannot be used for the locomotive to perform other operation, they should also be attributed to the work time, increasing the load of locomotives [6].

Then, with a fixed departure schedule, for example, of long-distance passenger trains and considering the actual volume of operations of station arrangements, the workload of the shunting locomotive will represent a certain dependence on the number of trains dispatched over a certain period of time (Pic. 1).

The analysis of the operation of shunting locomotives at passenger stations showed [7]:

– There are periods in their work (the period of intensive arrival and departure of trains, the period before and after technical maintenance (hereinafter referred to as TM) of the locomotive, operations to change locomotive crews), when the locomotive load is practically equal to one. Thus, at Moscow-passenger Kazanskaya station, the first locomotive with an average daily load of 0,72 has two periods of 3,5 hours, when there are not even small breaks in its work. The second locomotive with an average daily load of 0,65 also has two such periods lasting six and five hours. Moreover, there are practically no reserves for increasing the volume of work of locomotives.

– In order to reduce the locomotive load during such periods, the practices are to «prematurely» withdraw trains to the receiving and departure tracks (hereinafter referred to as RDT) or later withdraw trains from RDT for subsequent work with them in accordance with the technological process (along the technological



Pic. 1. General nature of changes in the load of a shunting locomotive depending on the number of passenger trains served at the station [performed by the authors].

line). Which, in turn, leads to an increase in the duration of RDT occupancy.

In practice, it is usually necessary to answer two questions at the same time:

1. Will the shunting locomotive cope with the given volume of work?

2. Is it possible to implement the «premature» withdrawal of trains to the receiving and departure tracks and/or the withdrawal of trains from RDT with a delay relative to the technological time, reducing the unproductive downtime of the shunting locomotive when performing a given volume of work, based on the existing number of RDTs and the arrival/departure schedule of passenger trains (the given number of passenger trains)?

The answer to these questions is actually a solution to the problem of checking the sustainability of shunting locomotives with a given volume of work, technical and technological capabilities of the station and a fixed schedule of arrival and departure of passenger trains to the station.

Let's consider the conditions for a shunting locomotive could perform a given volume of work.

Formally, the average daily load of a shunting locomotive can be determined by formula (1):

$$\psi_{loc} = \frac{\sum_{i=1}^{n} t_i^{oper}}{(1440 - T_{break}) \cdot K_r \cdot \alpha_{host}}, \qquad (1)$$

where

t_i^{oper} – operating time of a locomotive with the *i*-th train number, which is defined on the time axis, subject to the increase of technologically justified times for the start of operation with it (based on the arrival and departure times of passenger trains and mail and baggage trains, as well as the start times of TM operations and crew rotation);

n – the total number of trains arriving and departing per day, with which the shunting locomotive works (moving trains between RDT and the parking/equipment/repair/… tracks);

T_{break} –breaks in the operating time of a shunting locomotive (the time that the locomotive is not in the working yard during the day);

K_r – coefficient that considers possible interruptions in shunting operations due to failures of technical equipment (infrastructure reliability coefficient)[1];

α_{host} – the coefficient of hostility of movements, which considers the duration of operations that cause interruptions in the performance of shunting work, within the total duration of the day.

In cargo traffic, a queue of trains is formed to perform operations. It can be larger or smaller depending on the period of the day and the intensity of work with trains. In any case, when the locomotive load is less than 1,0 (and in intensive periods it can be more than 1,0), the locomotive on average performs a given volume per day, only increasing the average time of passage of trains along the technological line due to queues [8; 9].

In passenger traffic, trains are tied to arrival and departure times, and in all cases these schedules must be respected. Therefore, the condition $\psi_{loc} \leq 1$ is necessary (hereinafter – NC), but not sufficient.

Let us determine the sufficient conditions (hereinafter – SC) for performing operations with passenger trains for a shunting locomotive, subject to a given fixed schedule for performing operations.

Let be defined:

$t_i^{\text{start oper.}}$ – technologically justified times for the start of operations of a shunting locomotive with passenger, mail and baggage trains upon arrival and departure (based on the arrival and departure schedule of these trains);

i – the sequential number of the operation of a shunting locomotive with trains, determined in ascending order of times located on the time axis.

Then the daily time interval will be divided into (n + 1) intervals, the boundaries of which will be the adjacent start times of operations on the time axis, including the start and end times of the day.

Let us group the start times of the shunting locomotive operations with trains into separate non-overlapping calculation time periods T_{per}^{cal} (Pic. 2), the boundaries of which will be:

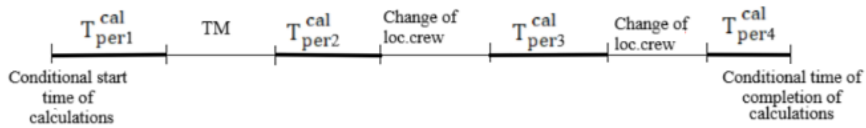
– start and end times of the total (cumulative) period – day;

– start and end times of periods when a shunting locomotive carries out work not related to working with trains (technical inspection, equipment, exchange of locomotive crews).

In the example shown in Pic. 2, there are four calculation periods in a day:

1. From zero minute (conditionally) until the start time of locomotive TM.

[1] Methodology for assessing and monitoring the efficiency of using shunting locomotives: approved by order of JSC Russian Railways, dated December 1, 2017, No. 2485/r.



Pic. 2. Periods of occupancy of a shunting locomotive during the day [performed by the authors].

2. From the time of completion of locomotive TM to the time of the start of the first rotation of the locomotive crew.

3. From the end of the first rotation of the locomotive crew to the start of the second rotation of the locomotive crew.

4. From the end of the second rotation of the locomotive crew to 1440th min (conditionally) – the end of the estimated day.

The start times of operation ($t_i^{\text{start.oper.}}$), located on the time axis segments that lack the technologically justified working time of the shunting locomotive in a given calculation period ($\mathrm{T}_{\text{per}}^{\text{cal}}$) (or they fall on a non-working time segment for the locomotive), are transferred to other calculation periods in accordance with the following rules:

– hours of operations with trains upon arrival – to the next (after the non-working period) calculation period: $t_{i.\text{arr}}^{\text{start oper.}} = T_{\text{per}}^{\text{start}}$;

– hours of operations with trains intended for departures – to the previous (before the non-working period) calculation period:

$t_{i.\text{dep}}^{\text{start oper.}} = \mathrm{T}_{\text{per}}^{\text{end}} - t_i^{\text{oper}}$.

NC for the execution of a given volume of operations by a shunting locomotive with unconditional respect of the schedule of arrival and departure of passenger trains should be considered as compliance with inequalities for each of such calculation periods ($\mathrm{T}_{\text{per}}^{\text{cal}}$). (2):

$$\frac{\sum_{i=1}^{k} t_i^{oper}}{T_{per}^{oper}} \le 1, \qquad (2)$$

where k – the total number of locomotive operating times within the time period of the calculation period, including those carried over to the given calculation period;

i – sequential number of the locomotive operation in the time segment of the calculation period, determined in ascending order of location on the time axis.

SC consists in the fact that within each calculation period ($\mathrm{T}_{\text{per}}^{\text{cal}}$). all time intervals free from operations are used by shifting the technologically justified times of the start of operations by a shunting locomotive with trains ($t_i^{\text{start oper}}$). within the permitted time interval, located to the left or to the right on the time axis from the technologically justified time of the start of operation depending on the nature of the operation (upon arrival or before departure).

SC must be fulfilled for all calculation periods. Otherwise, a recalculation should be carried out considering the transfer of the first times of the start of operation (intended for train departure) to the adjacent previous calculation period and the last times of the start of operation (intended for train arrival) to the adjacent subsequent calculationeriod. This can achieve a redistribution of the volumes of locomotive work within the calculation periods, but to the detriment of the total time of occupation of RDT trains.

Analytically, SC for the calculation period (without the permitted shift of the technologically justified time of the start of operations with the train) takes the form (3):

$$\frac{t_n^{\text{oper}}}{t_{n+1}^{\text{start oper}} - t_n^{\text{start oper}}} \le 1 \quad \forall n = k, k-1, \ldots.1,. \qquad (3)$$

where $t_{k+1}^{\text{oper}} = T_{\text{per}}^{\text{cal}}$.

Let us consider examples of the execution of NC and SC at $t_i^{\text{oper}} = 30$ min[2] for two adjacent calculation periods over a common time interval of 600 min (Pic. 3).

The first calculation period ($T_{\text{per1}}^{\text{cal}}$):

k = 5, necessary condition $\psi = \frac{150}{240} = 0{,}625$

n=5, $\frac{30}{240-210} = 1$;

n=4, $\frac{30}{210-210} = \infty$, SC is not met;

n=3, $\frac{30}{210-150} = 0{,}5$;

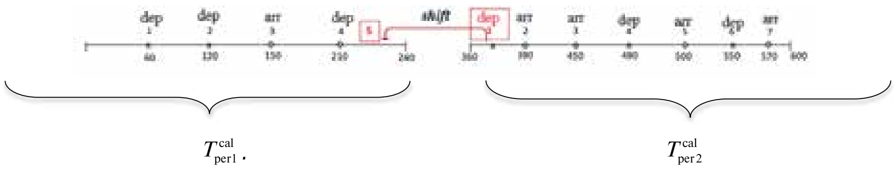
n=2, $\frac{30}{150-120} = 1$;

n=1, $\frac{30}{120-60} = 0{,}5$.

The second calculation period ($T_{\text{per2}}^{\text{cal}}$):

k = 7, necessary condition $\psi = \frac{180}{240} = 0{,}75$

[2] To solve the example, a random time is used to check the progress of the calculations.



Pic. 3. An example of performing NC and SC for adjacent calculation periods in a common time period [performed by the authors].

n = 6, $\frac{30}{600-570}=1$;

n = 5, $\frac{30}{570-550}=1{,}5$; SC is not met;

n = 4, $\frac{30}{550-500}=0{,}6$;

n = 3, $\frac{30}{500-480}=1{,}5$; SC is not met;

n = 2, $\frac{30}{480-450}=1$;

n = 1, $\frac{30}{450-380}=0{,}467$.

Failure to comply with SC (without the permitted shift in the technologically justified start time of operation with the train) does not mean that the shunting locomotive will not perform work in this calculation period. This will be possible if the shift is permitted subject to an increase in the time of occupancy of RDT by trains.

Let us assume (within the framework of this local problem) that an unlimited increase in the time of occupancy of RDT by trains is permissible (i.e., RDT load is small and is not limiting).

The use (or imibility of using) of free time intervals for the shunting locomotive to perform operation in the calculation period depends on:

– he specified schedule of arrival and departure of passenger trains at/from the station;

– the sequence of work in this period with trains upon arrival and before departure, since the possibility of shifting from the technologically justified time of work with the train in these variants is different (to the right or to the left on the time axis).

Then the problem of the shunting locomotive performing its operations is reduced to solving the problem of finding the minimum of a linear function under given constraints in the form of equalities and inequalities. Let us state this problem.

Designations:

n – number of passenger trains within the period of time under consideration;

m – number of other trains requiring locomotive shunting;

t_i^{arr} – the time of arrival of the *i*-th passenger train at the station (time can be counted from any time point). The duration of the calculation period can be any (*i* is a sequential (conditional) number of the train within a time period);

t_i^{dep} – departure time of the *i*-th passenger train from the station according to the schedule;

t_i^{rem} – time for removal (rearranging) of the *i*-th passenger train from RDT on the servicing (storage) track, including the time necessary for moving the locomotive to the train and returning it to the parking place, the time for rearranging the mail and baggage cars to the loading and unloading places (if necessary) and the time for additional assembling of the train if an idle time is planned for it;

t_i^{del} –time of delivery (relocation) of the *i*-th passenger train from the parking tracks to RDT, including the time of movement of the locomotive to the train and its return to the parking place, including the time for coupling mail and baggage cars to the passenger train and additional assembling of the train when it is relocated from the parking tracks;

$t_i^{\text{oper arr m-b}}$ –the time of operation of the locomotive with the arriving mail and baggage train, including the time of the locomotive moving to the train and returning it to its parking place upon the arrival of the train;

$t_i^{\text{arr m-b}}$, $t_i^{\text{dep m-b}}$ – schedule of arrival and departure of the *i*-th mail and baggage train, respectively;

$t_i^{\text{start oper m-b arr}}$ – the shortest on the number axis (the earliest) technologically justified time of starting work with the *i*-th mail and baggage train after its arrival (4):

$$t_i^{\text{start oper m-b arr}} = t_i^{\text{arr m-b}} + t_i^{\text{tech arr m-b}}, \qquad (4)$$

where $t_i^{\text{tech arr m-b}}$ – technologically necessary time from the moment of arrival of the *i*-th mail and baggage train until the moment the shunting locomotive starts operation with it;

$t_i^{\text{oper dep m-b}}$ – the time of operation of the locomotive with the mail and baggage train, including the time of the locomotive moving to the train and returning it to the parking place when the train departs;

$t_i^{\text{start oper m-b dep}}$ – the longest on the number axis (the latest) technologically justified time of the start of work with the *i*-th mail and baggage train upon its departure (5):

$$t_i^{\text{start oper m-b dep}} = t_i^{\text{dep m-b}} - t_i^{\text{tech dep m-b}}, \quad (5)$$

where $t_i^{\text{tech dep m-b}}$ – technologically necessary time from the moment of completion of operation with the *i*-th mail and baggage train until the moment of its departure;

$t_i^{\text{start oper arr}}$ – the shortest on the number axis (the earliest), technologically justified time of the start of work with the *i*-th passenger train when it is moved from RDT (6):

$$t_i^{\text{start oper arr}} = t_i^{\text{arr}} + t_i^{\text{disem}}, \quad (6)$$

where t_i^{disem} – the time of disembarking of passengers from the *i*-th passenger train (provided that it is limiting in relation to technical operations carried out with the train in parallel);

$t_i^{\text{start oper dep}}$ – the longest on the number axis (the latest) technologically justified time of starting work with the *i*-th passenger train by rearranging it to RDT (7):

$$t_i^{\text{start oper dep}} = t_i^{\text{dep}} - t_i^{\text{emb}} - t_i^{\text{tech}}, \quad (7)$$

where t_i^{emb} – time of embarking of passengers into the *i*-th passenger train;

t_i^{tech} – technologically necessary time from the moment of completion of boarding of passengers until the moment of departure.

In addition, if there may be hostility at the station when moving trains from RDT and back, they must be considered as additional waiting time (8):

$$T_{\text{w}} = M\left[n_{\text{w}}^{\text{cross}}\right] \cdot t_{\text{occ}}^{\text{cross}}. \quad (8)$$

For example, if $t_{\text{occ}}^{\text{cross}} = 0,2\,\text{hour}$,

$$M\left[n_{\text{w}}^{\text{cross}}\right] = \left(0,075 + \frac{0,1}{0,2}\right)\psi_{\text{cross}} = 0,575\psi_{\text{cross}}.$$

If $\psi_{\text{cross}} = 0,8$, then $M\left[n_{\text{w}}^{\text{cross}}\right] = 0,46$, $\text{T}_{\text{w}} = 0{,}092$ hour = 5,5 min.

If $\psi_{\text{cross}} = 0,7$, then $M\left[n_{\text{w}}^{\text{cross}}\right] = 0,4025$, $\text{T}_{\text{w}} = 0{,}08$ hour = 4,8 min.

If $\psi_{\text{cross}} = 0,76$, then $M\left[n_{\text{w}}^{\text{cross}}\right] = 0,437$, $\text{T}_{\text{w}} =$ 0,087 hour = 5,2 min.

This time should be considered when calculating the total time of movement of trains, if necessary.

The performance indicators of a single «service system», which means the movement of trains from RDT and back with significant traffic loads on the routes, can be calculated using the methodology described in [10–12] differentially for different time periods depending on the load of the route intersections, including «overload» periods, i.e., when the demand for using intersections exceeds their capacity. The corresponding calculations are given in (9):

$$M\left[n_w^{cross}\right] = \begin{cases} \dfrac{0,1(1+0,75t_{occ}^{cross})}{t_{зан}^{перес}}\psi_{cross}, npu\, 0 \le \psi_{cross} \le 0,8 \\ \dfrac{2(\psi_{cross}^2 - \psi_{cross} + 0,2) \cdot (1+0,75t_{occ}^{cross})}{t_{occ}^{cross}}, when\, 0,8 \le \psi_{cross} \le 1 \\ \dfrac{20\left(\psi_{cross}^2 - 1,75\psi_{cross} + 0,77\right)}{t_{occ}^{cross}} + 15\left(\psi_{cross}^2 - 2,3\psi_{cross} + 1,32\right), \\ when\, 1 < \psi_{cross} \le 1,2 \end{cases}, \quad (9)$$

where $M\left[n_{\text{w}}^{\text{cross}}\right]$ – the mathematical expectation (average number) of passenger trains waiting extra time due to hostility;

ψ_{cross} – load factor of the «service system» (hostile train routes);

$t_{\text{occ}}^{\text{cross}}$ – average time of occupation of a hostile route when moving trains, hours.

The duration of the periods of differentiated calculation of the time of waiting for a rearrangement may be less than the calculated periods of operation of shunting locomotives depending on the change in the intensity of occupancy of routes with hostility and is determined for specific passenger stations.

Let us set the problem for a single shunting locomotive.

During the considered period, the locomotive must perform the operation of moving (rearranging) of (*n*) passenger trains on the servicing (storage) track, of moving (rearranging) them to RDT, and perform operations with (*m*) mail and baggage trains upon arrival and before departure.

At the same time, the sum of deviations of the actual start times of operations with trains from those dictated by the technological processes of work with passenger and mail and baggage trains and the schedules of their arrival and departure must be minimal, which will ensure the optimal balance of occupancy of RDT and storage tracks with the unconditional fulfilment of the specified volume of work.

Thus, this problem is a linear programming problem of finding the minimum of the objective function under constraints of the types of equalities and inequalities type [13; 14].

The absence of a solution to this problem with the given initial data will indicate the impossibility of performing the given volume of operation for the locomotive under the conditions of a fixed schedule.

Let us represent the problem mathematically with the above notations.

Let us introduce the numbering of the start times of operations with arriving and departing passenger and mail and baggage trains [15]. From here on, as the *i*-th arriving (departing) train we will mean not a specific train that arrives at the station (departs from the station), but a train that arrives at the station (departs from the station) along the *i*-th «thread» and is serviced at the station in accordance with the accepted technology of work with a train arriving at the station (departing from the station) for a specific purpose of the *i*-th «thread» of the schedule. The count can be started from any moment accordingly with time passed.

The unknown (required) values in the problem will be the actual recommended start times for operation with passenger and mail-baggage trains (Pic. 3).

The total minimum number of unknowns is (2 m + 2 n) depending on the value of the period of time under consideration, i.e., x_i, i = 1,…, 2 (m + n) (here the numbering is as defined above).

The constraints in the problem will be:

1. For all times determining the start operation time of the shunting locomotive for removal of passenger trains from RDT (10):

$$t_i^{\text{start oper arr}} \le x_i \le t_i^{\text{start oper arr}} + \nu, \quad (10)$$

where Δ – the maximum possible (established by will or through other considerations) time interval during which a train must vacate RDT. This value may be established differentially for different time periods or for each train depending on the actual occupancy of the parking tracks at certain periods of time.

2. For all times that determine the start operation time of the shunting locomotive to deliver a passenger train to RDT (11):

$$t_i^{\text{start oper dep}} - \nu_1 \le x_i \le t_i^{\text{start oper dep}}, \quad (11)$$

where Δ_1 – the maximum possible (established by will or upon other considerations) time interval that allows for the early relocation of a train from the parking (servicing) track for subsequent departure from the station. This value can also be established differentially depending on the actual occupancy of RDT, as well as on the schedule of arrival and departure of trains, primarily of transit ones.

The value Δ = 0 and or $\Delta_1 = 0$ adopted in the calculations means that, due to the dense occupancy of RDT (parking) tracks, deviations from the technological schedule for operations with passenger trains are not permitted.

3. For all times that determine the start time of operations of the shunting locomotive with the arriving mail and baggage train (12):

$$t_i^{\text{start oper m-b arr}} \le x_i \le t_i^{\text{start oper m-b arr}} + \nu_2, \quad (12)$$

where Δ_2 – the maximum possible (established by will or upon other considerations) time interval during which operation must begin with the mail and baggage train upon arrival.

4. For all times that determine the start time of the shunting locomotive's operation to assemble and rearrange the mail and baggage train when it departs from the station (13):

$$t_i^{\text{start oper m-b dep}} - \nu_3 \le x_i \le t_i^{\text{start oper m-b dep}}, \quad (13)$$

where Δ_3 – the maximum possible (established by will or upon other considerations) time interval that allows for early commencement of operation with a mail and baggage train before departure. The explanations for the values Δ and Δ_1 may also fully apply to the values Δ_2 and Δ_3.

5. For all times that determine the start of technical maintenance, exchange of shunting locomotive crews (14):

$$t^{\text{end maint}} = t^{\text{start maint}} + C_j, \quad (14)$$

where C_i –fixed time for performing technical maintenance, changing locomotive crews for the *j*-th operation.

These values are not variables, they are given as initial information: $t^{\text{start maint}}$.

In some cases, a slight shift in time for the values of $t^{\text{start maint}}$ and the duration of the maintenance itself may be allowed.

The next group of constraints concerns ensuring the possibility for a shunting locomotive to start work with the next train after completing operations with the previous train.

6. For all times determining the start time of the shunting locomotive's operations with the next train after completion of operations with the previous train to remove (rearrange) the passenger train from RDT on the servicing (parking) track (15):

$$x_i + t_{\text{rem } i} \le x_{i+1}. \quad (15)$$

7. For all times determining the start time of the shunting locomotive's operations with the next train after completion of operations with the previous train to deliver (rearrange) a passenger train from the parking tracks to RDT (16):

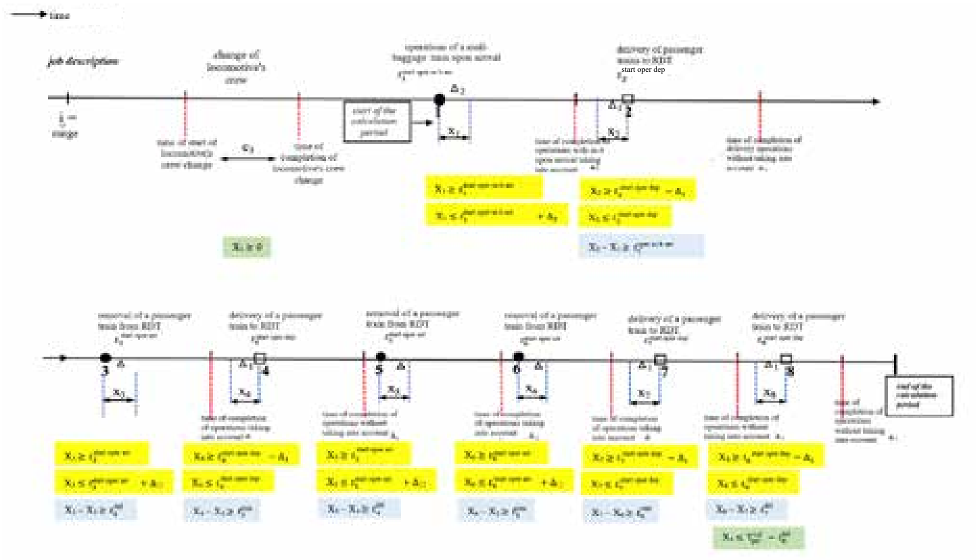
$$x_i + t_{\text{del } i} \le x_{i+1}. \quad (16)$$

Constraints 6, 7, 8 and 9 provide the necessary «sparseness» in the operation of the locomotive.

8. For all times determining the start time of the shunting locomotive's operations with the next train after completion of operations with the previous train (mail and baggage train) upon arrival (17):

$$x_i + t_i^{\text{oper m-b arr}} \le x_{i+1}. \quad (17)$$

9. For all times determining the start time of the shunting locomotive's work with the next train after completion of work with the previous



Pic. 4. Fragment of the placement of searched values on the time axis [performed by the authors].

train (mail and baggage train) before departure (18):

$$x_i + t_i^{\text{oper m-b dep}} \le x_{i+1}\,. \quad (18)$$

10. For all times defining the time before the start of technical maintenance (exchange of crews) of the locomotive (19):

$$x_i + t_i^{\text{oper(arr)dep}} \le T_{\text{per}}^{\text{cal}}\,. \quad (19)$$

Constraint 10 similarly (like all the constraints of this group) determines the possibility of completing operations with trains before the start of a technological break in the operation of the locomotive.

Let us consider a specific example that corresponds to the general pattern of train arrivals and departures shown in Pic. 3.

Initial data:

$t_i^{\text{del}} = t_i^{\text{rem}}$, i = 2,… ., 8 → 20 min.

Calculation is made for $\Delta = \Delta_1 = 0, 5, 10$ min.

$t_1^{\text{oper arr m-b}} = 60$ min.

$T_{\text{per}}^{\text{cal}} = 240$ min.

In this case, the formal load of the shunting locomotive is 0,83, and the direct operating time is 200 minutes.

The analytical criterion for this problem has the following form (20):

$$f\left(x_{i,}\ i = 1,\dots 2(m+n)\right) = \\ = min\left[\sum_{i \in M_{\text{del}}}\left(-x_i + t_i^{\text{start oper arr}}\right) + \sum_{i \in M_{\text{rem}}}\left(-t_i^{\text{start oper arr}} + x_i\right)\right], \quad (20)$$

where M_{sub} – a subset of natural numbers that define in a numerical sequence the numbers of the shunting locomotive's operations for delivery of passenger trains to RDT and the operations with mail and baggage trains for their dispatch for the calculation period;

M_{rem} – a subset of natural numbers that define in a numerical sequence the numbers of the shunting locomotive's operations on removal of passenger trains from RDT and operations with mail and baggage trains upon arrival for the calculation period.

In this example (21):

$$f\left(x_{i,}\ i = 1,\dots 8\right) = min\left[\begin{array}{l}\sum\limits_{i=2,4,7,8}\left(-x_i + t_i^{\text{start oper dep}}\right) + \\ + \sum\limits_{i=1,3,5,6}\left(-t_i^{\text{start oper arr}} + x_i\right)\end{array}\right], \quad (21)$$

$t_1^{\text{start oper arr}} = 0$; $t_2^{\text{start oper dep}} = 65$; $t_3^{\text{start oper arr}} = 80$;

$t_4^{\text{start oper dep}} = 110$; $t_5^{\text{start oper arr}} = 120$; $t_6^{\text{start oper arr}} = 140$;

$t_7^{\text{start oper dep}} = 175$; $t_8^{\text{start oper dep}} = 190$.

$T_{\text{per}}^{\text{cal}} = 240$ min.

Constraints when $\Delta = 10$:

$x_1 \ge 0$;

$x_1 \ge 0$, $x_1 \ge 0$, $x_1 \le \Delta$;

$x_2 \ge 65 - \Delta$, $x_2 \le 65$, $x_2 - x_1 \ge 20$;

$x_3 \ge 80$, $x_3 \le 80 + \Delta$, $x_3 - x_2 \ge 20$;

$x_4 \ge 110 - \Delta$, $x_4 \le 110$, $x_4 - x_3 \ge 20$;

$x_5 \ge 120$, $x_5 \le 120 + \Delta$, $x_5 - x_4 \ge 20$;

$x_6 \ge 140$, $x_6 \le 140 + \Delta$, $x_6 - x_5 \ge 20$;

$x_7 \ge 175 - \Delta$, $x_7 \le 175$, $x_7 - x_6 \ge 20$;

$x_8 \ge 190 - \Delta$, $x_8 \le 190$, $x_8 - x_7 \ge 20$; $x_8 \le 240–20 = 220$.

Criterion:

$(65 - x_2) + (110 - x_4) + (175 - x_7) + (190 - x_8)$ $(-0 + x_1) + (-80 + x_3) + (-120 + x_5) + (-140 + x_6)$.

The reduced form of the criterion and constraints (with a fixed value of Δ and finite values of the constant):

Table 1

Solution when $\Delta = 10$

$x_1 \geq 0$, $x_1 \leq 10$ $x_2 \geq 65-10$, $x_2 \leq 65$, $x_2 - x_1 \geq 20$; $x_3 \geq 80$, $x_3 \leq 80+10$, $x_3 - x_2 \geq 20$; $x_4 \geq 110-10$, $x_4 \leq 110$, $x_4 - x_3 \geq 20$; $x_5 \geq 120$, $x_5 \leq 120+10$, $x_5 - x_4 \geq 20$; $x_6 \geq 140$, $x_6 \leq 140+10$, $x_6 - x_5 \geq 20$; $x_7 \geq 175-10$, $x_7 \leq 175$, $x_7 - x_6 \geq 20$; $x_8 \geq 190-10$, $x_8 \leq 190$, $x_8 - x_7 \geq 20$; $x_8 \leq 240-20 = 220$.	$x_1 = 0$, $x_2 = 60$, $x_3 = 80$, $x_4 = 100$, $x_5 = 120$, $x_6 = 140$, $x_7 = 170$, $x_8 = 190$,

min: $x_1 - x_2 + x_3 - x_4 + x_5 + x_6 - x_7 - x_8 + 200$.

The solution is shown in Table 1.

Minimal value of the criterion: min $f = 20$.

The sufficient condition without the permissible shift ($\Delta = 0$) is not satisfied. With the permissible shift $\Delta = 5$, the solution to the problem for the given initial data is also absent.

Let's change the initial data, namely, reduce the calculation period: $T_{per}^{cal} = 200\,\text{min}$ and shift the start of operations with the departing passenger train $t_7^{\text{start oper dep}} = 170\,\text{min}$.

The solution is shown in Table 2.

Minimal value of the criterion: $f = 35$.

In this case $\psi_{shunt} = 1$. The locomotive operates at maximum capacity and any delay of the train can «break» the entire schedule of arrival and departure of passenger trains, and the time of additional occupation of RDT by passenger trains increases by 75 %.

When several locomotives are operating, it is necessary to distribute the work between them according to the following algorithm:

1. It is necessary to preliminarily determine the required (needed) number of shunting locomotives (22):

$$M_{loc} = \frac{\sum_{i=1}^{n} t_i^{oper} + \sum T_{break}^{day}}{1440}, \tag{22}$$

where n – number of all operations of shunting locomotives per day;

t_i^{oper} – duration of the i-th operation of the locomotive;

$\sum T_{\text{break}}^{\text{day}}$ – total time (per day) of breaks in the operation of shunting locomotives associated with their technical inspection and crew exchanges (possibly, variant of calculations will be required, since technical inspection is not carried out every day).

2. All start times of operations shall be marked on the daily time scale and transferred in relation to the calculation periods for the first shunting locomotive (see the previously specified rules).

3. For each calculation period for the first shunting locomotive, based on the analytical sufficient condition (3), it is necessary to carry out the removal and placement of the locomotive's operations from the time scale of the first locomotive to the time scale of the second locomotive according to the condition (23):

$$t_{\text{transfer } i}^{\text{oper}} = max_{i=2,\ldots,k}(t_{i-1}^{\text{start oper}} + t_{i-1}^{\text{oper}} - t_i^{\text{start oper}}. \tag{23}$$

In this way, the transfer of operations that are in the most «constrained» conditions is carried out. Such transfer for each calculation period (separately) is carried out until the analytically sufficient condition is met.

4. It is necessary to make iterative transfers from the time axis of the first locomotive to the time axis of the next locomotive until the work is distributed relatively evenly across all time axes of all locomotives used.

5. The distribution of operations must be confirmed by solving the problem for each locomotive and for each of its calculation periods for a certain permitted shift in operations' start times associated with the additional occupation of RDT.

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Under the conditions of mandatory performance of operations by shunting locomotives intended for the removal of passenger trains arriving at station's RDT and the rearrangement of passenger trains on RDT for departure at a high average daily load, it is necessary to resort to:

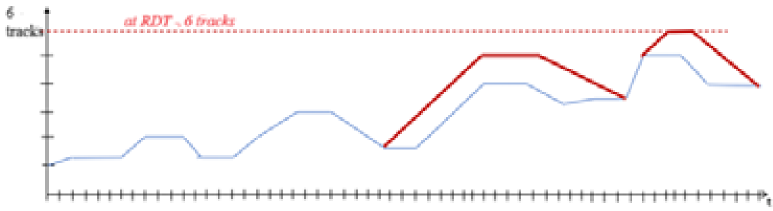
– a later time of removal of trains (in relation to the technologically justified time) from RDT tracks;

– an earlier time of delivery of trains (in relation to the technologically justified time) on RDT tracks.

Table 2

Solution when $\Delta = 10$

$x_1 \geq 0$, $x_1 \leq 10$	$x_1 = 0$,
$x_2 \geq 65 - 10$, $x_2 \leq 65$, $x_2 - x_1 \geq 20$;	$x_2 = 60$,
$x_3 \geq 80$, $x_3 \leq 80 + 10$, $x_3 - x_2 \geq 20$;	$x_3 = 80$,
$x_4 \geq 110 - 10$, $x_4 \leq 110$, $x_4 - x_3 \geq 20$;	$x_4 = 100$,
$x_5 \geq 120$, $x_5 \leq 120 + 10$, $x_5 - x_4 \geq 20$;	$x_5 = 120$,
$x_6 \geq 140$, $x_6 \leq 140 + 10$, $x_6 - x_5 \geq 20$;	$x_6 = 140$,
$x_7 \geq 170 - 10$, $x_7 \leq 170$, $x_7 - x_6 \geq 20$;	$x_7 = 160$,
$x_8 \geq 190 - 10$, $x_8 \leq 190$, $x_8 - x_7 \geq 20$;	$x_8 = 180$,
$x_8 \leq 200 - 20 = 180$.	



Pic. 5. Time axis [performed by the authors].
— The line corresponds to the occupancy of the tracks without the permitted shift (the locomotive does not respect the schedule).
— The line corresponds to the occupancy of the tracks with the permitted shift (the locomotive respects the schedule).

This leads to an increased (per day) time of the total occupation of RDT tracks by trains.

Such an increase in certain periods of time can lead to a shortage of RDT to fulfil the schedule of arrival and departure of both transit (passing) trains and passenger trains for which the station is final destination.

Therefore, after determining the start times of shunting locomotives with terminal trains, it is necessary to check the sufficiency of RDT capacity to fulfil the general schedule of arrival and departure of passenger trains at the station.

The initial data for such a check are:

1. *For the increase in the number of occupied tracks:*

– the arrival time of transit (passing) trains and passenger trains for which the station is final destination, adjusted for the time of preparation of the route for receiving a train on the track $t_i^{\text{transit arr}}$, $t_i^{\text{final arr}}$;

– the time of the actual start of the shunting locomotive's operations to move a train to RDT, adjusted for the time necessary for locomotive arrives to the train $t_i^{\text{final dep}}\left(x_i + t_{\text{arrival}}\right)$.

2. *For the decrease in the number of occupied tracks:*

– the departure time of (passing) trains and passenger trains for which the station is final destination, adjusted for the time necessary for the track to be free after the train starts moving $t_i^{\text{transit dep}}$, $t_i^{\text{final dep}}$;

– the actual start time of the shunting locomotive's operation to remove the passenger train for which the station is final destination from RDT, adjusted for the time necessary for the locomotive arrives to the train being removed: $t_i^{\text{final arr}}\left(x_i + t_{\text{arrival}}\right)$ and for the time necessary for the track to be free after the train starts moving.

All the specified times are plotted on the time axis (per day or per calculation period) in ascending order, and the graph displays the change in the number of occupied tracks over time (with a step of t = 6 min) as shown in Pic. 5.

If at certain intervals the specified number of tracks is exceeded (in this example – 6), then the schedule will also not be respected, and it is necessary to change the schedule of arrivals and departures of passenger trains.

CONCLUSIONS

The proposed methodology is aimed at mathematical formalisation of the process of determining the rational sequence in the operations of shunting locomotives with passenger trains at stations. It can be used to solve several operations' problems, including:

• adjusting the results of calculating the number of shunting locomotives at a passenger technical station under conditions of their maximum loads;

• improving and adjusting the schedule of arrival and departure of passenger trains when assigning additional passenger trains during periods of larger transportation periods;

• increasing the efficiency of shunting locomotives considering the use of additional backup locomotives provided for the periods when RDTs are unoccupied;

• determining reserves in the operation of shunting locomotives in case of possible delays in the arrival of passenger trains at the station.

The methodology allows solving these problems both fragmentarily in individual periods of time and for a «daily segment».

The paper formalises the concepts of a necessary condition and a sufficient condition for shunting locomotives to perform a given volume of operations, both in general and for individual busiest periods of trains' arrival and departure.

Using the linear programming apparatus allows solving both specific problems and conducting research to assess the level of influence of various factors on the performance indicators of passenger and passenger technical stations, determining the reserves for using shunting locomotives and their rational use in the context of the problem of a shortage of shunting locomotives.

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