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Features of Using the Method of Normalised Range and Fractal Analysis in Studying the Car Traffic Flow Intensity

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ABSTRACT

The relevance of studying the methods of studying traffic flows is due to the need to analyse their features, determine the allowable areas of their application in solving practical problems of the transport sector.

The objective of the work is to identify the results of application of modern methods of time series analysis that use the values of intensities of car traffic flows on the urban street-and-road network. The subject of the study is associated with the calculated Hurst and fractal dimension indices (fractal analysis), as well as with checking the validity of the quantitative relationship of these indicators, specified by B. Mandelbrot and used in applied research, on real data on intensity of car traffic flows. Digital data for the study were obtained using «Azimuth» stationary measuring software and hardware photo and video recording complexes, located on the street-and-road network of the city.

The study has found that anomalous values of key indicators are encountered when using the normalised range method and fractal analysis: the Hurst exponent takes values outside the usually defined range, the relationship between the fractal dimension and the Hurst exponent does not fully correspond to the known B. Mandelbrot set. It seems necessary to conduct a deep and thorough study of the results obtained when using the above and, possibly, other methods for studying the intensity of traffic flows on street-and-road networks.

Keywords: *traffic flow, car traffic intensity, Hurst exponent, fractal dimension, urban transport.*

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INTRODUCTION

The intensity of the traffic flow as a time series (function of time), along with other random processes, belongs to a special class of elements of functional spaces which, unlike smooth (differentiable) functions, exhibit fractal properties, or self-similarity, that is, with repeated changes in the scale of variables, the structure of the function already recorded on previous scales is reproduced [1, P. 36]. To describe the special (fractal) properties of such functions, a specially defined concept of fractal, or fractional dimension of Hausdorff–Besicovitch [2, P. 22] (of Minkowski [3, P. 129]) is used.

Fractal analysis is successfully used to study self-similar stochastic processes characteristic of network traffic and data on solar flares [4], to study and predict earthquakes [5]. Modelling [6] of the multimodal distribution of porosity of oil and gas fields made it possible to identify the relationship between the fractal index and the degree of oil saturation of the soil.

Analysis of fractal properties of time series modelling real stochastic processes is provided in the work [7]. Identification of time series based on fractal analysis considering chaos is proposed in [8]. Classification of stationary, non-stationary and quasi-periodic signals was developed by the author of [9]. The works [10; 11] recommend using the critical value of fractal dimension as an indicator of the crisis state (catastrophe) of the system under consideration. According to [12], to assess the structure and influence of long-term memory of time series it is advisable to use the value of fractal dimension of functions describing phase transformations in the mechanical and physical systems under consideration.

Fractal dimension can be used to analyse quote trends and manage investment portfolios, to study and identify the dynamics of time series of financial activity indicators [13], to forecast the volatility of the oil products market [14] and to assess the significance of financial assets [15] based on the assumption [16] on the fractal nature of the market and the presence of history of market process (long-term or short-term memory).

The fractal approach [17] made it possible to optimise the shape of the bus body to reduce resistance to movement and reduce fuel consumption. Analysis [18; 19] of fractal indicators of the car traffic flow in the urban street-and-road network (SRN), of probability distribution and correlation characteristics of traffic showed that empirical data on traffic flows correspond to fractal indicators and are satisfactorily described with their help. The study [20] found that the number of motor vehicles on the roads is largely determined not by the Poisson (simplest) process, but by factors of a fractal nature. To classify traffic flows, following their self-similarity and irregularity, it is advisable to use the hypothesis of the «multifractal» nature of the spectrum of the car traffic flow [21].

Along with the analysis of the fractal dimension, the method of normalised range is actively used, which is based on the calculation of the Hurst index. The Hurst index [22] is used to analyse the dynamics of electroencephalographic signals of patients to construct an index of healthy and abnormal brain activity. Quantitative assessment [23] of the morphological properties of the surface structures of tooth enamel based on the fractal dimension is used along with the traditionally determined index of surface roughness. The analysis of intensity of the car traffic flow in SRN of a metropolis using modern mathematical methods (statistical, wavelet and Fourier analysis, Hurst index) is described in [24–29].

For most time series generated by natural processes, direct (analytical) determination of fractal dimension is impossible, so special algorithms [30], the Hurst exponent [1, P. 348] or numerical procedures [31] must be used. The relationship between fractal dimension and the Hurst exponent has been studied in sufficient detail in the works [1, P. 353; 32–34].

The *objective* of the work is to reveal the results of modern methods of analysis of time series that use values of intensity of car traffic flows in street-and-road network obtained through an experiment with the help of measuring software and hardware systems.

DATA OBTAINED THROUGH FIELD OBSERVATIONS OF THE INTENSITY OF THE CAR TRAFFIC FLOW

Studies on characteristics of car traffic flow traditionally use the dependence of the traffic flow intensity on time [35; 36]. The intensity $N(t_i)$ of the car traffic flow at the section of the urban road network considered in the proposed study (using the city of Perm as an example) at an arbitrary time t_i was determined based on data

Table 1

Hurst exponent *H* and fractal dimension *D* of traffic flow intensity functions at sections of urban road networks [performed by the author]

№	Average intensity *N*, car/h	Hurst exponent *H*	Fractal dimension *D*	Pearson statistics χ^2
1	166	0,5392	1,6998	30,6
2	281	0,7330	1,6943	32,5
3	311	0,7612	1,7501	31,6
4	417	0,7433	1,7701	40,9
5	467	–0,0310	1,7259	45,4
6	599	0,0359	1,6414	325,8
7	719	0,0573	1,7081	36,7
8	830	0,0158	1,6928	3634,7
9	956	0,0085	1,6715	39,0
10	975	–0,0520	1,6823	32,0
11	1046	–0,0230	1,6801	56,3
12	1273	0,0200	1,6523	108,0
13	1377	–0,0590	1,6606	31,5
14	1398	0,0313	1,6463	71,5
15	1426	–0,0080	1,7021	31,4

received from the measuring software and hardware systems (MSHS) in real time, using the expression:

$$N(t_i) = \frac{n(t_i)}{\Delta}, \quad (1)$$

where $n(t_i)$ is number of vehicles at the entry point recorded during the time interval $[t_i – \Delta/2, t_i + \Delta/2_J$, $i = \overline{1, m}$;

Δ – selected time interval;

m – total number of observation intervals.

MSHS of «Azimuth» series[1] [37] allows recognising state registration plates, measuring the speed of a vehicle in the control zone, its average speed, the time of movement between the control lines and other characteristics.

To calculate the Hurst exponents and fractal characteristics of the functions of the intensity of the car traffic flow, 15 streets of the city of Perm (Vavilova Street, Lasvinskaya Street, Podlesnaya Street, Stakhanovskaya Street, Kosmonavtov Highway, Yuzhnaya Damba and others) with different values of the average traffic intensity (Table 1) were selected.

Pic. 1 shows the dependences of the intensities of traffic flows on time, calculated according to (1), with Δ = 5 min. The duration of observation of traffic flows varied from 350 to 700 minutes, during which the intensities $N(t)$ of car flows remained almost constant.

The normalised range method [1, P. 349; 24, 28] is based on the approximation of the dimensionless *R/S* index with a power-law dependence of the form at^H, where a is a constant, H is the Hurst exponent (constant), t is time. Here:

$$R(M) = \max_{t_1 \le t_i \le t_M} Z(t_i, M) - \min_{t_1 \le t_i \le t_M} Z(t_i, M) \quad (2)$$

– the range of deviations of —

$Z(t_i, M) = \sum_{i=1}^{M} \left[N(t_i) - \langle N \rangle \right]$ of random values of intensity $N(t_i)$ of the car traffic flow from the average value

$$\langle N \rangle = \frac{1}{M} \sum_{i=1}^{M} N(t_i); \quad (3)$$

$$S = \sqrt{\frac{1}{M} \sum_{i=1}^{M} \left[N(t_i) - \langle N \rangle \right]^2} \quad (4)$$

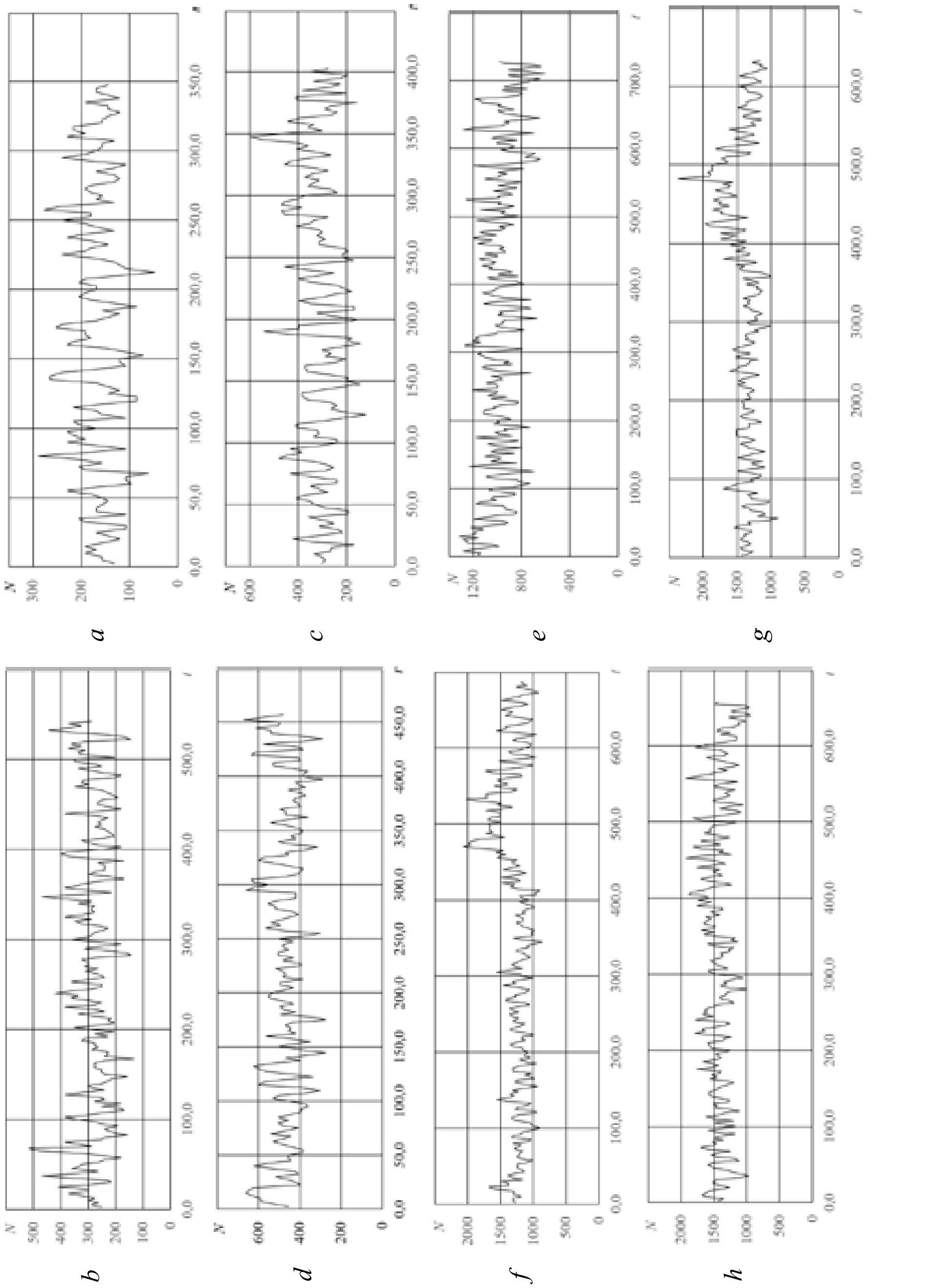
– standard deviation of the intensity $N(t_i)$ of the traffic flow.

The exponent (Hurst index H) of at^H function approximating the *R/S* dependence allows us to identify the main types of time series behaviour [1, P. 351; 38]: when $H = 0,5$, the values of the time series are considered random, that is, the terms of the series do not exhibit a trend dependence; in other words, the history of the appearance of the «early» terms of the series does not affect the values of the subsequent ones.

When $0 \le H < 0,5$, the time series is considered «anti-persistent», which reflects its instability.

When $0,5 < H \le 1,0$, the observed process turns out to be «persistent», that is, it has a trend component; the history of the formation of the

[1] «Azimuth 2» MSHS. Road safety technologies. [Electronic resource]: https://tbdd.ru/node/224. Last accessed 11.01.2024.

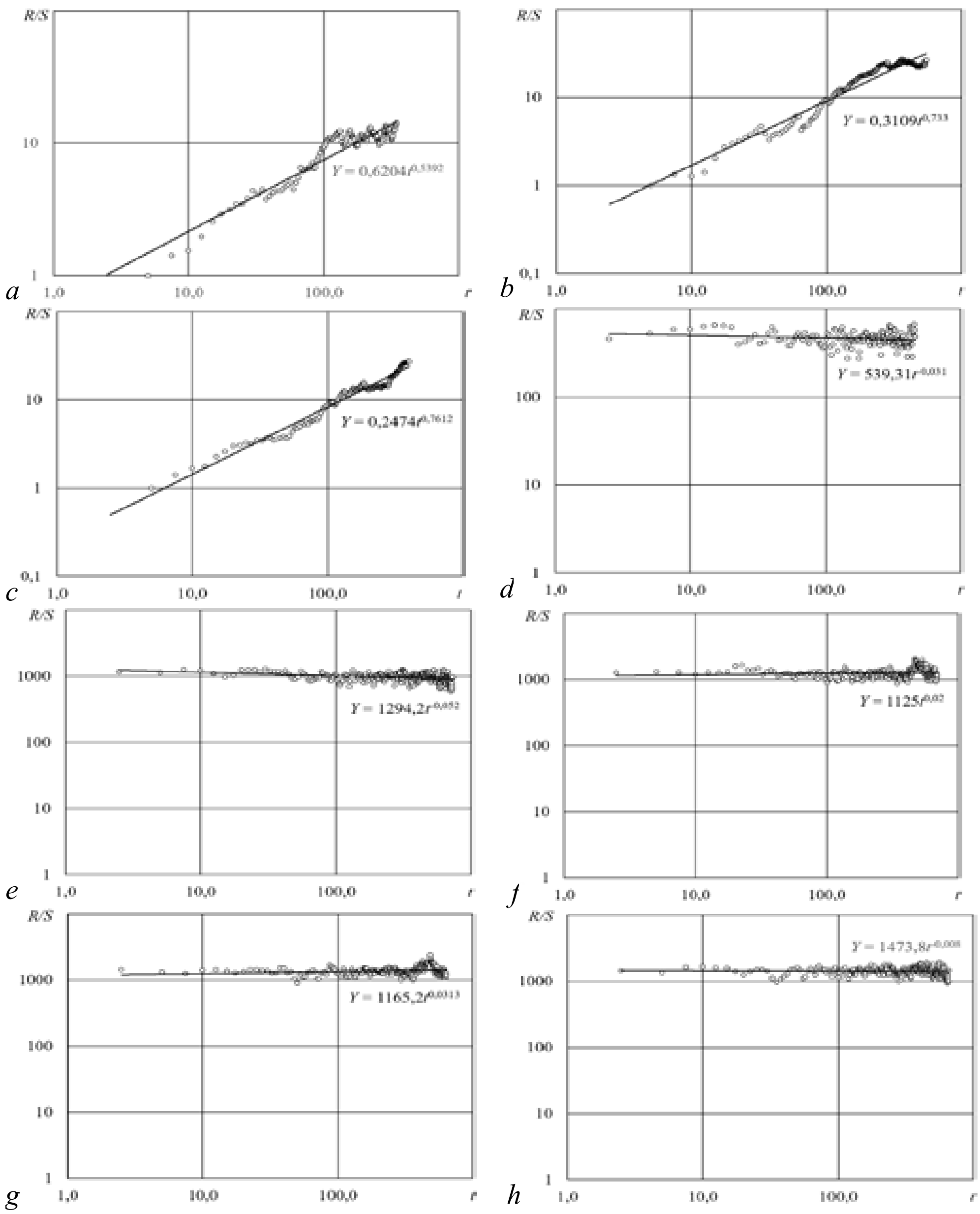


Pic. 1. Intensities N (car/h) of traffic flows as a function of time t (min); average values of intensities of traffic flows (car/h): 166 (a), 281 (b), 311 (c), 467 (d), 975 (e), 1273 (f), 1398 (g) and 1426 (h)[performed by the author].

«early» terms affects the values of the subsequent terms of the time series.

Pic. 2 presents in logarithmic coordinates the dependences of the functions of the normalised ranges R/S on time t for the functions of the intensity $N(t)$ of traffic flows at the considered road sections according to relations (2)–(4), as well as the approximation by the least squares method of these curves by power functions. The corresponding values H of the Hurst exponent are given in Table 1. For several time series of intensities $N(t_i)$ of traffic flows, the values H of the Hurst exponent take negative values, which is not provided for by the classical interpretation of this indicator, $0 \le H \le 1{,}0$ [1, P. 349; 37].

Calculation of fractal dimensions of the considered intensity curves (1) of traffic flows is



Pic. 2. Dependences of the indicators of the normalised range R/S of the functions N(t) of the intensity of transport flows on time t (min, logarithmic coordinates); average values of the intensity N of flows: 166 (a), 281 (b), 311 (c), 467 (d), 975 (d), 1273 (e), 1398 (g) and 1426 (h) [performed by the author].

made with the box-counting method [1. P. 46; 2, P. 197; 3, P. 137], the idea of which is based on determining Hausdorff–Besicovitch dimension.

It is assumed that within the considered observation interval $[t_1, t_m]$ the function $N(t)$ with a finite number of breaks is known (it is these functions $N(t_i)$ that are considered as time series obtained with MSHS of «Azimuth» series). Within the segment $[t_1, t_m]$ a grid region $\omega_m = \{t_i = \delta \cdot i, \quad i = \overline{1,m}\}$ is introduced,

where $\delta = (t_m - t_0) / m$ is a grid step.

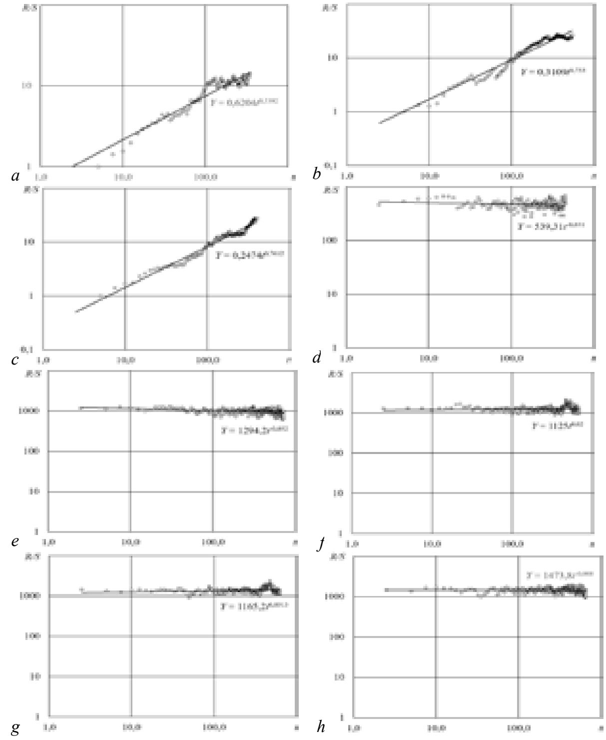
The considered function $N(t)$ is covered by square cells with a side of δ. As the size of δ decreases, the number of cells m covering the curve increases according to the power law [13]

$$m(\delta) \sim \left(\frac{1}{\delta}\right)^D, \tag{5}$$

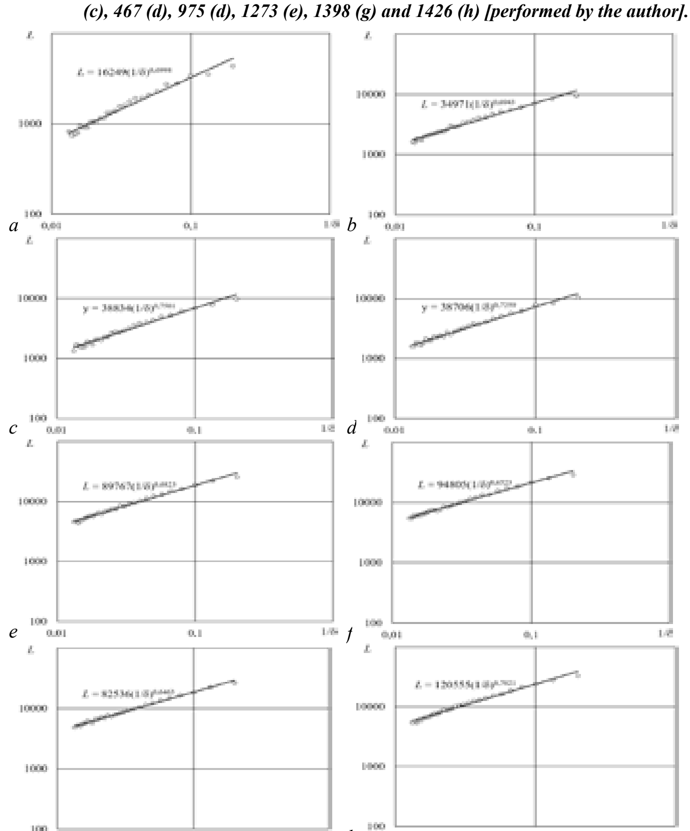
the exponent D defines the Hausdorff–Besicovitch dimension.

Taking the logarithm of expression (5) leads to the relation:

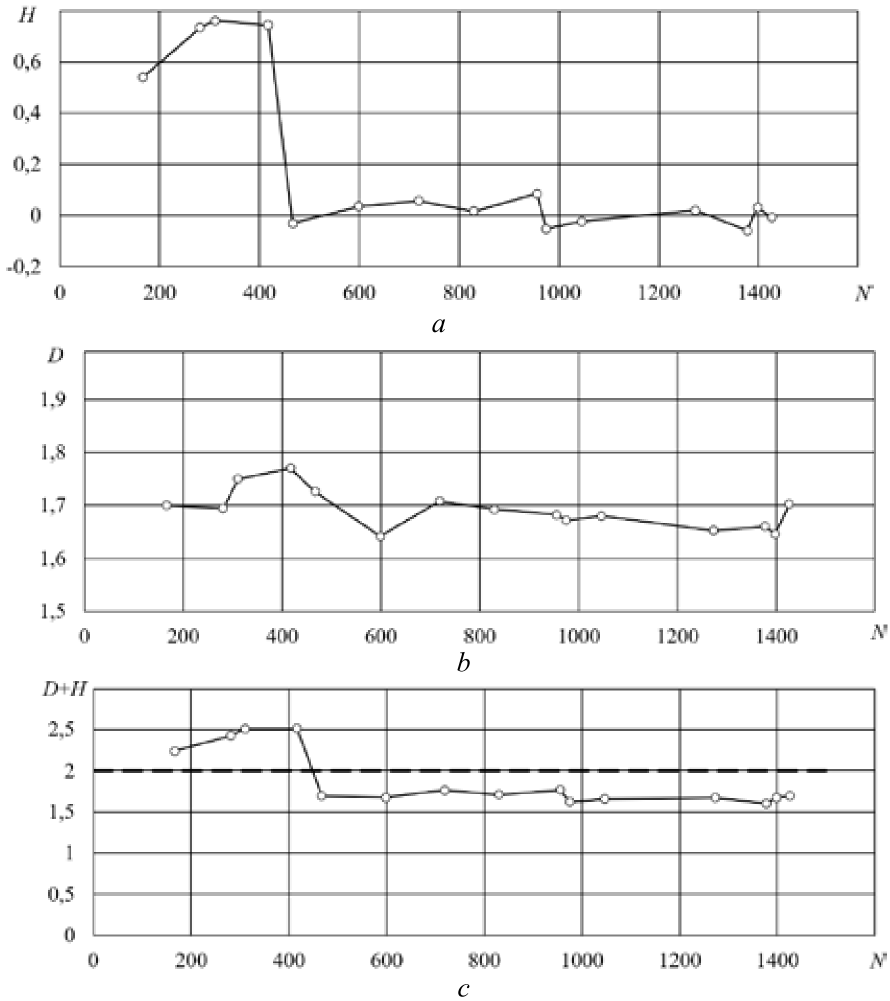
$$D = \lim_{\delta \to 0} \frac{\ln m(\delta)}{\ln(1/\delta)}. \tag{6}$$



Pic. 2. Dependences of the indicators of the normalised range R/S of the functions N(t) of the intensity of transport flows on time t (min, logarithmic coordinates); average values of the intensity N of flows: 166 (a), 281 (b), 311 (c), 467 (d), 975 (d), 1273 (e), 1398 (g) and 1426 (h) [performed by the author].



Pic. 3. Dependences of the length L of the functions N(t) of the intensity of traffic flows on the value of 1⁄δ (min–1, logarithmic coordinates $ln(1/\delta)$ and $lnL(\delta)$); average intensities of N flows: 166 (a), 281 (b), 311 (c), 467 (d), 975 (e), 1273 (f), 1398 (g) and 1426 (h) [performed by the author].



Pic. 4. Dependences of the Hurst exponent H (a), fractal dimension D (b) and the sum D + H (c) on the average value N (car/h) of traffic flow intensity [performed by the author].

To calculate the fractal dimension D, the plane on which the function $N(t)$ under consideration is defined is initially covered with square cells of size δ_1, and the number M_1 of cells covering the graph under study is counted. Then the size of the square cell is reduced to the value δ_2, the number M_2 of cells covering the graph under study with cells of size $2\delta_2$ is counted, and so on.

For practical calculations, instead of the number m of cells covering the graph under study, it is convenient to use the length of the measured curve $L(\delta) = m\delta$. In this case, expression (5) is reduced to the form:

$$L(\delta)=\delta m(\delta)\sim\delta\left(\frac{1}{\delta}\right)^{D}=\left(\frac{1}{\delta}\right)^{D-1}. \qquad (7)$$

The obtained empirical dependence $L(\delta)$ is approximated with the least squares method with a power function:

$$L(\delta) \approx a(1/\delta)^{d}. \qquad (8)$$

From the comparison of (7) and (8) it follows that the fractal dimension D is determined by the value of the exponent d of the approximating function,

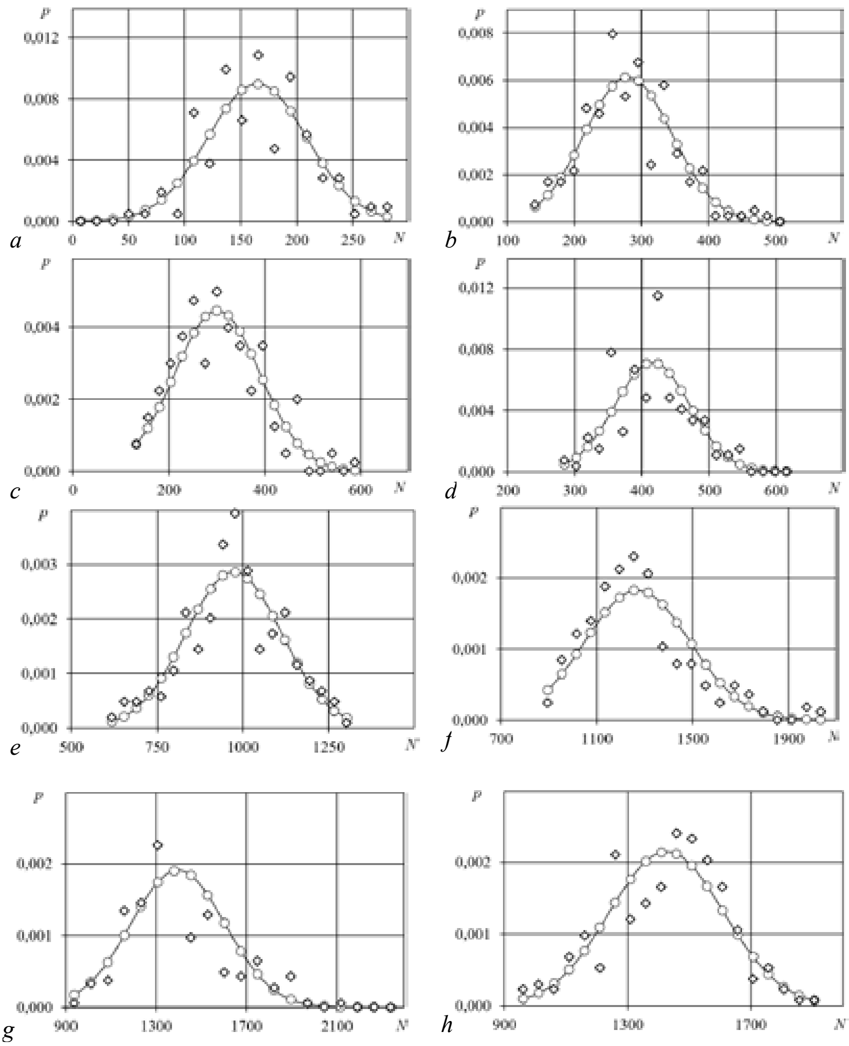
$$D = 1 + d. \qquad (9)$$

Pic. 3 shows the dependences of the length L of the functions $N(t)$ of the intensity of traffic flows at the considered road sections on the value $1/\delta$ (in logarithmic coordinates), determined according to the algorithm of the cell coverage method, as well as the approximation by the least squares method of these curves with power functions.

In the monograph [1, P. 353] B. Mandelbrot proposed a relationship between the Hurst index (exponent) H and the fractal dimension D:

$$D = 2 - H, \qquad (10)$$

which is actively used by researchers to calculate the index H from the known value of D and,



Pic. 5. Gaussian (–o–) and empirical (◊) distributions of the density of probabilities p of the intensities N of traffic flows; values of the average intensities ⟨N⟩ of car traffic flows (car/h): 166 (a), 281 (b), 311 (c), 467 (d), 975 (e), 1273 (f), 1398 (g) and 1426 (h) [performed by the author].

conversely, to determine the dimension of D from the found value of the index H. The use of relation (10) is found in theoretical studies [30; 34; 38], in publications on economics [11; 15; 16; 39], physics [12], medicine [23], network technologies [41–44], industry development [45–47], transport problems [20; 48] and other disciplines. The monograph [32] is devoted to the study of relation (10) using a large amount of experimental data; it notes the presence of a certain discrepancy between this relation and the results of field measurements.

The availability of data on real traffic flow intensities following collection of data of MSHS installed in the road network of the city of Perm allows us to further study the issue of the conformity of the ratio (10) with the results of field measurements. For this purpose, the Hurst indices H and the fractal dimension D were calculated at the selected sections of the street-and-road network (Table 1) with different traffic intensities.

Pic. 4 shows the dependences of the Hurst indices H (Pic. 4, *a*) and fractal dimensions D (Pic. 4, *b*), as well as the sum $D + H$ (Pic. 4, *c*) on the average intensities N of car traffic flows.

For values of the average traffic flow intensity $\langle N \rangle$ less than 420 car/h, the Hurst index takes values from 0,54 to 0,76, which characterises the time dependence of the traffic flow intensity as persistent, characteristic of natural processes [37].

At an average traffic flow intensity $\langle N \rangle$ of more than 420 car/h, the Hurst exponent drops to minimum values, including negative values (see data in Table 1, Pic. 2, *d, e*), characterising the time dependence of traffic flow intensity as antipersistent, i.e. as chaotic. It should be noted that during computational processing of field observation data, negative values of the Hurst exponent were obtained (Pic. 2, *d*, *e*, *h*; 4, *a*; Tab). This contradicts the statement contained in some of the publications listed above that the Hurst exponent cannot take negative values.

The fractal dimension D of the time series (Pic. 4, *b*) depends little on the average traffic flow intensity and varies within the range from 1,65 to 1,77.

The sum $D + H$ (Pic. 4, *c*) at $\langle N \rangle < 420$ car/h exceeds the value 2 predicted by the relation (10); at $\langle N \rangle > 420$ car/h, the sum $D + H$ is less than the expected value. The relative deviation of the total value of $D + H$ from the value of 2 reaches 25,7 %.

In the monograph [1, P. 350] B. Mandelbrot indicates that the relationship (9) is obtained under the assumption that the random process under study should correspond to the Gaussian (normal) probability distribution. Pic. 5 presents data on the distribution of theoretical and empirical distributions of the probabilities p of the intensities N of car traffic flows in Perm SRN.

Table 1 presents the numerical values of the statistics of the Pearson criterion χ^2 for all the road sections considered. The critical value of the Pearson criterion for a significance level of 0,95 is 30,1, which is less than all the values of the criterion χ^2 given in Table 1 for the functions $N(t_i)$ of the intensity of car traffic flows given in this study.

It should be recognised that in all the cases considered, the probability distribution of the values of the time series of intensities $N(t_i)$ of transport flows cannot be considered as Gaussian (normal). This, apparently, explains the discrepancy between the relationship (10), established by B. Mandelbrot, and the results of field observations (Pic. 4, *c*).

It becomes obvious that before using the relationship (10), one should make sure that the probability distribution of the components of the time series under study is Gaussian (or close to it). Otherwise, the error in using formula (10) may be significant.

CONCLUSIONS

Processing of data from MSHS installed in the road network of the city of Perm allows to obtain reliable and meaningful information on traffic flows in the street-and-road network of a modern city in real time.

Analysis of the data on traffic intensity obtained from MSHS revealed anomalies in the values of the Hurst exponent when using the normalised range method. It is traditionally believed that the Hurst exponent, which is a criterion for stability or instability of time series evolution trends, lies in the range from 0 to 1, whereas when calculating this indicator for the functions of traffic intensity in this study, the values obtained were beyond the specified range. The dependence of the Hurst exponent on the average value of traffic intensity was revealed: for small values of the average intensity, the Hurst exponent exceeds 0,5, which corresponds to the persistence of the observed process; for values exceeding 420 … 450 car/h, this indicator sharply decreases, which is a sign of anti-persistence of the process of car traffic flow, and can take negative values.

It has been established that the traditionally used B. Mandelbrot relation, which links the fractal dimension and the Hurst exponent, when performing field measurements of the intensity of vehicle flows, is violated with a relative error of up to 25 %. An additional study has shown that the reason for such an error may be the failure to meet the condition of the Gaussian (normal) probability distribution for the obtained time series of car traffic flow intensities.

It seems necessary in practical studies to check the compliance of the probability distribution of the components of the time series with the Gaussian (normal) distribution to justify the correctness of applying the B. Mandelbrot relation.

The detected anomalies require further in-depth analysis of the considered and other methods and approaches used to analyse transport processes in the street-and-road networks of large cities.

REFERENCES

1. Mandelbrot, B. The Fractal Geometry of Nature [Edition in Russian]. Moscow, Institute of Computer Research, 2002, 656 p. ISBN 5-93972-108-7.
2. Feder, J. Fractals [Edition in Russian]. Moscow, Mir publ., 1991, 254 p. ISBN 5-03-001712-7.
3. Crownover, R. M. Introduction to fractals and chaos [Edition in Russian]. Moscow, Postmarket publ., 2000, 354 p. ISBN 5-901095-03-0.
4. Grahovac, D., Leonenko, N. N., Taqqu, M. S. Scaling Properties of the Empirical Structure Function of Linear Fractional Stable Motion and Estimation of Its Parameters. *Journal of Statistical Physics*, 2015, Vol. 158 (1), pp. 105–119. DOI: 10.1007/s10955-014-1126-4.

5. Lyubushin, A. A. Great Japan Earthquake Forecast [*Prognoz Velikogo Yaponskogo zemletryaseniya*]. *Priroda*, 2012, Iss. 8 (1164), pp. 23–33. [Electronic resource]: https://elibrary.ru/download/elibrary_18042955_47303750.pdf. Last accessed 16.01.2024. EDN: PEVRQB.

6. Hasanov, A. B., Abbasova, G. G. Analysis of the fractal structure and stochastic distribution of pores in oil and gas reservoirs. *Perm Journal of Petroleum and Mining Engineering*, 2019, Vol. 19, Iss. 3, pp. 228–239. DOI: 10.15593/2224-9923/2019.3.3.

7. Smirnov, V. V., Spiridonov, F. F. Fractal models of stochastic processes [*Fraktalnie modeli stokhasticheskikh protsessov*]. *South Siberian Scientific Bulletin*, 2013, Iss. 1 (3), pp. 99–102. [Electronic resource]: https://elibrary.ru/download/elibrary_19107961_73506142.pdf. Last accessed 16.01.2024. EDN: QCDQDT.

8. Pashchenko, F. F., Amosov, O. S., Muller, N. V. Structural-parametric identification of time series using fractal and wavelet analysis [*Strukturno-parametricheskaya identifikatsiya vremennogo ryada s primeneniem fraktalnogo i veivlet-analiza*]. *Information Science and Control Systems*, 2015, Iss. 2 (44), pp. 80–88. [Electronic resource]: https://www.elibrary.ru/item.asp?edn=twnbmr&ysclid=m3ok83dhjp774512399. Last accessed 17.01.2024. EDN: TWNBMR.

9. Klikushin, Yu. N. Method of fractal classification of complex signals [*Metod fraktalnoi klassifikatsii slozhnykh signalov*]. *Zhurnal Radioelektroniki*, 2000, Iss. 4, 6 p. [Electronic resource]: https://www.elibrary.ru/item.asp?id=15111685. Last accessed 16.01.2024. EDN: MSRXQN.

10. Neganov, V. A., Antipov, O. I., Neganova, E. V. Fractal analysis of time series describing qualitative transformation of systems, including disasters [*Fraktalniy analiz vremennykh ryadov, opisyvayushchikh kachestvennie preobrazovaniya system, vklyuchaya katastrofy*]. *Fizika volnovykh protsessov i radiotekhnicheskie sistemy*, 2011, Vol. 14, Iss. 1, pp. 105–110. [Electronic resource]: https://elibrary.ru/download/elibrary_16337899_76189553.pdf. Last accessed 16.01.2024. EDN: NTNOXH.

11. Krivonosova, E. K., Pervadchuk, V. P., Krivonosova, E. A. Comparison of fractal characteristics of time series of economic indicators [*Sravnenie fraktalnykh kharakteristik vremennykh ryadov ekonomicheskikh pokazatelei*]. *Sovremennie problem nauki i obrazovaniya*, 2014, Iss. 6, 113 p. [Electronic resource]: https://elibrary.ru/download/elibrary_22877127_95621621.pdf. Last accessed 16.01.2024. EDN: TGQDAZ.

12. Barabash, T. K., Maslovskaya, A. G. Computer modelling of fractal time series [*Kompyuternoe modelirovanie fraktalnykh vremennykh ryadov*]. *Bulletin of Amur State University. Natural* and economic sciences, 2010, Iss.49, pphttps://elibrary.ru/contents.asp?id=33826785&selid=18986248. 31–38. [Electronic resource]: https://vestnik.amursu.ru/wp-content/uploads/2017/12/N49_7.pdf. Last accessed 16.01.2024.

13. Garafutdinov, R. V., Akhunyanova, S. A. Adapted box-counting method for assessment of the fractal dimension of financial time series. *Applied Mathematics and Control Issues*, 2020, Iss. 3, pp. 185–218. DOI: 10.15593/2499-9873/2020.3.10.

14. Gerogiorgis, D. I. Fractal scaling in crude oil price evolution via Time Series Analysis (TSA) of historical data. *Chemical Product and Process Modeling*, 2009, Vol. 4, No. 5. DOI: https://doi.org/10.2202/1934-2659.1370.

15. Nekrasova, I. V. The Hurst index as a measure for the fractal structure and long-term memory of financial markets. *International Research Journal*, 2015, Iss. 7 (38), pp. 87–91. [Electronic resource]: https://research-journal.org/media/PDF/irj_issues/7–3–38.pdf#page=87. Last accessed 15.01.2024.

16. Simonov, P. M., Garrafutdinov, R. V. Modelling and forecasting of financial instruments dynamics using econometrics models and fractal analysis. *Bulletin of Perm University. Economics*, 2019, Vol. 14, Iss. 2, pp. 268–288. DOI: 10. 17072/1994–9960–2019–2–268–288.

17. Generalova, A. A., Bychkov, D. S. Improving the aerodynamic properties of an intercity bus using fractal theory [*Uluchshenie aerodunamicheskikh svoistv mezhdugorodnego avtobusa s primeneniem teorii fraktalov*]. *Models, systems, networks in economics, technology, nature and society*, 2015, Iss. 2 (14), pp. 158–166. [Electronic resource]: https://elibrary.ru/download/elibrary_24107892_77708413.pdf. Last accessed 13.01.2024.

18. Karablin, O. V. On the fractal nature of city automobile traffic [*O fraktalnom kharaktere avtomobilnogo trafika goroda*]. *Economy: yesterday, today, tomorrow*, 2018, Vol.8, Iss. 9A, pp. 287–292. [Electronic resource]: http://www.publishing-vak.ru/file/archive-economy-2018–9/32-karablin.pdf. Last accessed 13.01.2024.

19. Pengjian Shang, Meng Wan, Santi Kama. Fractal nature of highway traffic data. *Computers and Mathematics with Applications*, 2007, Vol. 54, Iss. 1, pp. 107–116. DOI: doi.org/10.1016/j.camwa.2006.07.017.

20. Qiang Meng, Hooi Ling Khoo. Self-Similar Characteristics of Vehicle Arrival Pattern on Highways. Journal of Transportation Engineering, 2009, Vol. 135, Iss. 11, pp. 864–872. DOI: doi.org/10.1061/(ASCE)0733-947X(2009)135:11(864).

21. Xuewei Li, Pengjian Shang. Multifractal classification of road traffic flows. *Chaos, Solitons and Fractals*, 2007, Vol. 31, Iss. 5, pp. 1089–1094. DOI: doi.org/10.1016/j.chaos.2005.10.109.

22. David, S. A., Machado, J. A. T., Inacio, C. M. C., Valentim, C. A. A combined measure to differentiate EEG signals using fractal dimension and MFDFA-Hurst. *Communications in Nonlinear Science Numerical Simulation*, 2020, Vol. 84, 105170. DOI: https://doi.org/l0.l0l6/j.cnsns.2020.105170.

23. Erofeeva, E. S., Lyapunova, E. A., Oborin, V. A., Gileva, O. S., Naimark, O. B. Structural and functional analysis of dental hard tissues in assessing the quality of whitening technologies [*Strukturno-funktsionalniy analiz tverdykh tkanei zubov v otsenke kachestva tekhnologii otbelivaniya*]. *Russian Journal of Biomechanics*, 2010, Vol. 14, Iss. 2 (48), pp. 47–55. [Electronic resource]: https://www.elibrary.ru/download/elibrary_15105346_39403207.pdf. Last accessed 16.01.2024.

24. Boyarshinov, M. G. The method of normalised range for the analysis of traffic flow intensity [*Metod normirovannogo razmakha dlya analiza intensivnosti transportnogo potoka*]. *Bulletin of the State Budgetary Institution «Scientific Center for Life Safety»*, 2020, Iss. 2 (46), pp. 35–46. [Electronic resource]: https://elibrary.ru/download/elibrary_44788054_61375863.pdf. Last accessed 15.01.2024.

25. Boyarshinov, M. G., Vavilin, A. S. The deterministic component of the traffic flow intensity. IOP Conference Series: Materials Science and Engineering, International Conference: Actual Issues of Mechanical Engineering (AIME 2020) 27th – 29th October 2020, Saint-Petersburg, Russian Federation, 2021, 1111 (1), 012013(10 p). DOI: 10.1088/1757-899X/1111/1/012013.

26. Boyarshinov, M. G., Vavilin, A. S., Shumkov, A. G. Using the complex of photo and video recording of traffic violations to identify deterministic and stochastic components of the traffic flow intensity. *Intellekt. Innovacii. Investicii*, 2021, Iss. 3, pp. 61–71. DOI: 10.25198/2077-7175-2021-3-61.

27. Boyarshinov, M. G., Vavilin, A. S., Shumkov, A. G. Fourier analysis of the traffic flow intensity. *Intellekt. Innovacii. Investicii*, 2021, Iss. 4, pp. 46–59. DOI: 10.25198/2077-7175-2021-4-46.

28. Boyarshinov, M. G., Vavilin, A. S., Vaskina, E. V. Application of the Hurst index to research the traffic flow

intensity. *Intellekt. Innovacii. Investicii*, 2022, Iss. 2, pp. 68–81. DOI: 10.25198/2077-7175-2021-2-68.

29. Boyarshinov, M. G., Vavilin, A. S., Vaskina, E. V. Application of wavelet analysis to investigate traffic flow intensity. *Intellekt. Innovacii. Investicii*, 2022, Iss. 4, pp. 72–87. DOI: doi.org/10.25198/2077-7175-2022-4-72.

30. Grassberger, P., Procaccia, I. Characterization of Strange Attractors, Physical Review Letters, 1983, 50, pp. 346–349. [Electronic resource]: https://e-l.unifi.it/pluginfile.php/591014/mod_resource/content/0/PhysRevLett.50.346.pdf. Last accessed 16.01.2024.

31. Krylova, O. I., Tsvetkov, I. V. A software package and algorithm for calculating the fractal dimension and linear trend of time series. *Software products and systems*, 2012, Iss. 4, pp. 106–110. [Electronic resource]: https://swsys.ru/index.php?page=article&id=3320&lang. Last accessed 16.01.2024. EDN: OXSJMU.

32. Deshcherevsky, A. V. Fractal dimension, Hurst exponent and slope angle of the time series spectrum. Moscow, Institute of Seismology, Joint institute of Physics of the Earth named after O. Yu. Schmidt, Russian Academy of Sciences, 1997, 34 p. [Electronic resource]: https://www.elibrary.ru/download/elibrary_26650013_92379281.pdf. Last accessed 16.01.2024. EDN: WLESYZ.

33. Starchenko, N. V. Fractality index and local analysis of chaotic time series. Abstract of Ph.D. (Physics and Mathematics) thesis [*Indeks fraktalnosti i lokalniy analiz khaoticheskikh vremennykh ryadov. Avtoref. diss…kand. fiz-mat nauk*]. Moscow, MEPhI (State University), 2005, 24 p. [Electronic resource]: https://viewer.rusneb.ru/ru/000200_000018_RU_NLR_bibl_1085207?page=1&rotate=0&theme=white. Last accessed 16.01.2024.

34. Anisimov, I. A., Osipov, G. S. Comparison of classical and modified methods for calculating the fractal dimension of time series using the Hurst exponent [*Sravnenie klassicheskogo i modifitsirovannogo metodov rascheta fraktalnoi razmernosti vremennykh ryadov s pomoshchyu pokazatelya Hersta*]. *International Journal of Humanities and Natural Sciences*, 2020, Vol. 10–2 (49), pp. 6–10. DOI: 10.24411/2500-1000-2020-11104.

35. Drew, D. R. Traffic flow theory and control [Edition in Russian]. Moscow, Transport publ., 1972, 424 p.

36. Silyanov, V. V. Theory of traffic flows in road design and traffic organization [*Teoriya transportnykh potokov v proektirovanii dorog i organizatsii dvizheniya*]. Moscow, Transport publ., 1977, 303 p.

37. Sutcliffe, J., Hurst, S., Awadallah, A. G., Brown, E., Hamed, Kh. Harold Edwin Hurst: the Nile and Egypt, past and future. *Hydrological Sciences Journal*, 2016, Vol. 61, Iss. 9, pp. 1557–1570. DOI: 10.1080/02626667.2015.1019508.

38. Golub, Yu. Ya. Fractal dimensionality of time series multiplied by a number and multiplication of other time series [*Izychenie fraktalnoy razmernosti vremennogo riada na chislo i umnozheniya vremennykh riadov*]. *Science and Business: Ways of Development*, 2016, Iss. 5 (59), pp. 72–76. [Electronic resource]: https://www.elibrary.ru/download/elibrary_26384218_98248727.pdf. Last accessed 13.01.2024. EDN: WFJNCD.

39. Golub, Yu. Ya. Analytical Analysis of the Fractal Dimensionality of a Cross Rate of One Currency in Relation to Another [*Analiticheskoe rassmotrenie fraktalnoi razmernosti kross-kursov odnoi valyuty po otnosheniyu k drugoi*]. *Science and Business: Ways of Development*, 2014, Iss. 7 (37), pp. 42–45. [Electronic resource]: https://www.elibrary.ru/download/elibrary_22309341_13493325.pdf. Last accessed 14.01.2024. EDN: SUFBVT.

40. Mikhailov, V. V., Kirnosov, S. L., Gedzenko, M. O. Fractal model of processing streaming data in the task of forecasting weather conditions [*Fraktalnaya model obrabotki potokovykh dannykh v zadache prognozirovaniya uslovii pogody*]. *Problems of ensuring safety in elimination of consequences of emergency situations*, 2013, Vol. 2, Iss.1 (2), pphttps://www.elibrary.ru/contents.asp?id=34248283&selid=26293729. 43–46. [Electronic resource]: https://www.elibrary.ru/download/elibrary_26293729_82151009.pdf. Last accessed 13.01.2024. EDN: WDKAMR.

41. Li Li, Zhiheng Li, Yi Zhang, Yudong Chen. A Mixed-Fractal Traffic Flow Model Whose Hurst Exponent Appears Crossover. Fifth International Joint Conference on Computational Sciences and Optimization, Conference Publishing Service, 2012, pp. 443–447. DOI 10.1109/CSO.2012.103.

42. Kaklauskas, L., Sakalauskas, L. Study of on-line measurement of traffic self-similarity. CEJOR, 2013, Vol. 21, pp. 63–84. DOI 10.1007/s10100–011–0216–5.

43. Mehrvar, H. R., Le-Ngoc, T. Estimation of Degree of Self-Similarity for Traffic Control in Broadband Satellite Communications. Proceedings 1995 Canadian Conference on Electrical and Computer Engineering, Montreal, QC, Canada, 1995, Vol. 1, pp. 515–518. DOI: 10.1109/CCECE.1995.528187.

44. Glavatskiy, S. P. Statistical analysis of social media traffic [*Statisticheskiy analiz trafika sotsialnykh setei*]. *Naukovi pratsi ONAZ im. O. S. Popova*, 2013, Iss. 2, pp. 94–99. [Electronic resource]: https://www.elibrary.ru/item.asp?id=21597433. – Last accessed 14.08.2023.

45. Zhmurko, D. Yu., Osipov, A. K. Forecasting the development indicators of the sugar industry using fractal analysis methods [*Prognozirovanie pokazatelei razvitiya sakharnoi otrasli s primeneniem metodov fraktalnogo analiza*]. *Bulletin of Udmurt University. Economics and Law*, 2018, Vol. 28, Iss. 2, pp. 185–193. [Electronic resource]: https://www.elibrary.ru/download/elibrary_35078230_18865640.pdf. Last accessed 14.08.2023. EDN: LVBFMT.

46. Lopukhin, A. M. Application of fractal analysis methods to forecasting development indicators of coffee industry enterprises [*Primenenie metodov fraktalnogo analiza k prognozirovaniyu pokazatelei razvitiya predpriyatiyi kofeinoi otrasli*]. *Continuum. Mathematics. Computer science. Education*, 2020, Iss. 4, pp. 70–77. DOI: 10.24888/2500-1957-2020-4-70-79.

47. Shmyrin, A. M., Sedykh, I. A., Shcherbakov, A. P. Nonlinear analysis methods in investigating clinker production characteristics [*Metody nelineinogo analiza pri issledovanii kharakteristik proizvodstva klinkera*]. *Vestnik TGU*, 2014, Vol. 19, Iss. 3, pp. 923–926. [Electronic resource]: https://www.elibrary.ru/download/elibrary_21830477_25858437.pdf. Last accessed 14.08.2023. EDN: SJSQBH.

48. Can Ye, Huiyun Li, Guoqing Xu. An Early Warning Model of Traffic Accidents Based on Fractal Theory. 17th International IEEE Conference on Intelligent Transportation Systems (ITSC), Qingdao, China, 2014, pp. 2280–2285. DOI: 10.1109/ITSC.2014.6958055. ●

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