ЗАКЛЮЧЕНИЕ

Основными результатами теоретических расчетов, если суммировать содержание статьи, являются аналитическая оценка минимального интервала попутного следования поездов при построении систем обеспечения безопасности на базе радиоканала (класса CBTC), оценка величины минимального интервала по сравнению с минимально возможной потенциальной оценкой для «идеальной системы», анализ причин, влияющих на увеличение минимального интервала попутного следования.

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Координаты автора: Баранов Л. А. – baranov.miit@gmail.com.
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EVALUATION OF METRO TRAIN SUCCESSION TIME FOR SAFETY SYSTEMS BASED ON RADIO CHANNEL

Baranov, Leonid A., *Moscow State University of Railway Engineering (MIIT), Moscow, Russia.*

ABSTRACT

Improving safety of metro trains in conditions of growing intensity of passenger traffic requires improvement of management, a combination of automation and systems of information exchange by radio channel. The article substantiates with theoretical calculations the possibility of estimating minimum succession time for trains passing along metro lines, equipped with a safety system and information means of a class Communications-Based Train Control (CBTC). The base is the author's method that compares values obtained with potential values of a «perfect system».

Keywords: underground, metro, train, succession time, traffic safety system, analytical assessment, methods, mathematical calculation, algorithm, radio channel, information.

Background. *Meeting the requirements of comfort and safety of passenger transportation, especially in the high-intensity schedule involves improving systems of automatic or automated train traffic control. System of dispatch control, centralized automatic driving system, security systems should have a single unifying origin [1,2].*

Objective. *The objective of the author is to provide theoretical calculations and justification for estimating minimum succession time for trains, operating, mainly, in metro.*

Methods. *The author uses engineering and mathematical methods, analysis, coordinate method.*

Results.

General provisions

Implementation of high traffic intensity is associated with the need to have sufficient power of traction substations, reduction of traction networks resistance and increase in level of their protection, ability to distinguish between comparable load currents and remote short-circuit that are more and more important with increasing power of modern rolling stock.

Compliance with restrictions associated with power supply of given traffic intensity sets a task of implementing required succession time by safety systems and station centralization. On the domestic underground a system of automatic locking ARS- ALS is successfully operated, track circuits are used as train positioning sensors, detectors of running rails integrity, determinants of block section occupancy [3, 4]. However, application of systems to ensure traffic safety on the basis of the radio channel (class CBTC–Communications-Based Train Control) is widely debated [5]. Their obvious advantage is a lack of fixed block sections, as well as track circuits, for noise immunity of which filtering of noise resulting in traction drives and power installations is required.

However, systems of class CBTC do not provide control of running rails integrity, which is an obstacle to their use. To remove this obstacle, options are considered for development of individual control systems or combination of ARS and ALS and systems for sharing information over the radio channel.

Arguments defining virtues of this approach are the increase in traffic safety by parallel operating independent systems, implemented on different physical principles. While at the same time there is another problem: by parallel operating security system restriction is adopted for safer signal. Consequently, in these circumstances, advantages of the system based on CBTC principles and designed to maintain a minimum succession time will be in demand in the event of malfunction of classical ARS- ALS system.

Existing publications on CBTC systems describe principles of their construction [6], range and kind of solutions [7], requirements for radio channel [8].

This paper presents an assessment of minimum succession time of trains passing along metro lines, equipped with a traffic safety system on the basis of information exchange over radio channel (class CBTC). We used the technique to obtain potentially possible minimum succession time, developed by the author for the «ideal system» [9].

Algorithmization of system

Let's consider the algorithm of traffic safety system functioning on the basis of information exchange over radio channel. Let the information about coordinate and speed of foregoing train (first) is provided by radio channel to a server, which sends it to a train, following in the rear (second). If the second train in each sufficiently short time interval T, equal to the amount of time of information transmission over radio channel with account of the method of its construction, operation protocol and duration of information processing in the server, receives information only about the coordinate of the «rear end» of the first train, and it allows its instantaneous stop, then determining permissible safe speed of a train, following in the rear, is considered control on coordinate of the «rear end». When transmitting additional information about speed of foregoing train the same operation for the second train is performed in the light of emergency braking distance of the first one. The calculation of permissible speed is carried by onboard safety devices.

The results of calculation of minimum succession time, obtained in the course of control, taking into account emergency braking distance, are more common. Firstly, under these conditions, the succession time according to conditions of traffic safety is minimum possible; secondly, if in expressions defining minimum succession time, emergency braking distance of foregoing train is taken as zero (which is equivalent to the assumption on equality of deceleration infinity), then we have an option with control based on coordinate of the «rear end». The difference between values of succession time for these two principles of building traffic safety systems (hereinafter-TSS) will assess the effectiveness of the use of additional information about a foregoing train. This approach is used by the author to determine the potential minimum succession time at ideal TSS [9], in which information about coordinates and speed of trains has no errors, and data transmission is considered instant and error-free.

We introduce the following notations:

S_1, $\overset{0}{S}_1$ are respectively measured and true values of the «rear end» coordinate of the foregoing train;

S_2, $\overset{0}{S}_2$ are measured and true values of the «head end» of a train, following in the rear;

$\Delta S_1 = \overset{0}{S}_1 - S_1$ and $\Delta S_2 = \overset{0}{S}_2 - S_2$ are errors in measurement of train coordinates;

V_1, $\overset{0}{V}_1$ are measured and true values of speed of a foregoing train;

V_2, $\overset{0}{V}_2$ are measured and true values of speed of a train, following in the rear;

$\Delta V_1 = \overset{0}{V}_1 - V_1$ and $\Delta V_2 = \overset{0}{V}_2 - V_2$ are errors in measurement of train speed.

Since values of errors are random and can be either positive or negative, further selection of the error sign will depend on how it affects the calculation result. Thus, the «minus» sign in front of error value indicates that a measured value is greater than a true one, otherwise, a «plus» sign is used.

When trains are moving between the «rear end» of a foregoing train and «head end» of a train, following in the rear under safety conditions should not be less than the difference in the distance of service braking S_{T2} of a train, following in the rear, the speed of which is equal to V_2, and the distance of emergency braking of a foregoing train, the speed of which is V_1. If $\overset{0}{S}_1$ is coordinate of the «rear end» of the first train, and $\overset{0}{S}_2$ is coordinate of the «head end» of the train following it, the above formulated conditions can be written as:

$$\overset{0}{S}_1 - \overset{0}{S}_2 \geq S_{T2}(\overset{0}{V}_2) - S_{T1}(\overset{0}{V}_1)\ . \qquad (1)$$

In construction and operation of TSS measured values of corresponding quantities are used. We show that the replacement of true values with values measured errors without taking necessary amendments could result in emergency situations.

For example, if the coordinate of the «head end» of the second train is measured with an error of $+\Delta S_2$, and the coordinate of the «rear end» of the first train with an error ΔS_1, the true distance between the «head end» of the second and the «rear end» of the first trains is

$$S_1 - \Delta S_1 - (S_2 + \Delta S_2) = S_1 - S_2 - (\Delta S_1 + \Delta S_2). \qquad (2)$$

Hence the value $S_1 - S_2$, used by TSS for calculation of allowable speed, exceeds the true one by $\Delta S_1 + \Delta S_2$, which may lead to an increase in the allowable speed and, as a consequence, to an accident.

The same result occurs because of errors in speed measurement. Let the true speed of the second train be $\overset{0}{V}_2 = V_2 + \Delta V_2$, and of the first $\overset{0}{V}_1 = V_1 - \Delta V_1$; values V_1 and V_2 are taken by TSS in calculating the permissible speed as the original. Since $S_{T2}(V_2 + \Delta V_2) > S_{T2}(V_2)$, $S_{T1}(V_1 + \Delta V_1) > S_{T1}(V_1)$, then

$$S_{T2}(V_2) - S_{T1}(V_1) > S_{T2}(V_2 + \Delta V_2) - S_{T1}(V_1 - \Delta V_1).$$

If we denote

$$S_{T2}(V_2 + \Delta V_2) - S_{T2}(V_2) = \Delta S_{T2}(V_2 + \Delta V_2),$$

$$S_{T1}(V_1 + \Delta V_1) - S_{T1}(V_1) = \Delta S_{T1}(V_1 + \Delta V_1),$$ then

$$S_{T2}(V_2) - S_{T1}(V_1) < S_{T2}(V_2) + \Delta S_{T2}(V_2 + \Delta V_2) - S_{T1}(V_1) + \Delta S_{T1}(V_1 + \Delta V_1).$$

It follows that the replacement in (1) of values $\overset{0}{V}_2$ and $\overset{0}{V}_1$ with V_2 and V_1 decreases the right side by a value $\Delta S_{T2}(V_2 + \Delta V_2) + \Delta S_{T1}(V_1 + \Delta V_1)$, which in its turn leads to violations of safety requirements.

Safety conditions for above fixed values of errors are respected when

$$S_1 - S_2 \ge \left[S_{T2}(V_2) + \Delta S_2 + \Delta S_{T2}(V_2 + \Delta V_2)\right] - \left[S_{T1}(V_1) - \Delta S_1 - \Delta S_{T1}(V_1 + \Delta V_1)\right]$$

and

$$S_1 - S_2 \ge S_{T2}(V_2) - S_{T1}(V_1) + \left[\Delta S_1 + \Delta S_2 + \Delta S_{T1}(V_1 + \Delta V_1) + \Delta S_{T2}(V_2 + \Delta V_2)\right]. \quad (3)$$

From this expression it follows that to determine the allowable speed with respect to safety conditions while using measurement results of distance and speed with errors it is necessary to introduce a protective gap with length of

$$S_{prot} > \Delta S_1 + \Delta S_2 + \Delta S_{T1}(V_1 + \Delta V_1) + \Delta S_{T2}(V_2 + \Delta V_2), \quad (4)$$

determined by values and signs of corresponding errors.

Safety conditions (1), valid for an ideal system, in the case of real measuring channels change and have a form of

$$S_1 - S_2 \ge S_{T2}(V_2) - S_{T1}(V_1) + S_{T2}(V_2) - S_{T1}(V_1) + S_{prot}. \quad (5)$$

It should be separately noted that the choice of length of S_{prot} is directly related to the evidence base that allows to find maximum values of corresponding errors.

In the stochastic approach an evidence is required that a probability that a measurement result for a certain range does not exceed a value, set by a standard.

Thus, the selection of the permissible speed of the second train should be carried out within the framework of the control at the end of protective period, which moves along with the «rear end» of a foregoing train, taking into account (or without) its emergency braking distance.

Succession time at some point S_0 of the haul is determined by the formula

$$T_u(S_0) = T_2(S_0) - T_1(S_0), \quad (6)$$

where $T_1(S_0)$ and $T_2(S_0)$ are moments of passing of a point S_0 by «head ends» of respectively the first and the second trains.

If a «head end» of the first train is located at the point S_1 of the route (hence, «rear end» of this train is located at the point $S_1 - l_{tr}$, where l_{tr} is length of the train), «head ends» of the second train at the point $S_2 = S_1 - S_{prot} - S_{T2}(V_2) + S_{T1}(V_1)$ TSS system still does not limit the speed of movement V_2j of a train, following the rear. This can be written as follows:

$$T_2\left[S_2 = S_1 - S_{prot} - S_{T2}(V_2) + S_{T1}(V_1)\right] \ge T_1(S_1) + T_d, \quad (7)$$

where T_2 is time, when «head end» of the second train, which moves at speed V_2, is located at the point $S_2 = S_1 - S_{prot} - S_{T2}(V_2) + S_{T1}(V_1)$;

T_1 is time, when «rear end» of the first train, which moves at speed of V_1, is located at the point S_1;

T_d is time of delivery of information about the coordinate of the «rear end» and speed of the first train to the second train.

Assessment of the value of T_d will be given later. We assume that the time of delivery of information does not depend on coordinates of trains.

In inequation (7) coordinates S_2, S_1, V_1, V_2 are measured variables, which differ from true values. To compensate the effects of errors and the driver reaction time (or train device of automatic driving system) F_{min} is facultative, in particular, typically equal to 5 s for conditions of underground.

Therefore, according to (7):

$$F_{\min} \ge T_2\begin{bmatrix} S_2 = S_1 - S_{prot} - \\ -S_{T2}(V_2) + S_{T1}(V_1) \end{bmatrix} - T_1(S_1) - T_d. \quad (8)$$

If, respectively, from left and right sides (6) we deduct left and right sides (8) and transpose F_{min} to the right side, we get

$$T_u(S_0) \ge T_2(S_0) - T_1(S_0) + F_{\min} + T_d + T_1(S_1) - T_2\left[S_2 = S_1 - S_{prot} - S_{T2}(V_2) + S_{T1}(V_1)\right]. \quad (9)$$

Since succession time T_u satisfies safety conditions, if inequality (9) holds for all $\ge S_0$, T_u, the value of $T_{u\,min}$ is minimum succession time, which is determined by the maximum value of the right side of (9):

$$T_{u\min} = \max_{S_1}\left\{\begin{array}{l} F_{\min} + T_d + T_2(S_0) - \\ - T_2\begin{bmatrix} S_2 = S_1 - S_{prot} - \\ -S_{T2}(V_2) + S_{T1}(V_1) \end{bmatrix} + \\ + T_1(S_1) - T_1(S_0) \end{array}\right\}. \quad (10)$$

The point with coordinates S_1^* and respectively V_1^*, in which the maximum of the right side (9) is provided, is called limiting.

A necessary condition for the maximum of the right side of (9) has a form

$$\frac{d}{dS_1}\left\{\begin{array}{l} F_{\min} + T_d - T_1(S_0) + T_1(S_1) - \\ - T_2\left[S_2 = S_1 - S_{prot} - S_{T2}(V_2) + S_{T1}(V_1)\right] \end{array}\right\} = 0,$$

from which

$$\frac{dT_1(S_1)}{dS_1} - \frac{dT_2\begin{bmatrix} S_2 = S_1 - S_{prot} - \\ - S_{T2}(V_2) + S_{T1}(V_1) \end{bmatrix}}{dS_2} \cdot \left[1 + \frac{dS_{T1}(V_2)}{dS_1}\right] = 0$$

and

$$\frac{V_2[S_2 = S_1 - S_{prot} - S_{T2}(V_2) + S_{T1}(V_1)]}{1 + \frac{dS_{T1}(V_1)}{dS_1}} = V_1(S_1). \quad (11)$$

Putting in this expression $S_{prot} = 0$, we obtain a result, which coincides with appropriate conditions for a perfect traffic safety system [2].

We estimate $\frac{dS_{T1}(V_1)}{dS_1}$ for a model of uniformly decelerated motion during deceleration of emergency braking b_e from speed V_1, denoting time by t:

$$S_{T1}(V_1) = \frac{V_1^2(S_1)}{2b_e};$$

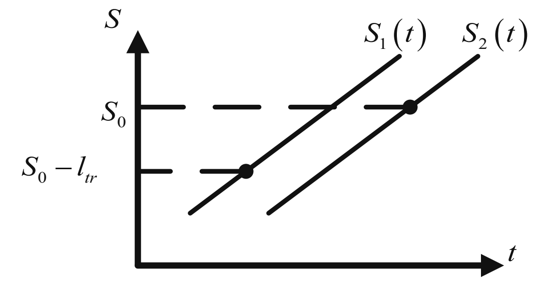
$$\frac{dS_{T1}(V_1)}{dS_1} = \frac{1}{2b_e} \times 2V_1(S_1)\frac{dV_1(S_1)}{dS_1} =$$

$$= \frac{1}{b_e}V_1(S_1)\frac{dV_1(S_1)}{dt}\frac{dt}{dS_1} =$$

$$= \frac{1}{b_e}\frac{dV_1(S_1)}{dt}\frac{1}{V_1(S_1)}V_1(S_1) = \frac{a_1}{b_e},$$

where $\frac{1}{V_1(S_1)}V_1(S_1) = \frac{a_1}{b_e}$,

and here $a_1 = \frac{dV_1(S_1)}{dt}$



Pic. 1. Dependence on time t of the coordinate S_1 of the «rear end» of a foregoing train and coordinate S_2 of the «head end» of a train, following in the rear while driving on the haul with constant speed $V_1 = V_2 = V$.

means acceleration of a foregoing train. Thus:

$$\frac{V_2[S_2 = S_1 - S_{prot} - S_{T2}(V_2) + S_{T1}(V_1)]}{1+\frac{a_1}{b_e}} = V_1(S_1). \qquad (12)$$

Equation (12) together with predetermined motion trajectories of trains enables to determine coordinates of a limiting point (S_1^*, V_1^*).

Following the procedure described in [2], we can show that the review process of an emergency braking as a uniformly decelerated motion hardly makes errors in the calculation of $T_{u\,min}$.

Determination of succession time during movement on the haul at a constant speed

Formed dependences on time t of coordinate of the «rear end» of a foregoing train and coordinate S_2 of the «head end» of a train, following in the rear, moving on the haul at a constant speed are shown in Pic. 1.

$S_2(t)$ $S_1(t)$ $S_0 - l_{tr}$ S_0 t S

Location of the limiting point S_1^* and speed V_1^* of a foregoing train at this point is determined from the equation (11). As trains move at constant speed V, then $a_1 = 0$. In this case $V_2 = V_1 = V^* = V$ and $T_{u\,min}$ do not depend on S_1. Putting in (11) $S_0 = S_1 + l_{tr}$ and $V_1 = V_2 = V$, we get

$$\begin{aligned} T_{u\min} &= F_{\min} + T_d + T_2(S_1 + l_{tr}) - \\ &- T_2\left[S_1 - S_{prot} - S_{T2}(V) + S_{T1}(V)\right] + \\ &+ T_1(S_1) - T_1(S_1 + l_{tr}) = \\ &= F_{\min} + T_d + \frac{l_{tr} + S_{prot} + S_{T2}(V) - S_{T1}(V)}{V}. \end{aligned} \qquad (13)$$

Dependences $S_{T2}(V)$ and $S_{T1}(V)$ can be obtained from traction calculations.

For a model of uniformly decelerated motion

$$S_{T2}(V) = \frac{V^2}{2_{b_s}};\; S_{T1}(V) = \frac{V^2}{2_{b_e}},$$

where b_s and b_e are respectively deceleration during service and emergency braking.

After substitution of $S_{T2}(V)$ and $S_{T1}(V)$ in (13) we obtain

$$T_{u\min} = F_{\min} + T_d + \frac{l_{tr} + S_{prot}}{V} + \frac{V}{2}\left[\frac{1}{b_s} - \frac{1}{b_e}\right]. \qquad (14)$$

Increase in the minimum succession time as compared to appropriate for an ideal system is

$$\Delta T_{u\min} = T_d + \frac{S_{prot}}{V}. \qquad (15)$$

Testing for extreme point the function $T_{u\,min}(V)$, we get speed value V_{opt}, minimizing $T_{u\,min}$:

$$\frac{dT_{u\min}}{dV} = -\frac{l_{tr} + S_{prot}}{V^2} + \frac{1}{2}\left[\frac{1}{b_s} - \frac{1}{b_e}\right] = 0$$

and $$V_{opt} = \sqrt{\frac{2(l_{tr} + S_{prot})}{\frac{1}{b_s} - \frac{1}{b_e}}}; \qquad (16)$$

$$\begin{aligned} T_{\min opt} &= F_{\min} + T_d + \\ &+ \frac{1}{2}\sqrt{\frac{2(l_{tr} + S_{prot})}{\frac{1}{b_s} - \frac{1}{b_e}}}\left[\frac{1}{b_s} - \frac{1}{b_e}\right] + \\ &+ \frac{l_{tr} + S_{prot}}{\sqrt{\frac{2(l_{tr} + S_{prot})}{\frac{1}{b_s} - \frac{1}{b_e}}}}. \end{aligned} \qquad (17)$$

While basing the control process on the coordinate of the «rear end» of a foregoing train $b_e \to \infty$ and

$$T_{U\min} = F_{\min} + T_d + \frac{l_{tr} + S_{prot}}{V} + \frac{V}{2b_s}; \qquad (18)$$

$$V_{opt} = \sqrt{2(l_{tr} + S_{prot})b_s}; \qquad (19)$$

$$\begin{aligned} T_{\min opt} &= F_{\min} + T_d + \frac{1}{2b_s}\sqrt{2(l_{tr} + S_{prot})b_s} + \\ &+ \frac{l_{tr} + S_{prot}}{\sqrt{2(l_{tr} + S_{prot})}}. \end{aligned} \qquad (20)$$

Interval between departing and arriving trains

Let the coordinate of the «head end» of the train standing at the station (otherwise – points of its stop) be $S = l_{tr}$. Then S = 0 is coordinate of the «rear end» of this train. Duration of its stop is denoted by us as T_{st}. The second train approaches the station, usually on slowing-down. Its speed with account for restrictions on permissible ascent and descent from the area with zero profile to 3% can be assumed to be constant and equal to V_{2H}. The assumption of constancy of V_{2H} will be used to simplify calculations $T_{u\,min}$.

The interval between the first and second trains we define at the point $S_0 = l_{tr}$ according to (6):

$T_u = T_{20}(S_0=l_{tr}) - T_{10}(S_0=l_{tr})$, (21)

where $T_{10}(S_0=l_{tr})$ and $T_{20}(S_0=l_{tr})$ are moments of departure of the first and the second trains from the station.

Since $T_{20}(S_0=l_{tr}) = T_2(S_2-l_{tr}) + T_{st}$, where $T_2(S_2=l_{tr})$ is moment of stop of the second train at the station, $T_{10}(S_0=l_{tr}) = T_1(S_1=0)$, where S_1 is coordinate of the «rear end» of the first train, then

$T_u = T_2(S_2-l_{tr}) - T_1(S_1=0) + T_{st}$. (22)

The speed of the first train at the limiting point S_1^* is determined from (12) under the condition of control with account of its emergency braking distance:

$$V_1\left(S_1^*\right) = \frac{V_{2H}}{1+\dfrac{a_1}{b_e}}, \quad (23)$$

where $a_1 = a_s$ is acceleration of speeding-up of a departing train. Coordinate of the limiting point S_1^* can be found from traction calculation as a distance, covered by a train from the moment of start-up to the speed V_1^*. Coordinate, at which «head end» of the second one can be located at a point, corresponding to safety conditions:

$$S_2^* = S_1^* - l_{tr} - S_{prot}. \quad (24)$$

Hence it is apparent that the point (limiting for the second train) is within the approach to the station.

For a model of uniformly decelerated motion of the first train

$$S_1^* = \frac{V_1^{*2}}{2a_s}. \quad (25)$$

Minimum interval according to (10):

$$T_{u\min} = F_{\min} + T_{st} + T_d + T_2\left(S_2 = l_{tr}\right) -$$
$$-T_2\left[S_1^* + S_{prot} + S_{T1}\left(V_1^*\right)\right] + T_1\left(S_1^*\right) - T_1(0). \quad (26)$$

Here it is taken into account that at $S_0=l_{tr}$ moment when the «head end» of the first train is located at the point S_0, its «rear end» is located at the point taken as the origin of coordinates for the route $S_0-l_{tr}=0$, i. e. $T_1^{\Gamma}\left(S_0 - l_{tr}\right) = T_1(0)$.

Let's transform formula (26) to a form convenient for calculations based on the assumption of constant speed V_{2H}:

$$T_{u\min} = F_{\min} + T_{st} + T_d + T_{1p}(V_1^*) + T_{2T}(V_{2H}) +$$
$$+\frac{l_{tr} + S_{prot} + S_{T2}(V_{2H}) - S_{2dtb}(V_{2H}) - S_{1p}(V_1^*) - S_{1e}(V_1^*)}{V_{2H}}, \quad (27)$$

Where $T_{1p}(V_1^*) = T_1\left(S_1^*\right) - T_1(0)$ is speeding-up time of a train, departing from a station, to a speed of V_1^*;

$T_{2T}(V_{2H})$ is time of target braking of the second train, starting from speed V_{2H};

$S_{2dtb}(V_{2H})$ is distance of target braking of the second train from speed V_{2H};

$S_{1p}(V_1^*)$ is speeding-up path of the first train to speed V_1^*;

$S_{1e}(V_1^*)$ is braking length of the first train at the moment of emergency braking at initial speed V_1^*.

All quantities in (27) can be obtained from traction calculations. Speed V_1^* is determined by the formula (23).

While basing control on the coordinate of the «rear end» of a foregoing train its length of emergency braking is assumed to be zero. Here $b_1 \to \infty$, $V_1^* = V_{2H}$, from (27) it follows:

$$T_{u\min} = F_{\min} + T_{st} + T_d + T_{1p}(V_{2H}) + T_{2T}(V_{2H}) +$$
$$+\frac{l_{tr} + S_{prot} + S_{T2}(V_{2H}) - S_{2dtb}(V_{2H}) - S_{1p}(V_{2H})}{V_{2H}} \quad (28)$$

If in (27) and (28) we assume $T_d = 0$; $S_{prot}=0$, we obtain an expression of a minimum time interval for a perfect system [9]. Therefore, the expression for calculating the minimum time interval assuming uniformly decelerated motion of trains can be obtained in the same way as in [2], considering that $T_d \neq 0$ and $S_{prot} \neq 0$:

$$S_{1p}\left(V_1^*\right) = \frac{V_1^{*2}}{2a_s},\ S_{1e}\left(V_{2H}\right) = \frac{V_1^{*2}}{2b_e};\ T_{1p}\left(V_1^*\right) = \frac{V_1^{*2}}{2a_s};$$

$$S_{2dtb}\left(V_{2H}\right) = \frac{V_1^{*2}}{2b_{tb}};\ T_{2T}(V_{2H}) = \frac{V_1^*}{b_{tb}};\ S_{T2}(V_{2H}) = \frac{V_{2H}^2}{b_s},$$

where b_{tb} and b_s are respectively deceleration at targeting and service braking of the second train.

Substituting these expressions into formulas for the minimum interval (27) and (28), we obtain:

– For control taking into account emergency braking path of a foregoing train

$$T_{u\min} = T_{st} + F_{min} + \quad {}_d + \frac{l_{tr} + S_{prot}}{V_{2H}} +$$
$$+\frac{V_{2H}}{2b_{tb}} + \frac{V_{2H}}{2a_s\left(1+\dfrac{a_s}{b_e}\right)} + \frac{V_2}{2b_s}; \quad (29)$$

– for control based on coordinate of the «rear end» of a foregoing train ($b_e \to \infty$)

$$T_{u\min} = T_{st} + F_{\min} + T_d + \frac{l_{tr} + S_{prot}}{V_{2H}} + \frac{V_{2H}}{2b_{tb}} + \frac{V_{2H}}{2a_s} + \frac{V_{2H}}{2b_s}. \quad (30)$$

Given the error ΔV_{2H} in measurement of V_{2H} provided that $\Delta V_{2H} << V_{2H}$:

$$\Delta T_{u\min} = \left[-\frac{l_{tr} + S_{prot}}{V_{2H}^2} + \frac{1}{2b_{tb}} + \frac{1}{2b_s} + \frac{1}{2a_s\left(1+\dfrac{a_s}{b_e}\right)}\right]\Delta V_{2H} <<$$
$$<< \left[\frac{l_{tr} + S_{prot}}{V_{2H}^2} + \frac{1}{2b_{tb}} + \frac{1}{2b_s} + \frac{1}{2a_s\left(1+\dfrac{a_s}{b_e}\right)}\right]\Delta V_{2H} = \quad (31)$$
$$= \left[\frac{l_{tr} + S_{prot}}{V_{2H}} + \frac{V_{2H}}{2b_{tb}} + \frac{V_{2H}}{2a_s\left(1+\dfrac{a_s}{b_p}\right)} + \frac{V_{2H}}{2b_s}\right]\frac{\Delta V_{2H}}{V_{2H}}.$$

The minimum succession time can be estimated as the sum of $T_{u\,min} + \Delta T_{u\,min}$.

Analysis of the expression (29) shows the possibility of determining the optimal speed of approach to the station via the criterion of minimum train-to-train interval:

$$\frac{dT_{u\min}}{dV_{2H}}=0 \text{ at } V_{2H_{opt}}=\sqrt{\frac{l_{tr}+S_{prot}}{\frac{1}{2в_{tb}}-\frac{1}{2a_s\left(1+\frac{a_s}{b_e}\right)+\frac{1}{2b_s}}}} . \quad (32)$$

For control based on the coordinate of the «rear end» of departing train ($b_e\to\infty$):

$$H_{2H_{opt}}=\sqrt{\frac{l_{tr}+S_{prot}}{\frac{1}{2b_{tb}}+\frac{1}{2a_s}+\frac{1}{b_s}}} . \quad (33)$$

Assessment of duration of the message delivery

Paths T are duration of message delivery to a train, following in the rear, in the absence of decoding refusal, P are probabilities of refusal of decoding of one message.

$P(T_d=T)=1-P$.

Probability that duration of delivery T_d is equal to iT:

$P(T_d=iT)=(1-P)P^{i-1}$ i-1, 2,......

From which average delivery time is:

$$\overline{T}_d=\sum_{i=1}^{\infty} iT(1-P)P^{i-1}=\frac{T}{1-P};$$

Variance of random variable T_d is:

$$\overline{T}_d^2=\sum_{i=1}^{\infty}(iT-\frac{T}{1-P})^2(1-P)P^{i-1}=\frac{PT^2}{(1-P)^2} .$$

Probability that message delivery time will not exceed nT is:

$$P(T_d\le nT)=\sum_{i=1}^{n}P^{i-1}(1-P)=1-P^n .$$

In that case, if the algorithm of TSS operation contains a mechanism of allowable timeout, i. e. period with no information updating, upon expiration of which the system switches to the state of protective refusal, the maximum information delivery time may be equal to T_d=nT. The choice of nT comes from the allowable probability $P(T_d>nT)=P^n$.

Thus, the minimum succession time under safety conditions essentially depends on the quality of the radio transmission channel.

Let T=0,3 s, $P=10^{-4}$. Then for n = 5 the value $P(T_d>nT)=10^{-20}$ and

T_d=5T=1,5 s. (34)

As an example we take calculation of $T_{u\,min}$ for the haul, at which during «peak» hours speed of approach to the station is V_{2H}= 45 km/h, rolling stock in operation has a length of l_{tr}= 176 m, a_s= 0,8 m/s², b_s= 0,85 m/s², b_{tb}= 0,8m/s², length of protective section is s_{prot}= 100 m, relative speed measurement error does not exceed 1,5%, F_{min}= 5 s, time of message delivery in accordance with (34) is equal to 1,5 s, duration of stop T_{st} = 25 s, control is performed basing on the coordinate of the «rear end» of a foregoing train (i. e. $b_e\to\infty$).

Calculation of $\Delta T_{u\,min}$ is performed according to the formula (31) for given initial data. As a result we get $\Delta T_{u\,min}$= 0,68 s. At model of uniformly decelerated motion time is determined by the formula (30): $\Delta T_{u\,min}$ = 79 s. Therefore, succession time can be assumed to be equal to 79 s + 0,68 s = 79,68 s ≈ 80 s, which is 12 s more than a minimum succession time in a «perfect system».

The relative difference between values of minimum intervals calculated by exact and approximate formulas (models of uniformly accelerated motion), at a speed of approach to the station from 40 to 30 km / h does not exceed 4.2%.

Conclusion. *The main results of theoretical calculations, if we summarize the contents of the article, are analytical evaluation of minimum succession time of trains in development of traffic safety systems based on radio channel (class CBTC), estimate of the minimum succession time as compared to the lowest possible potential estimate for the «perfect system», analysis of causes affecting the increase in the minimum succession time.*

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Information about the author:

Baranov, Leonid A. – D.Sc. (Eng.), professor of Moscow State University of Railway Engineering (MIIT), Moscow, Russia, baranov.miit@gmail.com.

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