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Numerical Method for Calculating Plane Flows of Viscous Gas in a Cylinder

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ABSTRACT

To solve scientific, technological and environmental problems in the field of road transport, it is necessary to develop new mathematical models. At this stage of development of internal combustion engines and their parts, it is necessary to model complex hydrodynamic technologies. This necessities revealing different forms of liquid and gas flows with arbitrary initial and boundary conditions for the region under consideration.

The work is devoted to modelling a turbulent flow over a plate located in a cylinder. Reynolds-averaged Navier-Stokes equations were used as a mathematical model of the flow. Turbulent viscosity was calculated using the one-equation Spalart-Allmaras turbulence model. To solve the system of hydrodynamic equations, the finite difference method was used. A numerical method for calculating plane flows of viscous gas in a cylinder using turbulence models is presented. Suitable options were selected based on physical assumptions used to develop the models.

Recommendations have been developed on the possibility of using the considered turbulence models.

Keywords: *road transport, modelling of turbulent flow, gas distribution mechanism, incompressible gas, turbulent viscosity, one-equation model, finite difference method, MacCormack method.*

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INTRODUCTION

Modern conditions of development of road transport, hydraulic structures and other industries require conducting research on a wider class of various issues referring to engines but different from each other in their features. Engine design is a time-consuming and expensive process due to the large number of variables that influence its design. Simulation helps analysing different engine configurations without actually building it and thus reducing cost and time during development stage. Engine modelling consists of two parts: fluid flow simulation and combustion and emptying simulation.

The development of modern engine building is associated with improvement of design and operating principle of the engine. Till now, the internal combustion engine (ICE) was mainly used as vehicles' power plant. To improve the economic and environmental performance of internal combustion engines, it is necessary to improve existing engines and their operating processes [1–3].

The work [4] presents simulations to study the in-cylinder gas flow and heat transfer characteristics of a free-piston compression-ignition engine-generator. An experiment was conducted to obtain accurate piston motion to simulate heat transfer and gas flow, and simulation results presented showed good similarity with the experimental data. The work [5] considered problems of modern piston engine building, the theory of working processes and methods for modelling processes in internal combustion engines.

The article [6] generalises the artificial compressibility method for calculating stationary viscous gas flows, which is suitable for arbitrary values of the Mach number. The paper [7] outlines the current state and areas of applicability of various approaches to turbulence modelling, presents the analysis and content of the most common semi-empirical turbulence models, considers examples of the use of these models for calculating steady flow in a plane channel.

It is known that the working principle of classical internal combustion engines is based on conversion of the thermal energy of fuel combustion into mechanical energy. All car engines operate on the same principle: the air-fuel mixture enters the cylinders, the pistons compress the mixture and it explodes. Gas pressure is generated, which pushes the pistons, while the gas distribution mechanism (GDM) is responsible for filling the cylinders with the working mixture and releasing exhaust gases. The GDM opens and closes the intake and exhaust valves in accordance with the engine's operating cycles. Modern engine building is developing at a rapid pace thanks to the increase in the average effective pressure and the speed of the pistons. High reliability, service life, fuel efficiency and environmental performance are the main criteria that determine the quality of GDM operation [8].

The main disadvantages of all existing internal combustion engines with valve-type gas distribution mechanisms are the increased aerodynamic resistance of the inlet and outlet annular sections, which forms vertical axial mixing of the valve head plates relative to the gas exchange windows. In addition, the designs of GDM are often quite complex, have a low efficiency of heat exchange, reduced coefficients of filling the cylinders with a fresh charge and cleaning them from combustion products.

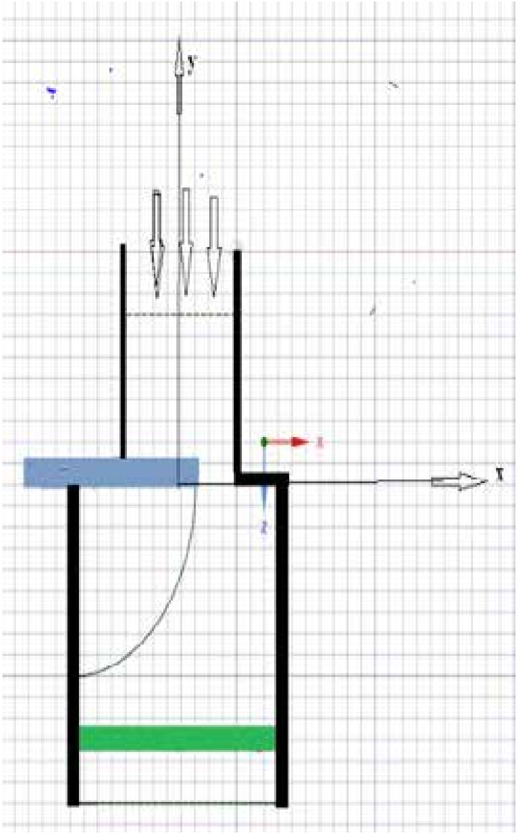
The internal combustion engine contains a cylinder with upper intake and exhaust ports, a gas distribution shaft with eccentrics, kinematically connected to the crankshaft, while each port is equipped with a slat damper. The creation of a highly efficient engine with damper-type GDM is associated with solving a number of issues, primarily ensuring compliance with modern environmental requirements.

The gas exchange process in the gas distribution mechanisms of the engine, which have the least hydraulic resistance, is reduced to the problem of the theory of compressible gas jets [9–11].

Thus, optimisation of the parameters of the intake and exhaust systems in internal combustion engines is considered the main task of studying gas-dynamic phenomena and is determined by the following criteria: an indicator of the gas exchange process (air flow, fresh mixture consumption) and an indicator of the combustion process [3].

Theoretical study of gas-dynamic processes in internal combustion engines applies various models of gas flow. Taking into account the capacity of computing technology, the choice of the appropriate mathematical model is determined depending on the research tasks.

Interest in the phenomenon of turbulence is due to its practical significance. This has led to many theories, hypotheses and studies aimed at solving problems associated with turbulent flows. Since there are still no general approaches to



Pic. 1. Flow in a cylindrical tube [performed by the authors].

describing such movement of liquid and gas, various approaches to describing the phenomenon of turbulence have been developed for a long time.

The work [12] briefly reviews A. N. Kolmogorov's hypotheses regarding the theory of turbulence and their consequences. Modern semi-empirical turbulence models are used to obtain a complete system of Reynolds equations. Numerical finite difference method was used to solve a closed system of equations of a viscous incompressible fluid in the variables velocity-pressure for an unsteady flow [13]. Different turbulence models have been developed based on different assumptions and approaches to perform practical calculations.

Therefore, it is relevant to develop numerical methods for solving systems of equations of unsteady turbulent flow based on finite-difference schemes with high accuracy.

The *objective* of the paper is to examine the applicability and accuracy of the results obtained using semi-empirical RANS-method models based on the Reynolds-averaged Navier-Stokes equation.

RESULTS

Formulation of the Problem

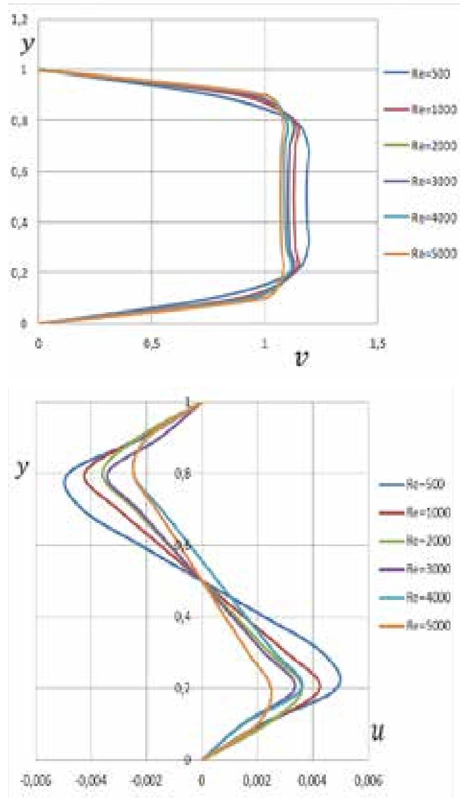
Study refers to a turbulent jet flowing around a plate installed in a cylinder with a turbulent flow of incompressible gas at a subsonic speed (Pic. 1). The flow is isothermal; external and surface forces are not considered. At infinity, the gas velocity is parallel to the *Oy* axis. It is assumed that the source is located at point A. In the same formulation, an analytical solution for the flow of an ideal gas in a cylinder was obtained [14].

Using the grid method, approximations for the stream function can be obtained and the components of the velocity vector can be calculated for the problem under consideration [15].

Basic Equations of Gas Motion

The unsteady Reynolds-averaged Navier-Stokes equations, in Cartesian coordinates, are used as a mathematical model of the flow. The one-equation Spalart-Allmaras (SA) turbulence model is used to calculate the turbulent viscosity [16]. The system of hydrodynamic equations for a)

b)



Pic. 2. Distribution of speed profiles for values x = 5
a) v – longitudinal component; b) u – transverse component.

an incompressible medium in Cartesian coordinates has the form[1]:

$$\begin{cases} \dfrac{\partial u}{\partial x}+\dfrac{\partial \vartheta}{\partial y}=0, \\ \dfrac{\partial u}{\partial t}+u\dfrac{\partial u}{\partial x}+\vartheta\dfrac{\partial u}{\partial y}+\dfrac{1}{\rho}\dfrac{\partial p}{\partial x}=\dfrac{\partial}{\partial y}\left(\left(v+\tilde{v}\right)\dfrac{\partial u}{\partial y}\right), \\ \dfrac{\partial \vartheta}{\partial t}+u\dfrac{\partial \vartheta}{\partial x}+\vartheta\dfrac{\partial \vartheta}{\partial y}+\dfrac{1}{\rho}\dfrac{\partial p}{\partial y}=\dfrac{\partial}{\partial y}\left(\left(v+\tilde{v}\right)\dfrac{\partial \vartheta}{\partial y}\right), \\ \dfrac{d\tilde{v}}{dt}=\mathrm{Pv}-\mathrm{D\,v}+\dfrac{1}{\sigma}\left[\dfrac{\partial}{\partial y}\left(\left(v+\tilde{v}\right)\dfrac{\partial \tilde{v}}{\partial y}\right)\right]+\dfrac{C_{b2}}{\sigma}\left(\dfrac{\partial \tilde{v}}{\partial y}\right)^2, \\ \mathrm{Pv}=C_{b1}\left(1-f_{t2}\right)\tilde{S}\tilde{v},\ \mathrm{Dv}=\left[C_{w1}f_w-\dfrac{C_{b1}}{k^2}f_{t2}\right]\left(\dfrac{\tilde{v}}{d}\right)^2. \end{cases} \qquad (1)$$

Here u, ϑ are the longitudinal and vertical components of the flow velocity vector, respectively, p is the hydrostatic pressure, Re is the Reynolds number, and $\tilde{v}$ is the linear eddy viscosity.

Turbulent eddy viscosity is calculated using the formula: $v_t=\tilde{v}f_{v1}$. The remaining values are the same as for the «standard» model, presented in [16].

On all stationary solid boundaries for the longitudinal and transverse components of velocity we have:

$u\big|_B=0$ and $\vartheta\big|_B=0$,

where B are solid boundaries.

For convenience, we rewrite the system of equations (1) in the following form:

$$\frac{\partial \Phi}{\partial t}+U\frac{\partial \Phi}{\partial x}+V\frac{\partial \Phi}{\partial y}=\frac{\partial}{\partial y}\left(q\frac{\partial \Phi}{\partial y}\right)+w.$$

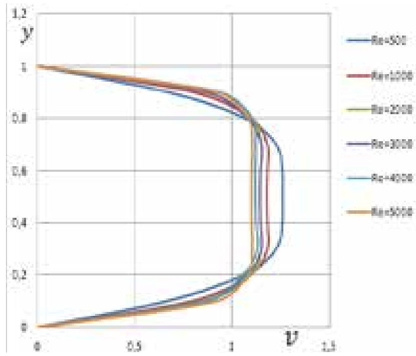
$$\Phi=\begin{bmatrix} U \\ V \\ \tilde{v} \end{bmatrix}, \qquad q=\begin{bmatrix} \left(\nu+\nu_t\right) \\ \left(v+\tilde{v}\right) \end{bmatrix},$$

$$w=\begin{bmatrix} \dfrac{1}{\rho}\dfrac{\partial P}{\partial x} \\ \dfrac{1}{\rho}\dfrac{\partial P}{\partial y} \\ \mathrm{Pv}-\mathrm{D\,v}+\dfrac{C_{b2}}{\sigma}\left(\left(\dfrac{\partial \tilde{v}}{\partial y}\right)^2\right) \end{bmatrix}.$$

To solve the system of nonstationary hydrodynamic equations (1), the finite difference method was applied. Matching the velocity and pressure fields is a complex process and presents a number of challenges. For this reason, a grid with a spaced layout structure was used. This

[1] Loytsyansky, L. G. Mechanics of liquid and gas: Textbook. 7th ed., rev. Moscow, Drofa publ., 2003, 840 p. ISBN 5-7107-6327-6.

a)



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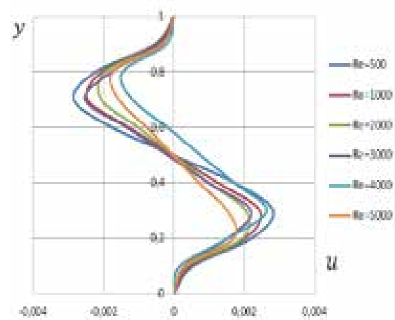

Pic. 3. Distribution of speed profiles for values x = 9,5
a) v – longitudinal component; b) u – transverse component.

means that the velocity and pressure components are defined at different nodal points. This approach provides certain advantages when calculating the pressure field. The layout of cells and nodes is similar to the layout used with the SIMPLE method [17].

To solve the problem numerically, we use the following boundary conditions:

• Normal pressure gradient is zero.

• Symmetry conditions were used on a vertical plane relative to the axis *y*.

• On the free boundary the pressure is constant.

To numerically solve this calculation scheme, we use a two-stage (predictor-corrector) MacCormack technique. This technique has second-order accuracy in time and length $O\left((\Delta t)^2,(\Delta x)^2,(\Delta y)^2\right)$. The condition for stability is $\frac{\Delta t}{\min(\Delta x,\Delta y)} \le 1$.[2] This technique is a variant of the two-stage Lax-Wendroff methods that does not require numerical calculation of the values of the desired function at points *j + 1/2* and *j – 1/2* [18, 19]. To solve the above nonlinear partial differential equations, it is also convenient to use the MacCormack method.

[2] Samarsky, A. A. Theory of difference schemes: Study guide. Moscow, Nauka publ., 1983, 616 p.

For the given problem, the equations of hydrodynamics in discrete form can be represented as:

predictor —

$$\overline{\Phi^{n+1}_{i,j}} = \Phi^{n}_{i,j} - U^{n}\frac{\Phi^{n}_{i+1,j}-\Phi^{n}_{i,j}}{\Delta x}\Delta t - V^{n}\frac{\Phi^{n}_{i,j+1}-\Phi^{n}_{i,j}}{\Delta y}\Delta t + \frac{0,5\Delta t}{\Delta y^2}\bullet \bullet\begin{bmatrix}(q^{n}_{j+1}+q^{n}_{j})(\Phi^{n}_{i,j+1}-\Phi^{n}_{i,j})- \\ -(q^{n}_{j}+q^{n}_{j-1})(\Phi^{n}_{i,j}-\Phi^{n}_{i,j-1})\end{bmatrix} - w^{n}_{i,j}.$$

corrector —

$$\Phi^{n+1}_{i,j} = \frac{1}{2}\left[\begin{matrix}\Phi^{n}_{i,j} + \overline{\Phi^{n+1}_{i,j}} - \overline{U^{n+1}}\dfrac{\overline{\Phi^{n+1}_{i+1,j}}-\overline{\Phi^{n+1}_{i,j}}}{\Delta x}\Delta t - \\ -\overline{V^{n+1}}\dfrac{\overline{\Phi^{n+1}_{i,j+1}}-\overline{\Phi^{n+1}_{i,j}}}{\Delta y}\Delta t\end{matrix}\right. + $$

$$\left. + \frac{0.5\Delta t}{\Delta y^2}\begin{bmatrix}(\overline{q^{n+1}_{j+1}}+\overline{q^{n+1}_{j}})(\overline{\Phi^{n+1}_{i,j+1}}-\overline{\Phi^{n+1}_{i,j}})- \\ -(\overline{q^{n+1}_{j}}+\overline{q^{n+1}_{j-1}})(\overline{\Phi^{n+1}_{i,j}}-\overline{\Phi^{n+1}_{i,j-1}})\end{bmatrix} - \overline{w^{n+1}_{i,j}}\right].$$

Calculation Results

Calculations were carried out at the following values of the main flow parameters: kinematic viscosity of air $\nu = 0{,}133$ cm²/s, density $\rho_o = 1{,}209$ kg/m³, pressure $p_0 = 0{,}1$ MPa. Pics. 2 and 3 show the calculation results (velocity profiles) of the problem of gas flow in a cylinder, obtained using an implicit scheme, for various values of the Reynolds number.

CONCLUSIONS

The article has presented methods for obtaining models of the motion of turbulent flow in a cylinder. Determining the profile of turbulent motion in a cylinder requires lengthy calculations. It is convenient to obtain the turbulent motion profile using the method proposed by P. R. Spalart and S. R. Allmaras.

From the above, we can conclude that with increasing Reynolds number, the velocity profile v changes from laminar to turbulent. Moreover, for laminar flow $\upsilon_{max} = 2\upsilon_{av}$, and for turbulent flow $\upsilon_{max} = 1{,}23\upsilon_{av}$. As can be seen from the graphs in Pic. 2 and 3, the velocity increases with laminar flow and decreases with the transition to turbulent flow.

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