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Lateral Force Evaluation in Vehicle's Wheel Interaction with a Rut

Ivan Yu. TSUKANOV

Anastasia N. LYUBICHEVA

Denis I. KOVALEV

Ivan Yu. Tsukanov[1], Anastasia N. Lyubicheva[2], Denis I. Kovalev[3]

[1, 2] Ishlinsky Institute for Problems in Mechanics of the Russian Academy of Sciences IPMech RAS), Moscow, Russia.
[3] Russian University of Transport, Moscow, Russia.
✉ *[2] lyubicheva@mail.ru.*

ABSTRACT

The operation of roads under modern conditions associated with high speeds and traffic density results in wear of pavement and formation of a rut. When crossing the ruts, instability can occur when relatively small steering inputs lead to large changes in the trajectory. The lateral force acting on a wheel of a car is of particular importance in the study of stability and controllability. The magnitude of this force is determined by various factors: tire stiffness, speed, change in the angle of rotation of the wheel, the geometry of the transverse profile of the rut.

The objective of the study refers to the problem of determining the angles of inclination of side walls of the rut that are safe for movement at different speeds. The paper considers the interaction of a car wheel with the road surface, proposes a mathematical model for interaction of a tire with ruts, considers the angle of the wheel running on the rut wall. To describe the elastic response of a tire in contact with a rut, an elastic foundation model and a plane section method are used. The lateral force was estimated based on equations of motion of the wheel when crossing the rut. The dependences of the ratio of the lateral force to the normal load for various rut depths and angles of inclination of its walls on the angle of rotation of the wheel and the speed of the vehicle were obtained. It is shown that at a rut inclination angle exceeding five degrees, the lateral force begins to increase significantly with an increase in the angle of rotation of the wheel, as well as the speed of the vehicle, and may exceed the applied normal load. According to the condition of vehicle stability against skidding on a wet surface, limiting average angles of inclination of the side walls of the rut are estimated for various values of the vehicle speed and wheel rotation angles.

Keywords: *road transport, rut, traffic safety, car wheel, lateral force, wheel steering angle, tire stiffness.*

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INTRODUCTION

The development of various deformations on the road surface is primarily associated with a constant increase in speeds and in the density of transport flows. Formation of ruts on the road surface is the most common among the damages, and it leads to a decrease in the traffic safety. When a vehicle crosses a rut, additional forces arise that affect vehicle's controllability and the stability of its movement. Instability manifests itself when relatively small external influences lead to large changes in the trajectory of movement [1–3], there emerges a danger of skidding up to the point of car overturning.

The works studying the stability of the vehicle, including when crossing the rut [1–4], examine the lateral force acting on the wheel of the car. The magnitude of this force is influenced by various factors: tire stiffness, speed, change in the angle of rotation of the wheel, the geometry of the transverse profile of the rut. The geometry of the transverse profile, as a rule, is characterised by the depth of the rut[1,2], without considering the shape of its cross section, which also significantly affects the magnitude of the lateral force. In the first approximation, this shape can be quantified by the average angles of inclination of the side walls of the rut.

Several works, e.g., [5; 6], are devoted to construction of analytical models on the interaction of a tire with a rut on the road surface. E.g., the work [5] studies the lateral stability of a car behaviour in a rut. The presence of oscillatory instability is shown using the model of a single-track vehicle with a higher stiffness of the rear tire as compared to the front one, and at a sufficiently high speed of the vehicle. An analytical expression is obtained that describes a stability boundary. A significant number of works are devoted to the study of tire hydroplaning, a review of such works can be found in [6]. The same paper suggests a method for calculating the maximum rut depth, which allows choosing its values depending on the requirements for the coefficient of adhesion of a tire with the pavement and for the speed of movement along a section of the road, on the depth of the water layer in the rut and the pavement roughness parameters.

The *objective* of the work is to build a *mathematical model* of interaction of a car wheel with the road surface in case of movement along a trajectory crossing the rut. When interacting with the surface of an obstacle, the tire deflections change depending on the shape of the rut. This circumstance leads to redistribution of the contact pressure, a change in the magnitude and direction of the reaction forces that arise between the tire and the road surface. The model will make it possible to evaluate the lateral force acting on the wheel, as well as to estimate the limiting average angles of inclination of the side walls of the rut at various values of the vehicle speed. To do this, the condition of vehicle stability against skidding on a wet surface will be adopted as a criterion for acceptable values of the coefficient of lateral force [3; 7].

RESULTS

Model of Interaction of a Car Wheel with the Road

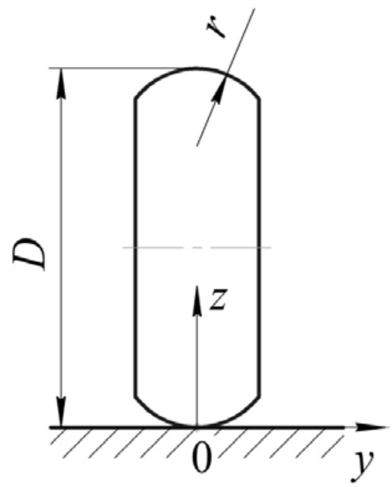
The deformation of a real tire occurs under the influence of many factors depending on the properties of the materials of tire components, tread geometry and tire design, which can be taken into account in finite element modelling for specific types of tires. Due to the interaction with road surfaces of different sizes and brands of car tires, which are not known in advance, to study the effect of the dimensions of the road rut on the lateral load on the wheel, we will use a simplified phenomenological model of a car tire based on an elastic foundation model [8]. When using such a simplified model, the *method* of plane sections can be used to determine the dimensions of the contact area between the wheel and the road surface [8].

To determine the tire stiffness in the model, let's consider the interaction of a car's wheel with a horizontal plane. A vertical load P is applied to the centre of the wheel, corresponding to a fourth of the car's weight. Let us assume that the driven wheels are in free rolling mode. The pavement will be considered non-deformable.

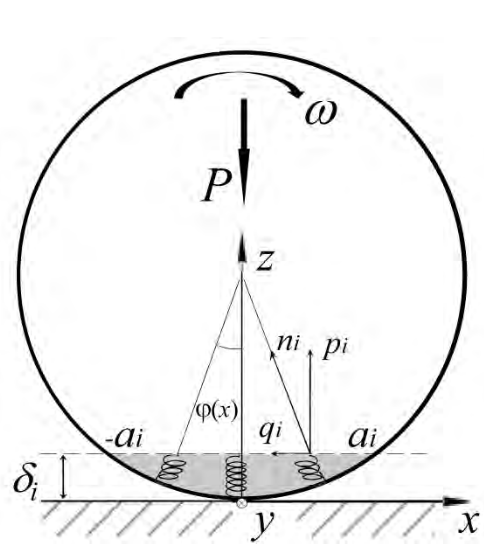
Let us introduce a local coordinate system $Oxyz$, its origin is located at the lower most protruding point of the central section of the wheel, the axis Ox coincides with the direction

[1] GOST [State standard] 32825-2014. Public motor roads. Road surfaces. Methods for measuring the geometric dimensions of damage. Moscow, Standartinform publ., 2019, 15 p. [Electronic resource]: https://docs.cntd.ru/document/1200117775. Last accessed 11.10.2022.

[2] GOST R [Russian state standard] 50597-2017. Motor roads and streets. Requirements for the operational state, admissible under the terms of ensuring road safety. Control methods. Moscow, Standartinform publ., 2017, 27 p. [Electronic resource]: https://docs.cntd.ru/document/1200147085. Last accessed 11.10.2022.



a



b

Pic. 1. Model of interaction of the wheel with the road: (a) geometric model of the wheel; (b) tire deformation model in the i-th section [performed by the authors].

of rolling, Oy is parallel to the axis of rotation of the wheel. Let us represent the outer surface of the wheel as a function (Pic. 1a):

$$z_1(x,y) = R + r - \sqrt{R^2 - x^2} - \sqrt{r^2 - y^2}\ , \qquad (1)$$

where $R = D/2$; D – the maximum outer diameter of the wheel;

r – is the curvature radius of the tread track ($r \le 1{,}65H$ [9]; here H is the profile height [9]).

We cut the contact area Ω by planes perpendicular to the axis of rotation of the wheel (Oy), with a uniform step $\Delta y = y_N/N$, where y_N is the boundary of the contact area in the direction of the axis Oy; $2N$ is the number of sections of the contact area.

In each section Ω_i in accordance with the indicated assumptions, we consider a two-dimensional contact problem, representing the deformation of the tire as the deformation of an elastic foundation with radially arranged springs (Pic. 1b) [8].

Let us write down the reaction of the foundation, directed radially to the surface, n_i and its projection p_i, normal to the horizontal surface:

$$\begin{aligned} &n_i(\varphi(x), y_i) = k_r u_r(\varphi(x), y_i),\\ &p_i(x, y_i) = k_r u_r(\varphi(x), y_i)\cos\varphi(x). \end{aligned} \qquad (2)$$

Here u_r is the radial displacement of the foundation at the point, depending on the angle $\phi(x)$; k_r is the coefficient of tire radial stiffness.

Let us note that for a symmetrical contact area, the tangent projection of the reaction q_i at each point of the section is nonzero, but the integral value of the tangent component of the reaction in each section is equal to zero.

The projections of radial displacements on the normal direction in the i-th section of the contact area have the form:

$$u_z(x, y_i) = u_r(\varphi(x))cos\varphi(x). \qquad (3)$$

Thus, the reaction component of the elastic foundation normal to the horizontal surface (contact pressure) is related to vertical displacements by the expression:

$$p_i(x, y_i) = k_r u_z(x, y_i). \qquad (4)$$

Deformation of the surface (deflection) of the tire at $x = 0$, $y = 0$ will be denoted by δ_0. The contact condition of the surfaces in the section then has the form:

$$z_i(x, y_i) + u_z(x, y_i) = \delta_0. \qquad (5)$$

Therefore, the displacements in the i-th section are equal to:

$$u_z(x, y_i) = \sqrt{R^2 - x^2} + \sqrt{r^2 - y_i^2} + \delta_0 - R - r, \qquad (6)$$

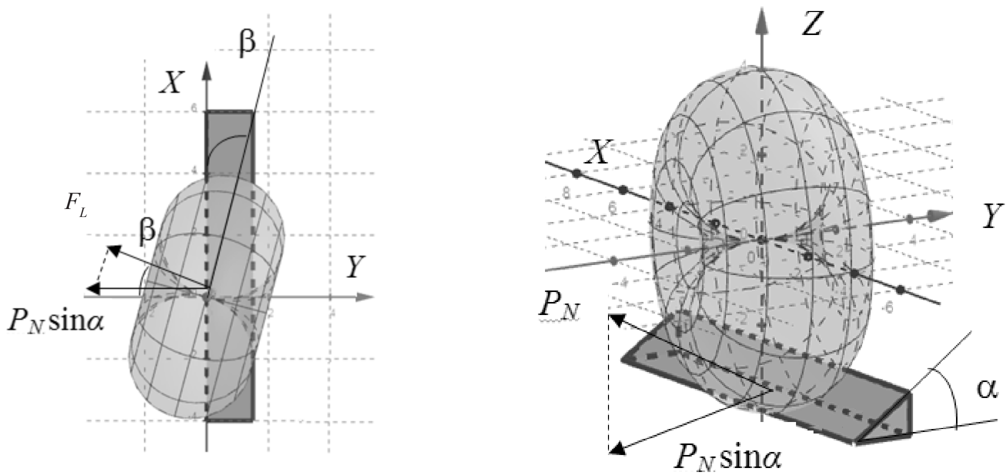
where $y_i = \Delta y(i - 1)$; $\Delta y = y_{\max} / N$, $i = 1\ldots N$. (7)

Let's denote the deformation in the i-th section by the expression:

$$\delta_i = \delta_0 - r + \sqrt{r^2 - y_i^2}. \qquad (8)$$

Let us determine the half-width of the contact area in the section a_i and the boundary $y_{\max} = y_N$ in the direction of the Oy axis, considering that the pressure and elastic displacements are equal to zero at the edges of the interaction area:

$$\begin{aligned} &u_z(a_i, y_i) = 0, \quad u_z(0, y_N) = 0,\\ &a_i = \sqrt{R^2 - (\delta_i - R)^2},\\ &y_N = \sqrt{2r\delta_0 - \delta_0^2}. \end{aligned} \qquad (9)$$



a *b*

Pic. 2. Scheme of contact of the wheel with the rut:
(a) isometric projection; (b) top view [performed by the authors].

To determine the amount of tire deflection on a flat road in the absence of reliable information about the design and properties of tire materials, we assume the hypothesis [10; 11] that most of the tire compression work is spent on compressing the air in it. According to [10; 11] about 60 % of the total work is spent on compressing the air in the tire when the tire is deformed, i.e.:

$$0{,}6\ W = W_{air}, \qquad (10)$$

where W is the work spent on tire compression;

W_{air} is the work spent on air compression in a tire.

Work done on air compression can be defined according to [10] as:

$$W_{air} = \sigma \Delta V, \qquad (11)$$

here ΔV – the change in the internal volume of the tire;

σ – the internal pressure in the tire.

The value of ΔV can be approximately determined by the volume of an elliptical segment with semi-axes defining the contact area:

$$\Delta V = \frac{1}{2}\pi a_0 y_N \delta_0 = \frac{1}{2}\pi \delta_0^2 \sqrt{2R\left(2r - \delta_0\right)}\ . \qquad (12)$$

The work done by the normal force to deform the wheel can be defined as:

$$W = \int_0^{\delta_0} P d\delta = P\delta_0\ . \qquad (13)$$

Using expressions (10)–(13), we obtain an equation for determining the tire deflection δ_0:

$$\frac{1}{2}\pi\sigma\delta_0\sqrt{2R\left(2r - \delta_0\right)} = 0{,}6P\ . \qquad (14)$$

The solution of the equation (14) is as follows:

$$\delta_0 = \frac{1}{6}\left(\begin{array}{l} \dfrac{8r^2}{-8r^3 + \frac{3}{2}\left(9K + \sqrt{81K^2 - 96r^3K}\right)^{1/3}} + \\ +1{,}6\left(-16r^3 - 3\left(\left(9K + \sqrt{81K^2 - 96r^3K}\right)^{1/3}\right)\right) - 4r \end{array} \right), \qquad (15)$$

where the coefficient K is equal to:

$$K = \frac{1{,}44P^2}{2\pi^2\sigma^2 R}\ . \qquad (16)$$

The tire radial stiffness coefficient in the elastic foundation model is determined by the ratio of the vertical load and tire deflection:

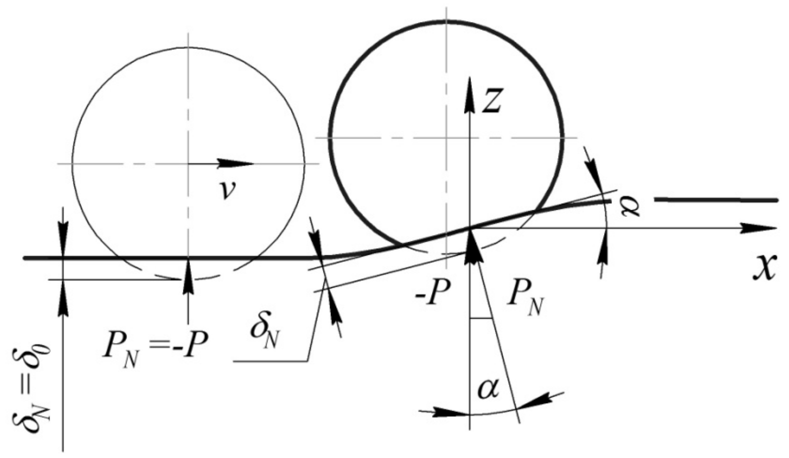
$$k_r = \frac{P}{\delta_0}\ . \qquad (17)$$

Interaction of the Wheel with the Side Wall of the Rut

Now let us consider the contact of the wheel with the rut, modelled by an inclined plane. Let us assume that the contact area is located entirely on the slope of the rut (Pic. 2).

The contact scheme in Pic. 2 is shown in the global coordinate system *Oxyz*, coinciding with the centre of the wheel; the *Ox* axis is directed along the edge of the obstacle. Let us denote the angle between the inclined plane and the horizontal plane as α, and the angle under which the wheel is running on the edge of the obstacle as β.

In the local coordinate system *Oxyz*, the approach of the wheel centre to the inclined surface (tire deflection) δ_N is directed along the normal to the contact area (Pic. 3), relations (5–9) are satisfied with the difference that the reaction normal to the surface is calculated according to the expression:



Pic. 3. Scheme of crossing the rut in the central section at y = 0 [performed by the authors].

$P_N = -P/\cos\alpha$. (18)

The lateral force F_l, acting on the wheel when hitting an inclined plane (see Pic. 2) is determined by the projection of the normal reaction in space:

$F_l = P_N \sin\alpha\cos\beta$. (19)

Determining the Lateral Force Acting on the Wheel When Crossing the Rut

Let us consider a shock-free passage by a wheel over a certain rut, provided that it is sufficiently long. We will use the following function as the cross-section profile of the rut:

$$z(x) = \frac{A}{2}\left(1 - \cos\frac{\pi x}{L}\right),\ x \in [0; L], \quad (20)$$

where A – the rut depth;

L – the cross-sectional width of the rut.

In practice, as a rule, the depth and the angle of inclination of the rut to the horizontal plane α are set. Since the track profile has a curvilinear shape, then by the angle α we mean its maximum value α_m. Based on the definition of the profile derivative, we get:

$$\mathrm{tg}\alpha = z'(x) = \frac{\pi}{2}\frac{A}{L}\sin\frac{\pi x}{L}. \quad (21)$$

The maximum value of the derivative of the profile occurs at the point $x = L/2$ and is equal to 1. Then the width of the cross-section of the rut will be determined as $L = \pi A / 2\mathrm{tg}\alpha_m \sin\beta\sqrt{a^2 + b^2}$.

Considering the rotation of the wheel through the angle β, the rut length will change. Neglecting the change in the curvature of the rut profile, we can write that:

$$L = \frac{\pi A}{2\mathrm{tg}\alpha_m \sin\beta}. \quad (22)$$

The elastic response of the tire acting as an elastic link with stiffness k_r occurs in the normal direction to the rut surface. Considering the model of vibrations of the wheel centre with a single mass [12] under the action of a perturbing force P_N and without considering friction in the contact, the equation of motion of the wheel centre along the plane normal to the rut surface has the form:

$$m\ddot{\delta}_N = -k_r\delta_N - P_N. \quad (23)$$

The force of reaction from the road on an elementary inclined area with an angle α, which, in turn, depends on the coordinate x, can be written as $P_N = -P/\cos\alpha\,(x)$. Since $x = vt$, where v is the horizontal speed of the wheel centre, we write:

$$z(t) = \frac{A}{2}\left(1 - \cos\gamma vt\right), \quad (24)$$

where $\gamma = (2\mathrm{tg}\alpha_m \sin\beta)/A$.

Expressing the cosine of the angle α in terms of the derivative of the rut profile function, equal to tgα, and using trigonometric identities, we obtain:

$$m\ddot{\delta}_N = -k_r\delta_N + P\sqrt{1 + 0{,}5A\gamma v\sin\gamma vt}. \quad (25)$$

The solution of the equation (25) has the form [13]:

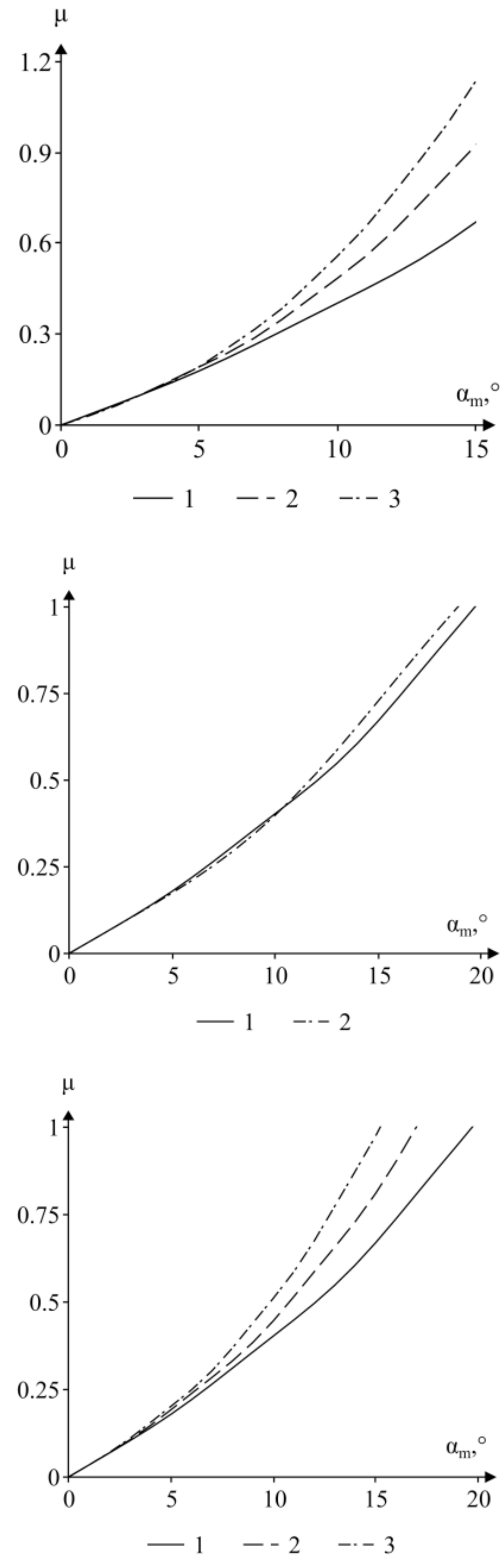
$$\delta_N(t) = \frac{P}{mf}\int_0^t \sqrt{1 + \left(0{,}5A\gamma v\sin\gamma v\tau\right)^2}\,\sin f(t-\tau)\,d\tau, \quad (26)$$

here $f = \sqrt{k_r/m}$ – the circular frequency of free oscillations of the wheel.

The integral in the expression (26) is not to be calculated analytically, so we will determine its values numerically. In fact, we are interested in the value of the maximum dynamic force $P_{N\max} = -k_r \bullet \delta_{N\max}$. Calculations show that the maximum normal deformation of the tire is achieved at $t \approx \pi/f$. Considering a

b

c



Pic. 4. Dependence of the ratio of the maximum value of the lateral force to the normal load on the wheel μ on the maximum angle of inclination of the rut profile to the horizontal plane (α_m): (a) at A = 50 mm, v = 60 km/h, 1 – β = 10°; 2 – β = 20°; 3 – β = 30°; (b) at β = 10°, v = 60 km/h, 1 – A = 50 mm; 2 – A = 100 mm; (c) at β = 10°, A = 50 mm, 1 – v = 60 km/h; 2 – v = 90 km/h; 3 – v = 120 km/h [performed by the authors].

formula (19), it is possible to obtain the maximum value of the ratio of the lateral force to the vertical load μ when the wheel leaves the rut:

$$\mu = \frac{F_{l\max}}{P} = \frac{\delta_N \pi}{\delta_0 f} \sin\alpha \cos\beta\ . \tag{27}$$

RESULTS AND DISCUSSION

Pic. 4 shows the charts of dependences of the parameter μ on the rut inclination angle α, which means its maximum value α_m for the rut profile (20). Calculations were made for the following initial data typical for passenger cars: maximum outer diameter of a wheel (tire) D = 520 mm; tire profile height H = 100 mm (ratio between profile height and tire width – 50 %); radius of curvature of the tread track r = 1,3 H.

The calculation results shown in Pic. 4 show that changes in factors such as vehicle speed and wheel angle have almost no effect on the parameter μ at $\alpha_m < 5°$. With such a small value of the rut profile inclination angle, the maximum lateral force does not exceed 25 % of the load applied to the wheel.

With a larger rut inclination angle, the lateral force begins to increase significantly with an increase in the angle of rotation of the wheel and the speed of the vehicle and may exceed the applied load. Therefore, with a large angle of inclination of the rut (obstacles), the angle of rotation of the wheel and the speed of the car should be the minimum allowable, which is confirmed by driving practices.

The rut depth, according to the above calculations, has a smaller effect on the value of μ with increasing α_m, which may be due to the assumptions made in the model, e.g., such as single-point contact, one-dimensional elastic model of the tire, neglect of friction and response of the elastic system of the car.

It should also be noted that with an increase in the rut depth, the considered shock-free model of dynamic interaction may not be applicable due to the possible separation of the wheel from the obstacle surface.

CONCLUSION

The proposed analytical model of the interaction of the car wheel with the road surface when overcoming the rut allows, in the first approximation and in the absence of detailed data on the tire structure, vehicle parameters, to estimate the value of the lateral force acting on the wheel depending on the main parameters of the car and the rut. The elastic tire model can also be integrated into more complex dynamic vehicle models to account for driving on uneven road surfaces.

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Information about the authors:

Tsukanov, Ivan Yu., *Ph.D. (Eng), Senior Researcher at the Tribology Laboratory of Ishlinsky Institute for Problems in Mechanics of Russian Academy of Sciences (IPMech RAS), Moscow, Russia, ivan.yu.tsukanov@gmail.com.*

Lyubicheva, Anastasia N., *Ph.D. (Physics and Mathematics), Senior Researcher at the Tribology Laboratory of Ishlinsky Institute for Problems in Mechanics of Russian Academy of Sciences (IPMech RAS), Moscow, Russia, lyubicheva@mail.ru.*

Kovalev, Denis I., *Lecturer-Researcher at Russian University of Transport, Moscow, Russia, kov_deni@mail.ru.*

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