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New Approaches to Pricing Management of Transport Services

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ABSTRACT

Development of new approaches to formation of analytics mechanisms for the purpose of pricing management of services is an important aspect of increasing the efficiency of transport management processes.

Research aimed at improving the tools for determining the optimal parameters of the ratio of quality and price of service for formation of a competitive and efficient tariff policy continues to remain relevant and in demand in modern market conditions.

The objective of the study, presented in the article, is to analyse and evaluate the prospects for implementation of the areas to improve the apparatus for assessing the price elasticity of demand for railway passenger transport services as the transition to the use of non-linear parameters in terms of customer behaviour modelling functions, as well as introduction of the most effective algorithms from the set of modern global mathematical optimisation tools.

The research conclusions are based on the use of system analysis mechanisms, methods of economic and mathematical modelling and optimisation, as well as of non-parametric statistics tools.

The results based on the use of an array of data on the demand of passengers of branded trains include: a comparative assessment of quality of modelling the price elasticity of demand using 15 functions that are nonlinear in terms of parameters; the most promising tools of the search for unknown parameters for non-smooth nonlinear functions for modelling the behaviour of railway customers are identified based on a three-stage procedure for comparative analysis of the performance of more than 60 optimisation algorithms (including the calculation of minima and medians for the sums of squares of modelling errors, bootstrap analysis, Kruskal–Wallace and Mann–Whitney tests, as well as the calculation of a metric specially developed by the authors for assessing the degree of superiority of one algorithm over another within the framework of non-parametric analysis).

The findings seem able to be successfully used in relation to other modes of transport in solving similar problems of developing an effective toolkit for managing the prices of transport services.

Keywords: *transport management, passenger transportation, tariff policy, price elasticity of demand, economic and mathematical models, heuristic optimisation algorithms, revenue management system.*

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INTRODUCTION

Studies devoted to development of approaches to managing the prices of transport services based on implementation of effective mechanisms for modelling customer behaviour to find the optimal parameters for the ratio of quality and cost of service are in demand and relevant.

The implementation of comprehensive programs to improve efficiency, the growth of customer focus and improvement of quality of services provided continue, in accordance with the provisions of the Long-Term Development Program [1], to remain important areas of application of the strategy for ensuring high competitiveness of JSC Russian Railways in today's market conditions.

The railway transport management in the passenger transportation market, over the past 15 years, has significant progressed in terms of development of a methodology for modelling customer behaviour for pricing purposes based on a study of price elasticity of demand, the main milestones of which include:

• Development of a mechanism for determining the optimal cost intervals for solvent demand for additional services in trains, based on a scenario analysis of respondents' answers about marginal prices using ranking scales of price ranges [1, pp. 45–47; 2, pp. 128–132].

• Development of tools for estimating optimal tariffs for suburban season tickets by modelling the process of customers' decision to choose a trip [3, pp. 51–63],

• Substantiation of the concept and approaches to implementation of dynamic pricing and management of profitability of long-distance passenger transportation [4, pp. 27–30].

• Development of an apparatus of economic and mathematical models for managing profitability of domestic long-distance passenger transportation within the framework of a dynamic pricing system [5, pp. 10–15; 6, pp. 33–39].

• Development of tools for determining the optimal cost of passenger service based on economic and mathematical modelling using linear parameters [2, pp. 123–128; 7, pp. 10–20] and, later on, essentially non-linear [8, pp. 50–59] forms of models for assessing price elasticity of demand.

• Improvement of the apparatus of economic and mathematical models for managing economic efficiency based on the combination of dynamic pricing tools with mechanisms for analysing competitors' tariffs [9, pp. 53–62] and approaches to determining the optimal number of cars within passenger trains [10, pp. 343–350].

The *objective* of the study, the results of which are presented in the article, is further consistent development of mechanisms for managing the prices of transport services based on improving the processes of predicting the behaviour of passengers by choosing new promising functions for analysing the price elasticity of demand, as well as searching for and applying the most effective algorithms for global mathematical optimisation.

RESULTS

Alternative Models for Analysing Price Elasticity of Demand

The work [8, p. 51] examined five non-linear, in terms of parameters, forms of economic-mathematical models of price elasticity analysis for use in automated yield management systems: Gompertz, Pearl–Reed, Verhulst models, as well as two variants of Weibull model: respectively two-parameter (W2b) and four-parameter (W4b) variants. However, there are other, widely used in practice, alternative functional forms of nonlinear models, the study of which may be promising for improving the efficiency of the analysis of the elasticity of demand in the framework of managing the prices of transportation and related services in railway transport:

• Model of Little, also known as ADBUDG [11, pp. 484–485]:

$$y(x) = \beta_1 + (\beta_2 - \beta_1)x^{\beta_3} / (\beta_4 + x^{\beta_3}), \qquad (1)$$

where $y(x)$ is simulated dependence of the demand for a service on its cost;

β – coefficients obtained during optimisation.

• Logit model [12, pp. 78–79]:

$$y(x) = \beta_1 \exp(\beta_2 + \beta_3 x) \big/ \left(1 + \exp(\beta_2 + \beta_3 x)\right). \qquad (2)$$

• Ratio model [13, p. 1].

• Rat-03 model:

$$y(x) = \beta_1 / (x^3 + \beta_2 x^2 + \beta_3 x + \beta_4). \qquad (3)$$

• Rat-12 model:

$$y(x) = (\beta_1 x + \beta_2) / (x^2 + \beta_3 x + \beta_4). \qquad (4)$$

• Rat-13 model:

$$y(x) = (\beta_1 x + \beta_2) / (x^3 + \beta_3 x^2 + \beta_4 x + \beta_5). \qquad (5)$$

• Rat-21 model:

$$y(x) = (\beta_1 x^2 + \beta_2 x + \beta_3) / (x + \beta_4). \qquad (6)$$

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[1] Long-term development program of JSC RZD until 2025 (approved by the order of the Government of the Russian Federation dated March 19, 2019, No. 466-r). [Electronic resource]: https://docs.cntd.ru/document/553927831?marker=64U0IK. Last accessed 01.05.2021.

• Rat-22 model:

$$y(x)=\left(\beta_1 x^2+\beta_2 x+\beta_3\right)/(x^2+\beta_4 x+\beta_5)\,. \qquad (7)$$

• Ratkowsky's model [14, pp. 1–5]:

$$y(x)=\beta_1/\left(1+\exp\left(-\beta_2-\beta_3 x\right)\right)\hat{}\left(1/\beta_4\right)\,. \qquad (8)$$

• Richards model [15, pp. 290–300]:

$$y(x)=\beta_1/\left(\left(1+\beta_2\exp\left(-\beta_3 x\right)\right)\hat{}\left(1/\beta_4\right)\right)\,. \qquad (9)$$

• Model of an exponent with double stretching, further called DWM (Double Weibullian Model) [16, pp. 139–159; 17, pp. 4402–4412]:

$$y(x)=\beta_1\cdot\exp\left(\left(x/\beta_2\right)^{\beta_3}-\left(x/\beta_4\right)^{\beta_5}\right)\,. \qquad (10)$$

The authors decided to compare the effectiveness of the above models (forms 1–10) with other five models previously presented in [8, p. 51], when used in relation to the array of information on price elasticity of demand, obtained from the results of a questionnaire survey of 3,5 thousand passengers of branded trains (a detailed description of the process of obtaining and processing the initial data is presented in [7, pp. 12–16]), run on Moscow–St. Petersburg route, in the context of studying the following eight data clusters characterising the demand for:

1) «Internet» service in compartment carriages of branded trains.

2) «Internet» service in the SV [extra comfort] carriages of branded trains.

3) «Communication» service in compartment carriages of branded trains.

4) «Communication» service in the SV carriages of branded trains.

5) Economy class compartment tickets of train No. 3/4 «Branded Express».

6) SV business class tickets of train No. 3/4 «Branded Express».

7) Economy class compartment tickets of train No. 5/6 «Nikolaevsky Express».

8) SV business class tickets of train No. 5/6 «Nikolaevsky Express».

The work [8, p. 55] according to the results of a comparative analysis of quality of estimation of the optimal parameters of the studied forms of models, identified Levenberg–Marquardt algorithm (hereinafter LM) as the most effective, therefore it was originally chosen to search for unknown parameters of the studied models (formulas 1–10). This algorithm, described in detail in [18, pp. 105–116], is based on the criterion of minimising the sum of squared errors of the model (hereinafter SSE), which, for the purposes of comparability, was implemented in all the considered optimisation algorithms [8, p. 52].

In the framework of this article, the authors decided to use the SSE minimum not only as a criterion for optimising unknown parameters, but also to use it as guideline when comparing quality of the optimisation algorithms, as well as effectiveness of the studied functional forms for pricing purposes. Nevertheless, it is worth noting that, before introducing pricing into practice, it is still advisable to make comparisons using indicators that are more convenient for management purposes, including [8, pp. 52–53] «integral criterion for the comparative error of the model in the studied group», «adjusted index of determination», «module parameter mean average deviation» (hereinafter MAD).

The resulting values of the minimum SSE obtained by the authors based on the results of 15 runs of the LM algorithm, which characterise effectiveness of using various functional forms of modelling in each of eight clusters studied (Table 1), allow us to conclude that the choice of Pearl–Read model as a starting point in the analysis of price elasticity demand for transport services is still an excellent solution (in [8, pp. 54–55] this model has already been noted as one of the most promising, demonstrating the highest efficiency in terms of the minimum MAD criterion).

At the same time, the LM algorithm did not converge for Richards and DWM models in any of the studied clusters, which is associated with gradient estimation problems due to non-smoothness of these nonlinear functional forms. To solve this problem, the authors, taking into account the analysis of the properties of these functions and the specifics of using models in the practice of price elasticity analysis, limited the areas of starting search points for unknown variables (in the first approximation, from –10 to 10 for unknown power parameters and from -50 to 50 for the remaining unknown quantities), after which it became possible to use optimisation algorithms based on construction of trust regions by the following methods: of the inner point (Trust Region Inner Point Method, hereinafter TRIPM, a detailed description of which is presented in [19, pp. 578–583]) and of sequential quadratic

Table 1

Results of optimising the parameters of nonlinear forms of modelling price elasticity of demand (LM algorithm) [performed by the authors]

Model form	Studied data cluster in the array of the initial information								
	1	2	3	4	5	6	7	8	Total
Minimum SSE based on the results of 15 runs, units									
Gompertz	0,0101	0,0329	0,0301	0,0690	0,0685	0,0990	0,0282	0,0283	0,3661
Pearl–Read	**0,0072**	**0,0106**	**0,0095**	**0,0162**	**0,0498**	**0,0674**	**0,0282**	**0,0198**	**0,2088**
Verhulst	0,0080	0,0254	0,0256	0,0590	0,0508	0,0732	0,0317	0,0373	0,3110
W2b	0,0078	0,0261	0,0261	0,0590	0,0564	0,0843	0,0329	0,0450	0,3376
W4b	0,0071	0,0170	0,0172	0,0448	0,0527	0,0724	0,0304	0,0381	0,2796
Little	0,0084	0,0098	0,0127	0,0286	0,0457	0,0422	0,0686	0,0529	0,2689
Logit	0,0080	0,0254	0,0256	0,0590	0,0508	0,0317	0,0732	0,0373	0,3110
Rat-03	0,0075	0,0096	0,0127	0,0226	0,0623	0,0497	0,0925	0,0653	0,3221
Rat-12	0,0074	0,0096	0,0130	0,0178	0,0908	0,0671	0,1021	0,0694	0,3772
Rat-13	**0,0072**	**0,0096**	0,0122	**0,0096**	**0,0459**	**0,0403**	0,0728	0,0423	**0,2400**
Rat-21	0,0159	0,0403	0,0746	0,0733	0,3957	0,1889	0,1949	0,1870	1,1706
Rat-22	**0,0072**	**0,0080**	0,0129	**0,0049**	0,0533	**0,0446**	0,0778	0,0351	**0,2439**
Ratkowsky's	0,0172	0,0457	0,0880	0,0857	0,0454	**0,0282**	0,0683	0,0283	0,4068
Richards	–	–	–	–	–	–	–	–	–
DWM	–	–	–	–	–	–	–	–	–

Note: SSE values are rounded to the fourth decimal places.

programming (hereinafter referred to as TRSQP, described in [19, pp. 546–554]). Since out of 15 runs of TRIPM and TRSQP in terms of clusters, less than 30 % were successful in terms of convergence, the MultiStart method was applied, when 30 starting points were used in each run, selected from specified intervals based on a uniform distribution using the pseudo-random number generator, Mersenne twister ([20, pp. 3–30]). As a result, for the considered data array within the framework of each of the 15 runs for each of the clusters, it was possible to obtain solutions suitable for further comparative analysis (Table 2).

It should be noted that TRIM and TRSQP, considering the peculiarities of their construction, do not guarantee obtaining global minima in the case of non-smooth functions. Moreover, even when using MultiStart, it is still impossible to completely exclude a possibility that the algorithms will not give a single acceptable solution within the given time intervals, which can be a serious obstacle to their application in the practice of managing the prices of transport

Table 2

Results of optimising the parameters of nonlinear forms of modelling price elasticity of demand (algorithms: TRIPM and TRSQP) [performed by the authors]

Model form	Studied cluster of data in the array of initial information								
	1	2	3	4	5	6	7	8	Total
Minimum SSE based on the results of 15 runs of TRIPM with multistart, units									
Richards	0,0073	**0,0101**	0,0155	0,0238	**0,0454**	**0,0683**	**0,0282**	0,0278	**0,2264**
DWM	**0,0069**	0,0140	0,0191	**0,0106**	0,0518	0,0784	**0,0291**	**0,0197**	**0,2296**
Minimum SSE based on the results of 15 runs of TRSQP with multistart, units									
Richards	0,0073	**0,0101**	0,0155	0,0238	**0,0454**	**0,0683**	0,2615	0,0921	0,5240
DWM	0,0074	0,0204	0,0243	0,0519	0,0555	0,0838	0,0318	0,0388	0,3139

Note: SSE values are rounded to fourth decimal places.

Table 3

General information about heuristic optimisation algorithms [compiled by the authors]

Full name and description of the algorithm methodology	Abbreviation
«Artificial bee colony algorithm» [21, pp. 19–30]	ABC
«Ant colony optimization for continuous domains» [22, pp. 1155–1173]	ACOR
«Artificial ecosystem-based optimization» [23, pp. 9383–9425]	AEO
«Autonomous groups particle swarm optimization» [24, pp. 4683–4697]	AGPSO
«Antlion optimizer» [25, pp. 80–89]	ALO
«Aquila optimizer» [26, pp. 1–16]	AO
«Bat algorithm» [27, pp. 313–315]	BAT
«Biogeography-Based optimization» [28, pp. 702–713]	BBO
«A modified bees algorithm with statistics-based tuning parameters» [29, pp. 287–301]	BeAm
«The standard bees algorithm» [30, pp. 2919–2938]	BeAs
«Bacterial foraging optimization» [31, pp. 52–67]	BFO
«Black-Hole-Based optimization» [32, pp. 879–888]	BHBO
«Cultural algorithm» [33, pp. 187–192]	CA
«Chaos Game optimization» [34, pp. 917–1004]	CGO
«The clonal selection principle optimization» [35, pp. 239–251]	CLONALG
«Coyote optimization algorithm» [36, pp. 2633–2640]	COA
«Constriction coefficient particle swarm optimization» [37, pp. 58–73]	CPSO
«Cuckoo search algorithm» [27, pp. 306–312]	CS
«Dragonfly algorithm» [38, pp. 1053–1073]	DAO
«Differential evolution» [39, pp. 341–359]	DE
«Hybrid particle swarm with differential evolution» [40, pp. 629–640]	DEPSO
«Earthquake optimization algorithm» [41, pp. 78–86]	EQOA
«Equilibrium optimizer» [42, pp. 1–19]	EOA
«Firefly algorithm» [43, pp. 209–218]	FA
«Flower pollination algorithm» [27, pp. 315–318]	FPA
«Real-coded genetic algorithm» [44, pp. 2276–2280]	GA
«Generalized normal distribution optimization» [45, pp. 1–21]	GNDO
«Grasshopper optimisation algorithm» [46, pp. 30–47]	GOA
«Gaussian Quantum-behaved particle swarm» [47, pp. 1676–1683]	GQPSO
«Generalized simulated annealing optimization» [48, pp. 216–220]	GSA
«Grey wolf optimizer» [49, pp. 46–61]	GWO
«Heap-based optimizer» [50, pp. 1–17]	HBO
«Harris hawks optimization» [51, pp. 849–872]	HHO
«Harmony search» [33, pp. 182–186]	HS
«Imperialist competitive algorithm» [21, pp. 51–65]	ICA
«An improved grey wolf optimizer» [52, pp. 1–37]	IGWO
«Adaptive differential evolution with optional external archive» [53, pp. 945–958]	JADE
«Inertia based particle swarm optimization» [54, pp. 32–41]	IPSO
«Invasive weed optimization» [55, pp. 355–366]	IWO
«Jaya optimization» [56, pp. 19–34]	JAYA
«Mexican Axolotl Optimization» [57, pp. 1–20]	MAO
«Marine predators algorithm» [58, pp. 1–23]	MPA
«Manta ray foraging optimization» [59, pp. 1–20]	MRFO
«Multi-Verse optimizer» [60, pp. 495–513]	MVO
«Neighborhood consensus continuous optimization» [61, pp. 115–141]	NCCO
«Nelder Mead optimization algorithm» [62, pp. 973–980]	NM
«Standart particle swarm optimization» [33, pp. 232–237]	SPSO
«Queuing search algorithm» [63, pp. 464–490]	QSO
«Real-coded adaptive simulated annealing optimization» [27, pp. 287–290]	SA
«Shuffled complex-self adaptive hybrid evolution algorithm» [64, pp. 215–235]	SC-SAHEL
«Shuffled complex evolution» [65, pp. 501–521]	SCE-UA
«Sunflower optimization» [66, pp. 619–626]	SFO
«Salp swarm agorithm» [67, pp. 163–191]	SSA
«A two-stage state transition algorithm» [68, pp. 1–13]	STA
«Time-varying asymmetric acceleration particle swarm optimization» [69, pp. 2134–2139]	TACPSO
«Teaching-Learning-Based optimization algorithm» [21, pp. 41–49]	TLBO
«Tug of war optimization algorithm» [21, pp. 123–135]	TWO
«Vibrating particles system optimization algorithm» [21, pp. 153–165]	VPS
«Water evaporation optimization algorithm» [21, pp. 138–152]	WEO
«Whale optimization algorithm» [70, pp. 51–67]	WOA

services in railway transport. Therefore, as an alternative approach to finding optimums for Richards and DWM models (taking into account the prospects for their use, according to the results presented in Table 2), it was decided to consider the use of global optimisation heuristic algorithms, which were originally designed in such a way that, with a limited computation time, to find steadily good solutions, even in the case of non-smooth nonlinear functions in a multidimensional environment of unknown variables.

Selection of Heuristic Global Optimisation Algorithms for Use in Pricing Management of Transport Services

In [8, p. 52], only three variants of heuristic numerical optimisation were considered, and Nelder–Mead method (hereinafter NM) turned to be the best of them (in terms of the minimum SSE in all clusters), but it, nevertheless, was significantly inferior in efficiency to LM algorithm [8, p. 57]. However, in recent decades, there has been significant progress in development of the theory of global optimisation heuristics, which resulted in the emergence of many publicly available (without restraints on their refinement and commercial use) algorithms, often accompanied by examples of source code descriptions in programming languages (R, Python, Matlab, Ruby). The convenience of practical work with them in currently widespread free interactive software development environments: Rstudio, Spyder, GNU Octave (supporting Matlab syntax), as well as a significant increase in the computing power of modern computers, provide broad prospects for rapid development and integration of the best solutions into existing software complexes serving numerous processes of sectoral management, including pricing. Therefore, to determine the most effective approach to optimising the parameters of non-smooth nonlinear models used to assess the price elasticity of demand, the authors conducted a comparative analysis of the work of 60 stochastic heuristic optimisation algorithms (Table 3), which included three main stages: selection of the best 25 % by the minimum SSE; down to 10 % list shortening based on SSE median analysis; final selection of several most promising algorithms based on non-parametric methods of statistical analysis.

To reduce the volume of calculations, it was decided to first select the most promising algorithms based on optimisation of DWM model and only after that to optimise for Richards model.

The results of estimating the minimum SSE in the context of the studied clusters based on the results of 15 runs of heuristic optimisation algorithms for DWM model (with a change in initialisation value of the pseudo-random number generator Mersenne twister during each of them), as well as a run time limit (no more than 12 seconds) for optimisation within the run are presented in Table 4.

The results of ranking the studied global optimisation heuristics (Table 4) show that the total minimum SSE of the studied clusters for each of the 15 best algorithms (25 % of the total) is lower than that of TRIPM algorithm with MultiStart (Table 2), which indicates high potential for their practical use.

The criterion for the minimum value of SSE, achieved by the results of 15 runs, reflects such aspects of efficiency of the optimisation algorithm as moving in the space of problem dimensions and searching in promising local areas, but for practical use in pricing, given the stochastic nature of all considered heuristic algorithms, the stability criterion is most important, reflecting thus a probability that the algorithm can find the minimum SSE in fewer runs. Therefore, to consider the stability criterion, a comparison of SSE medians was carried out, the results of which for the 18 best algorithms are presented in Table 5.

The results of comparing SSE medians (Table 4) how that the CS algorithm is the most stable. It should be noted that in terms of the total minimum SSE, it did not take a leading position only because of the results of work in the fourth studied cluster. The top five also included MRFO, FPA and TLBO, which, given their leadership in minimum SSE, makes them clear favourites. At the same time, SC-SAHEL, although it achieved the lowest SSE values compared to other algorithms, did so during only a small number of runs (i. e., with a lower probability), demonstrating significantly less competitive results in other cases, which is especially

Table 4

Algorithm ranking by minimum SSE for DWM model [performed by the authors]

Algorithm	Studied cluster of data in the array of initial information								
	1	2	3	4	5	6	7	8	Total
Minimum SSE based on the results of 15 runs of the algorithm, units									
SC-SAHEL	0,0069	0,0072	0,0191	0,0110	0,0509	0,0278	0,0748	0,0197	**0,2174**
MRFO	0,0069	0,0072	0,0192	0,0109	0,0510	0,0278	0,0748	0,0197	**0,2174**
FPA	0,0069	0,0072	0,0191	0,0106	0,0509	0,0284	0,0748	0,0197	**0,2177**
TLBO	0,0069	0,0072	0,0191	0,0108	0,0510	0,0284	0,0748	0,0197	**0,2179**
CS	0,0069	0,0072	0,0191	**0,0118**	0,0509	0,0278	0,0748	0,0197	**0,2182**
CGO	0,0069	0,0072	0,0191	0,0103	0,0509	0,0284	0,0748	0,0213	**0,2190**
EOA	0,0069	0,0072	0,0193	0,0133	0,0510	0,0278	0,0749	0,0206	**0,2210**
FA	0,0069	0,0072	0,0193	0,0141	0,0511	0,0278	0,0750	0,0197	**0,2211**
WEO	0,0069	0,0072	0,0196	0,0142	0,0511	0,0279	0,0749	0,0200	**0,2218**
AEO	0,0069	0,0072	0,0205	0,0103	0,0509	0,0279	0,0748	0,0233	**0,2218**
VPS	0,0069	0,0072	0,0192	0,0141	0,0511	0,0291	0,0749	0,0206	**0,2231**
GNDO	0,0069	0,0140	0,0191	0,0104	0,0509	0,0278	0,0748	0,0197	**0,2236**
STA	0,0069	0,0076	0,0194	0,0144	0,0520	0,0278	0,0760	0,0197	**0,2238**
AGPSO	0,0069	0,0072	0,0193	0,0124	0,0511	0,0279	0,0750	0,0248	**0,2246**
JADE	0,0069	0,0072	0,0191	0,0103	0,0509	0,0318	0,0748	0,0262	**0,2272**
CPSO	0,0069	0,0141	0,0206	0,0117	0,0510	0,0307	0,0749	0,0206	0,2307
GA	0,0069	0,0074	0,0196	0,0221	0,0520	0,0278	0,0759	0,0197	0,2314
GWO	0,0069	0,0073	0,0196	0,0148	0,0527	0,0318	0,0753	0,0239	0,2323
MVO	0,0070	0,0072	0,0197	0,0227	0,0524	0,0294	0,0757	0,0220	0,2360
IGWO	0,0071	0,0072	0,0194	0,0117	0,0531	0,0318	0,0818	0,0249	0,2370
ICA	0,0069	0,0147	0,0197	0,0195	0,0517	0,0285	0,0756	0,0206	0,2371
TACPSO	0,0069	0,0072	0,0192	0,0302	0,0510	0,0279	0,0749	0,0197	0,2371
DEPSO	0,0069	0,0072	0,0191	0,0104	0,0509	0,0318	0,0748	0,0382	0,2393
ACOR	0,0069	0,0073	0,0200	0,0174	0,0540	0,0286	0,0814	0,0250	0,2406
QSO	0,0069	0,0072	0,0193	0,0119	0,0510	0,0748	0,0318	0,0382	0,2411
MPA	0,0071	0,0072	0,0198	0,0135	0,0526	0,0308	0,0750	0,0353	0,2412
GSA	0,0069	0,0072	0,0198	0,0116	0,0509	0,0748	0,0318	0,0382	0,2413
ABC	0,0069	0,0147	0,0197	0,0182	0,0513	0,0318	0,0752	0,0262	0,2440
DE	0,0069	0,0145	0,0208	0,0276	0,0513	0,0284	0,0754	0,0197	0,2447
SCE-UA	0,0069	0,0140	0,0191	0,0103	0,0509	0,0307	0,0748	0,0388	0,2455
JAYA	0,0069	0,0077	0,0198	0,0145	0,0521	0,0318	0,0752	0,0384	0,2466
HHO	0,0071	0,0073	0,0196	0,0196	0,0531	0,0318	0,0787	0,0298	0,2470
SPSO	0,0069	0,0072	0,0194	0,0401	0,0511	0,0279	0,0750	0,0197	0,2472
NM	0,0069	0,0072	0,0200	0,0098	0,0555	0,0308	0,0809	0,0363	0,2474
HS	0,0069	0,0153	0,0205	0,0272	0,0525	0,0284	0,0765	0,0205	0,2479
WOA	0,0072	0,0076	0,0196	0,0192	0,0532	0,0318	0,0825	0,0304	0,2515
BeAm	0,0069	0,0090	0,0218	0,0259	0,0526	0,0290	0,0750	0,0363	0,2566
COA	0,0069	0,0075	0,0199	0,0262	0,0523	0,0318	0,0761	0,0385	0,2592
IWO	0,0074	0,0080	0,0197	0,0269	0,0538	0,0318	0,0767	0,0388	0,2632
CA	0,0069	0,0146	0,0212	0,0410	0,0518	0,0318	0,0756	0,0237	0,2667
GA	0,0074	0,0179	0,0206	0,0276	0,0519	0,0318	0,0750	0,0388	0,2710
HBO	0,0069	0,0072	0,0196	0,0404	0,0514	0,0318	0,0753	0,0384	0,2711
BHBO	0,0071	0,0182	0,0243	0,0412	0,0541	0,0318	0,0756	0,0222	0,2745
GQPSO	0,0076	0,0087	0,0220	0,0251	0,0560	0,0327	0,0841	0,0384	0,2745
AO	0,0075	0,0204	0,0198	0,0259	0,0555	0,0312	0,0832	0,0315	0,2750
IPSO	0,0069	0,0141	0,0192	0,0400	0,0510	0,0318	0,0749	0,0382	0,2763
DAO	0,0073	0,0168	0,0241	0,0294	0,0536	0,0318	0,0785	0,0388	0,2804
BBO	0,0069	0,0146	0,0212	0,0410	0,0518	0,0318	0,0756	0,0382	0,2812
BeAs	0,0070	0,0124	0,0224	0,0375	0,0542	0,0318	0,0773	0,0388	0,2813
NCCO	0,0069	0,0146	0,0212	0,0410	0,0518	0,0318	0,0756	0,0388	0,2818
CLONALG	0,0071	0,0114	0,0233	0,0397	0,0551	0,0320	0,0785	0,0386	0,2855
TWO	0,0072	0,0161	0,0234	0,0461	0,0533	0,0307	0,0769	0,0363	0,2900
SSA	0,0073	0,0184	0,0243	0,0466	0,0538	0,0318	0,0762	0,0388	0,2972
GOA	0,0074	0,0204	0,0214	0,0431	0,0560	0,0319	0,0837	0,0388	0,3027
BAT	0,0074	0,0124	0,0243	0,0519	0,0549	0,0319	0,0829	0,0388	0,3045
BFO	0,0072	0,0203	0,0242	0,0519	0,0555	0,0333	0,0776	0,0388	0,3089
SFO	0,0074	0,0203	0,0243	0,0519	0,0548	0,0318	0,0828	0,0388	0,3120
ALO	0,0074	0,0204	0,0243	0,0519	0,0555	0,0318	0,0838	0,0388	0,3138
MAO	0,0103	0,0223	0,0316	0,0600	0,0555	0,0483	0,0875	0,0636	0,3792
EQOA	0,0209	0,0468	0,0453	0,0242	0,1100	0,0597	0,1238	0,0531	0,4837

Note: SSE minimum values are shown rounded to fourth decimal places.

Table 5

Algorithm ranking by SSE median sum for DWM model [performed by the authors]

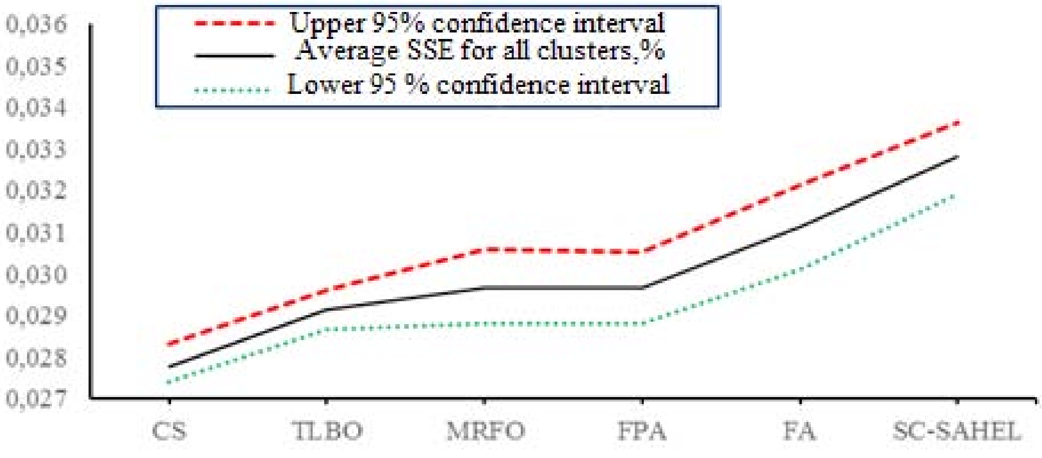
Algorithm	Studied cluster of data in the array of initial information								
	1	2	3	4	5	6	7	8	Total
SSE median sum according to 15 algorithm runs, units									
CS	0,0069	0,0072	0,0193	0,0119	0,0509	0,0278	0,0748	0,0197	**0,2186**
MRFO	0,0069	0,0073	0,0206	0,0130	0,0510	0,0279	0,0749	0,0213	**0,2229**
FA	0,0069	0,0072	0,0194	0,0142	0,0521	0,0296	0,0750	0,0218	**0,2261**
TLBO	0,0069	0,0072	0,0191	0,0112	0,0518	0,0318	0,0748	0,0250	**0,2278**
FPA	0,0069	0,0140	0,0205	0,0165	0,0510	0,0284	0,0748	0,0197	**0,2319**
WEO	0,0069	0,0092	0,0205	0,0199	0,0516	0,0294	0,0752	0,0224	0,2352
DEPSO	0,0069	0,0140	0,0191	0,0130	0,0518	0,0318	0,0748	0,0382	0,2496
VPS	0,0069	0,0141	0,0206	0,0141	0,0519	0,0296	0,0809	0,0382	0,2565
JAYA	0,0069	0,0081	0,0200	0,0230	0,0534	0,0318	0,0765	0,0384	0,2582
MPA	0,0074	0,0074	0,0203	0,0152	0,0550	0,0318	0,0835	0,0388	0,2594
IGWO	0,0077	0,0080	0,0199	0,0145	0,0555	0,0320	0,0828	0,0391	0,2595
DE	0,0069	0,0146	0,0208	0,0407	0,0524	0,0291	0,0754	0,0218	0,2617
EOF	0,0069	0,0142	0,0207	0,0401	0,0520	0,0304	0,0784	0,0206	0,2634
COA	0,0070	0,0092	0,0203	0,0290	0,0534	0,0318	0,0785	0,0386	0,2678
CGO	0,0074	0,0204	0,0243	0,0104	0,0555	0,0318	0,0808	0,0388	0,2694
HHO	0,0075	0,0076	0,0207	0,0248	0,0555	0,0318	0,0838	0,0388	0,2704
SC-SAHEL	0,0069	**0,0140**	0,0191	**0,0399**	0,0518	0,0284	0,0748	**0,0382**	0,2731
GWO	0,0074	0,0204	0,0213	0,0192	0,0539	0,0318	0,0825	0,0388	0,2754

Note: SSE median values are rounded to fourth decimal places.

noticeable regarding the results of its work in clusters: 2, 4 and 8 (Table 2).

As a result, as part of the analysis using non-parametric statistics, it was decided to compare the results for the following six algorithms: SC-SAHEL, CS, MRFO, TLBO, FPA, and FA (which turned out to be in the top three in terms of the minimum sum of SSE medians and entered the top ten in terms of minimum SSE).

The results of bootstrap estimation (the mechanism of which is described in detail in [71, pp. 11–77]) of the average SSE values for the studied algorithms based on the results of modelling 5000 stratified samples based on data on the results of 15 optimisation runs



Pic. 1. Bootstrap evaluation of DWM model optimisation results [performed by the authors].

Algorithm =>	MRFO	TLBO	MRFO	TLBO
pM-W for cluster 1 (at HKW=35,848; p<0,0001)			DTMDαβ for cluster 1	
CS	0,0000	0,0000	-1,00	-0,93
MFRO		0,0002		+0,80
pM-W for cluster 2 (at HKW=10,191; p=0,006)			DTMDαβ for cluster 2	
CS	0,0081	0,6318	-0,56	-0,10
MFRO		0,0045		+0,61
pM-W for cluster 3 (at HKW=32,289; p<0,0001)			DTMDαβ for cluster 3	
CS	0,0003	0,0050	-0,78	+0,60
MFRO		0,0001		+0,87
pM-W for cluster 4 (at HKW=32,369; p<0,0001)			DTMDαβ for cluster 4	
CS	0,0322	0,0321	-0,46	+0,46
MFRO		0,0051		+0,60
pM-W for cluster 5 (at HKW=29,366; p<0,0001)			DTMDαβ for cluster 5	
CS	0,0000	0,0000	-1,00	-1,00
MFRO		0,9174		-0,02
pM-W for cluster 6 (at HKW=19,371; p<0,0001)			DTMDαβ for cluster 6	
CS	0,0106	0,0000	-0,54	-0,84
MFRO		0,0174		-0,44
pM-W for cluster 7 (at HKW=20,365; p<0,0001)			DTMDαβ for cluster 7	
CS	0,0000	0,0014	-0,89	-0,68
MFRO		0,0922		+0,36
pM-W for cluster 8 (at HKW=10,289; p=0,006)			DTMDαβ for cluster 8	
CS	0,2297	0,0004	-0,25	-0,75
MFRO		0,1908		-0,28

Pic. 2. Results of non-parametric analysis of DWM optimisation results [performed by the authors]. HKW test statistics of Kruskal–Wallace; p – statistical significance of Kruskal–Wallace test; pM-W – statistical significance of Mann–Whitney test.

in each of the clusters (Pic. 1), characterising the overall assessment of efficiency (in addition to the previously obtained estimates based on the sum of minima and medians), indicate that at the initial stages of searching for unknown parameters of nonlinear models for the purposes of analysing price elasticity, it is preferable not to start optimisation using FA and SC-SAHEL. Also, the disadvantages of these algorithms include the need to set a large number of starting meta optimisation parameters (eight for FA and more than ten for SC-SAHEL), in contrast to two for TLBO, and three for CS, MFRO and FPA.

To draw the final conclusion, for the three best (according to the results of the bootstrap evaluation) algorithms, the statistical significance of the SSE differences (according to the results of 15 runs) was assessed based on the calculation of the non-parametric Kruskal–Wallace test [72, pp. 559–581] followed by pairwise comparisons based on the Mann–Whitney method [72, pp. 540–550]. In addition, for the convenience of assessing the degree of superiority of one algorithm over another (in pairwise comparisons within the framework of analytical matrices), the authors developed (formulae 11) a specialised DTMDαβ metric, the values of which are in the range from -1 (when all SSE values of the algorithm α are below than algorithm β) up to +1 (when all SSE values of algorithm β are less than those of algorithm α).

$$DTMD_{\alpha\beta}=\frac{\sum_{i=1}^{n}v_{\alpha i}-\sum_{i=1}^{n}h_{\beta i}}{\sum_{z=n+1}^{2n}z-\sum_{z=1}^{n}z}=\frac{\sum_{i=1}^{n}v_{\alpha i}-\sum_{i=1}^{n}h_{\beta i}}{\frac{n(n+1+2n)}{2}-\frac{n(n+1)}{2}}=\frac{\sum_{i=1}^{n}v_{\alpha i}-\sum_{i=1}^{n}h_{\beta i}}{n^2},\qquad(11)$$

Table 6

Ranking algorithms by minimum SSE for Richards model [performed by the authors]

Algo-rithm	Studied cluster in the array of data of initial information								
	1	2	3	4	5	6	7	8	Total
	Minimum SSE* based on the results of 15 runs of the algorithm, units								
CS	0,0073	0,0082	0,0155	0,0202	0,0454	0,0282	0,0683	0,0278	**0,2209**
MRFO	0,0073	0,0082	0,0155	0,0202	0,0454	0,0282	0,0683	0,0278	**0,2210**
TLBO	0,0075	0,0082	0,0155	0,0202	0,0454	0,0287	0,0683	0,0313	**0,2252**

Note: SSE minimum values are rounded to fourth decimal places.

where $\sum_{i=1}^{n} v_{ai}$, $\sum_{i=1}^{n} h_{bi}$ – the sums of ranking places obtained by the Mann–Whitney method by pairwise comparison of the SSE values in the samples of algorithm α (reflected in the analytical matrix along the vertical) and algorithm β (reflected in the analytical matrix along the horizontal);

$\sum_{z=n+1}^{2n} z - \sum_{z=1}^{n} z$ – the maximum difference between the sums of ranking places when combining two samples of the same size *n*, observed in the case when all SSE values in one of the samples are lower than any SSE value in the other sample.

The results of non-parametric analysis (Pic. 2) indicate that in all clusters for the compared algorithms, the differences in the average SSE scores are not random since the statistical significance of Kruskal–Wallace test is below the threshold level of 0,05. The analysis of the scores of Mann–Whitney tests using a threshold level of 0,05/3, according to the Bonferroni correction (see [72, pp. 565–566]), together with the results of calculating DTMDαβ, allows us to conclude that the CS algorithm is the most preferable, as it demonstrates statistically significantly better results in clusters 1, 5, 6, 7, and is not inferior to other algorithms in clusters 2 and 8.

The results of estimating the SSE minimum for the Richards model (Table 6) also confirm the effectiveness of the CS algorithm, although, in this case, its advantage over MRFO turns out to be insignificant.

Based on a comparison of the SSE minima (Table 4 and Table 6) obtained using heuristic optimisation algorithms, we can conclude that the DWN functional form looks somewhat more preferable than the Richards model. Also, although the total minimum SSE for DWM model when optimised by CS algorithm is slightly higher than that of the Pearl–Read model when optimised by LM algorithm, comparing the results separately in each of the clusters (Table 1 and Table 4) shows that DWM gives more efficient scores in five out of eight clusters, which makes this model more universal. At the same time, the use of the considered heuristic optimisation algorithms for Pearl–Read model turns out to be less effective than the initial use of LM algorithm (Table 7).

Thus, the functional form of DWM with parameter optimisation with the CS algorithm can be recommended as a competitive

Table 7

Efficiency of heuristic optimisation for the Pearl–Read model compared to using LM algorithm [performed by the authors]

Algorithms	Studied cluster of data in the array of initial information							
	1	2	3	4	5	6	7	8
	Ratio of SSE minima (based on the results of 15 runs), %							
CS/LM	100,3	100,0	**132,3**	**126,3**	100,0	100,0	102,3	100,3
MRFO/LM	100,0	100,8	**147,3**	100,0	100,0	100,0	**113,1**	101,4
TLBO/LM	103,5	100,1	**147,1**	**127,3**	100,1	100,0	**113,1**	**106,8**

alternative in the analysis of price elasticity of demand, in case when the use of the Pearl–Read model has not provided the efficiency parameters specified by management.

CONCLUSIONS

As part of development of mechanisms for managing the prices of transport services, the prospects for improving the apparatus for modelling customer behaviour are considered based on the use of various forms of nonlinear functions for analysing the price elasticity of demand, as well as the transition to the use of global optimisation heuristic tools.

Based on the use of the array of data on passenger demand, the prospects for using 15 functions, which are nonlinear in parameters, for modelling the price elasticity of demand are considered, in addition, to identify the most promising tools for assessing their parameters, the effectiveness of more than 60 optimisation algorithms is compared.

Despite the use in the analysis of data on the price elasticity of demand for passengers of branded trains, it seems that the conclusions drawn about the promising directions for development of mechanisms for managing the prices of transport services can be successfully used in relation to other modes of transport.

An important direction for further research is development of mechanisms for integrating the proposed approaches with specialised methods of competitive analysis and tools for assessing the optimal composition of trains for their subsequent joint use within the framework of a dynamic pricing system to improve the efficiency of the transport industry.

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