ORIGINAL ARTICLE
DOI: https://doi.org/10.30932/1992-3252-2021-19-6-2

World of Transport and Transportation, 2021,
Vol. 19, Iss. 6 (97), pp. 141−146

Mathematical Model of Transport Behaviour Based on Transport Macrosystems Theory

Igor E. AGUREEV

Andrey V. AKHROMESHIN

Igor E. Agureev[1]*,*
Andrey V. Akhromeshin[2]
[1, 2] *Tula State University, Tula, Russia.*
✉ [1]*agureev-igor@yandex.ru.*

ABSTRACT

Continuing the study, the purpose of which was to develop a new approach to determining the transport behaviour of residents of urban agglomerations, to outline the main ways of its development, a new description of transport behaviour is proposed based on various scientific disciplines and the theory of macrosystems.

This, according to the authors, will make contribution to laying the foundation for creation of a currently missing theory of transport behaviour.

A developed mathematical model of transport behaviour based on the provisions of academician Yu. S. Popkov's theory of macrosystems, uses the entropy approach to determine the equilibrium state of the transport system. At the same time, the model identifies an initial list of parameters responsible for describing the «transport behaviour». The latter is considered as a collective phenomenon that creates a deterministic representation resulting from interaction of many stochastically acting elements (road users). A compiled preliminary scheme can be used to solve the problem of finding unknowns in the system of equations and inequalities within the model.

Keywords: *transport system, macrosystems theory, transport behaviour, mathematical model, entropy approach.*

Acknowledgements. *This work was supported by the grant of Russian Foundation for Basic Research 19-48-710015\20 r_a.*

For citation: *Agureev, I. E., Akhromeshin, A. V. Mathematical Model of Transport Behaviour Based on Transport Macrosystems Theory. World of Transport and Transportation, 2021, Vol. 19, Iss. 6 (97), pp. 141−146. DOI: https://doi.org/10.30932/1992-3252-2021-19-6-2.*

The text of the article originally written in Russian is published in the first part of the issue.
Текст статьи на русском языке публикуется в первой части данного выпуска.

INTRODUCTION

Problem Statement

The article [1] presented a new approach to definition of the transport behaviour of the population of urban agglomerations, outlined the main routes of the development of the topic, and proposed a «new approach to describing transport behaviour from the standpoint of various scientific disciplines, of the theory of macrosystems, which lays the basis for creating the theory of transport behaviour that is currently missing» [1]. The objective implies that «the study of transport behaviour is carried out at the level of the «system as a whole», and not of its individual elements, which makes it possible to form general approaches to management of transport behaviour to achieve optimal characteristics of functioning of the transport system of a city (agglomeration). The mathematical description of the transport system, presented in the work, has a general form, requires expansion and addition» [1].

The construction of mathematical models based on the theory of macrosystems, assuming that any transport system is a multicomponent heterogeneous open system, «has scientific novelty and is promising to be further studied and implemented in the transport industry. An urgent task on this path is to study the nature of collective behaviour from the standpoint of the influence of motives when deciding on a trip and of the emergence of spatial structures of movement of passengers and vehicles, i.e., formation of passenger flows and of transport vehicle flows, as well as consideration of the concept of transport behaviour from the point of view of psychology, sociology, and urban studies» [1]. At the same time, construction of an appropriate mathematical model should reflect in a certain manner the features of individual behaviour at the level of the entire system.

This work is a continuation of work [1].

The formation of an idea about movements of the population consists currently of four types of trips [2–5]: «trips from places of residence to places of employment and back (the so-called trips for work purposes); trips from places of residence to cultural and public facilities (shops, etc.) and back; trips between workplaces (business or job trips); trips between cultural and community facilities. The following methods of calculating origin-destination trip matrix are most widely represented in the literature» [2]:

- Normative [standard-based] methods [6].
- Statistical methods [6].
- Gravitational [gravity] model [3].
- Entropy models [4; 5].
- Models of self-organising flows [7].
- Method of competing centres [8].
- Stouffer [Samuel A. Stouffer] intermediate capabilities models [8], etc.

The following conclusions can be drawn from the publications discussed above:

1) Transport behaviour can be considered from various standpoints, namely, from the point of view of factors that determine the choice of a mode of travelling in accordance with certain purposes of a trip.

2) Transport behaviour can be studied by various methods, in particular, the general picture of mobility of individuals by purpose, time, travel distance and mode of transport used is described using the method of connectivity graphs.

3) Collective transport behaviour is the result of interaction of individual transport processes of each of the passengers.

4) There is a constitutive term «pattern of daily individual activity of a passenger», which underlies the study of patterns in the passenger behaviour and is consistent with the concept of a transport system of individual trips (TSIT) proposed earlier by the authors in [1].

5) The expediency of studying transport behaviour with the help of the theory of transport macrosystems is confirmed.

It is important to note that the domestic and foreign scientific literature contains little information about how exactly the passenger decides whether he will make a trip or not, that is, behavioural models are practically not studied.

Brief Information about the Current State of Research on Transport Behaviour [9–14]

«Transport behaviour has a complex nature, the description of this term is associated with the definition of the concepts of «transport mobility (or mobility) of the population» [1].

«Transport behaviour as a phenomenon is distinguished by the following properties that relate to the collective level of its description:

1) Stochastic nature.

2) Multitude of participants that determine the characteristics of transport behaviour.

3) Plurality of factors influencing the choice and shaping the dynamics of the phenomenon.

4) Interdisciplinarity as a knowledge base for the study of the subject. The problem of transport behaviour should also be considered comprehensively, explored not only within the framework of transport science (the theory of passenger transportation), but also of related fields of knowledge, such as: sociology, urban studies, economics» [1], etc.

In this work we are primarily interested in the measurable parameters of transport behaviour, which could be included in the mathematical model as an initial data or be calculated with its help. These parameters should include:

1) Factor of splitting of passenger flows by modes of transport within the simulated period of time.

2) Factor of splitting of passenger flows by purpose of the trip.

3) Distribution of origin-destination trips along specific routes.

4) Interchange (transfer) factor.

5) Distribution of trips by range (distance).

6) Distribution of waiting time for passengers at embarkation/disembarkation points.

7) Exchanges of passenger flows at embarkation/disembarkation points as a function of time, etc.

The necessary and sufficient list of parameters that unambiguously characterise the transport behaviour within a mathematical model will have to be identified and substantiated as a result of further research and calculations.

RESULTS

Mathematical Model of Transport Behaviour (Version)

Transport behaviour in its entirety can be described using the mathematical apparatus of macrosystems theory, because the transport behaviour of an individual passenger (as an element of the TSIP) «has a random nature, is practically not subject to description, and in some cases could not be described even logically. The behaviour of a single passenger is a particular case», while «the description of the transport behaviour of groups of passengers is of practical interest for the purposes of management, planning and development of the transport system of a city (agglomeration) to issue practical recommendations» [1].

In the article [1] a mathematical model is given, which describes the transport system regarding the most general case:

$$\begin{cases} \widetilde{Gr} = \widetilde{Gr}(t); \\ \rho = \rho(t); \\ q = q(t); \\ \tau_0 \le t \le \tau_0 + \Delta\tau; \Delta\tau = \sum_{k=1}^{K} \Delta\tau_k; \\ V(t) = \left\{ V_1, \dots, V_\alpha, \dots, V_p : V_\alpha = \sum_{v=1}^{v_f} n_v(t) \mid v \in p_\alpha \right\}; \\ v = 1, \dots, v_f(t); \\ \pi_v = \pi_v(t); \\ P(t) = \left\{ \pi_1(t), \dots, \pi_\beta(t), \dots, \pi_{\upsilon_f}(t) \right\}; \\ G(t) = g_v \otimes P(t) \le G^*; \\ H\left(V \bullet (\Delta\tau_k)\right) = -\sum_{n=1}^{m} V_n \ln \frac{V_n}{ap_n} - (G_n + V_n)\ln(G_n + V_n) \to \max, \end{cases} \quad (1)$$

where k is index (individual number) of a time interval $\Delta\tau_k$;

K is total number of time intervals $\Delta\tau_k$;

$\tilde{Gr}$ is street-road network graph;

ρ is matrix of transport connectivity (connections);

q is matrix of effective carrying (transit) capacities;

t is continuous time;

$V(t)$ is a set, each of the elements of which is equal to the number of cars (vehicles) that are on the route (or participating in origin-destination trip) α at the time t;

v is car index (its unique identifier);

$v_f = N_a$ is the largest car index corresponding to the number of vehicles at the current time;

p is total number of routes (origin-destination connections);

n_v is Boolean variable, which is determined by relation (2) and equals 1 if the vehicle is on the route p_α and 0 otherwise;

p_α is α-th route;

π_v is equation of the transport process for a car, which determines the share of the completed transport process (transport work);

P(t) is a set consisting of separate equations of the transport process;

β is transport process index;

G(t) is vector function of resource(s) spending;

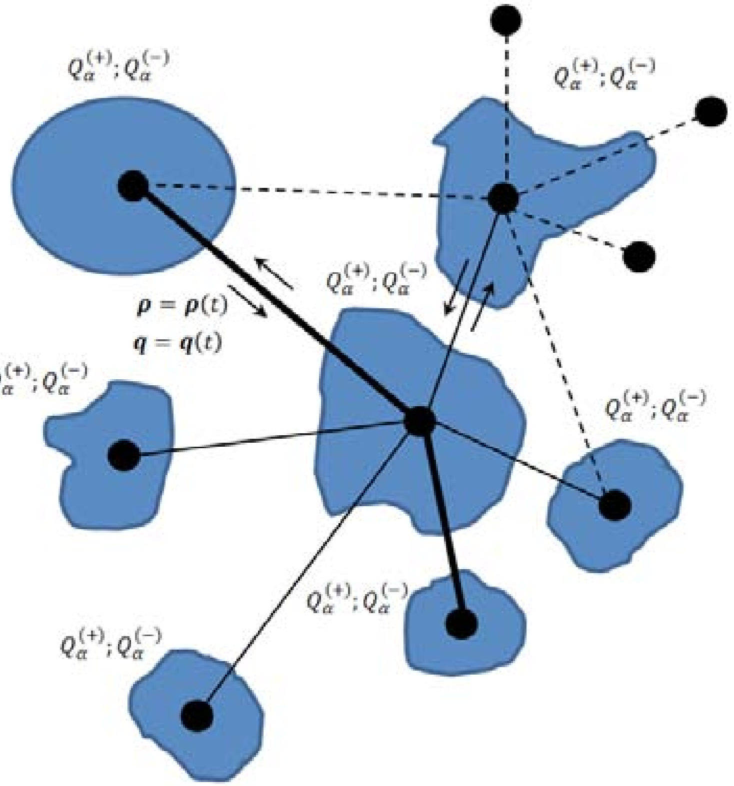
g_v is vector of specific resource costs for each vehicle;

H is information entropy of the transport system;

G* is vector of constraints regarding resource consumption;

ap_n is a priori probabilities of finding an element in state n;

G_n is capacity of state n;



Pic. 1. Scheme of transport connections (origin-destination trips) between districts [performed by the authors].

n is ordinal number of the state of the elements;

m is total number of different states.

In the theory of macrosystems, there are several ways to fill the states of the system (Yu. S. Popkov [5]). To calculate the information entropy *H* in system (1), the Bose–Einstein statistics was chosen as the most preferable one. This statistic means that a large (up to infinite) number of elements can be in the same state. For example, there may be a sufficiently large number of vehicles on the same route at the same time to warrant the use of this statistic.

The system of equations and constraints (1) is solved by numerical methods to find an equilibrium state (maximum information entropy), which gives the distribution of elements of the set *V(t)* over correspondences. These elements are the main unknowns of system (1).

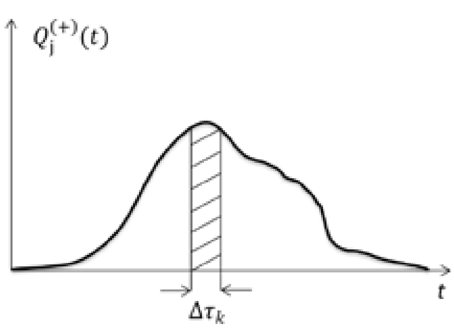
The scheme of origin-destination trips in the system is shown in Pic. 1.

Then,

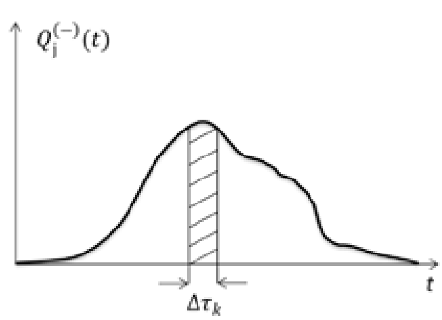
$$n(t) = f(x) = \begin{cases} 1, v \in p_\alpha \\ 0, v \notin p_\alpha. \end{cases} \quad (2)$$

To close the system (1), it is necessary to add a set of elements to it:

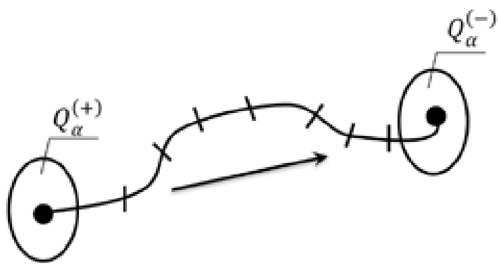
1) The characteristics of transport sources can be written in the form (3) and make it possible



Pic. 2. Dependence of intensity of source of vehicles [performed by the authors].



Pic. 3. Dependence of intensity of vehicles' runoff [performed by the authors].



Pic. 4. Chart of the route [performed by the authors].

to calculate the number of vehicles «generated» for any time interval (Pics. 2 and 4):

$$Q_i^{(+)} = Q_i^{(+)}(t). \quad (3)$$

2) Characteristics of the runoff (absorption centre, centre of mass gravity) located at the end of route α (Pic. 3);

3) Equation of balance of cars, which are on the route α:

$$\mathbf{V}_\alpha\Big|_{\Delta\tau_k} = \mathbf{V}_\alpha\Big|_{\Delta\tau_{k-1}} + \Delta\mathbf{V}_\alpha^{(+)}\Big|_{\Delta\tau_k} - \Delta\mathbf{V}_\alpha^{(-)}\Big|_{\Delta\tau_k}, \quad (4)$$

where the number of cars which are on the route within the time interval $\Delta\tau_k$ is considered (Pic. 2).

$$\Delta\mathbf{V}_\alpha^{(+)}\Big|_{\Delta\tau_k} = \int_{\Delta\tau_k} Q_\alpha^{(+)}(t)dt. \quad (5)$$

The number of cars, which terminated the transport process of the route within the time interval $\Delta\tau_k$ (Pic. 3):

$$\Delta V_\alpha^{(-)}\Big|_{\Delta\tau_k} = \int_{\Delta\tau_k} Q_\alpha^{(-)}(t)dt. \quad (6)$$

Thus, the variables requiring calculations in model (1) are the members of the sets: a) V(t); b) P(t).

To search for unknowns, it is necessary at each step $\Delta\tau_k$ to solve a problem containing the calculation:

1) $Q_i^{(+)}$.

2) $Q_i^{(-)}$,that is, the volumes of departures and arrivals for all transport districts.

3) H_{max} that is maximum value of entropy, with an appropriate distribution of the elements of the sets V(t) along the routes p_α, presumably considering only those vehicles that have reached the trip destination point ($\pi_v = 1$).

4) Elements of the set P(t), which requires to introduce additional conditions on the speed of vehicles.

5) A priori probabilities that presumably reflect the generalised cost (price) of the trip.

Thus, it is additionally required to enter into the description a formula for calculating a priori probabilities $p_1, p_2, \ldots, p_\alpha$, as well as a method of accounting of C_β* (generalised trip cost (price)).

CONCLUSIONS

The mathematical model of transport behaviour developed in this work within the framework of the macrosystem approach [15] requires at this stage knowledge of the following parameters as of a priori information:

1) Factor of splitting of passenger flows by modes of transport within the simulated period of time.

2) Factor of splitting of passenger flows by purpose of the trip.

3) Distribution of origin-destination trips along specific routes.

It is assumed that this model will allow to determine as a result of calculations:

1) Interchange (transfer) factor.

2) Distribution of trips by range (distance).

3) Distribution of waiting time for passengers at embarkation/disembarkation points.

4) Exchanges of passenger flows at embarkation/disembarkation points as a function of time, etc.

Certainly, answers should be obtained in the near future to questions about how the origin-destination trip matrix will be calculated for the proposed model [16–18], as well as on the applicability of entropy methods for solving problems of the equilibrium of the transport system to modern urban systems [19– 23].

REFERENCES

1. Agureev, I. E., Akhromeshin, A. V. Approaches to formalisation of the concept of transport behaviour of the population of urban agglomerations [*Podkhody k formalizatsii ponyatiya transportnogo povedeniya naseleniya gorodskikh aglomeratsii*]. *Intellekt. Innovatsii. Investitsii*, 2021, Iss. 2, pp. 60–70. DOI: 10.25198/2077-7175-2021-2-60.

2. Aliev, A. S., Strelnikov, A. I., Shvetsov, V. I., Shershevsky, Yu. Z. Modelling transport flows in a large city as applied to Moscow agglomeration [*Modelirovanie transportnykh potokov v krupnom gorode s primeneniem k moskovskoi aglomeratsii*]. *Avtomatika i telemekhanika*, 2005, Iss. 11, pp. 113–125. [Electronic resource]: http://www.mathnet.ru/php/getFT.phtml?jrnid=at&paperid=1465&what=fullt&option_lang=rus. Last accessed 19.11.2021.

3. Carrothers, G. A. P. An Historical Review of the Gravity and Potential Concepts of Human Interaction. *Journal of the American Planning Association*, 1956, Vol. 22, pp. 94–102. DOI: 10.1080/01944365608979229.

4. Wilson, A. G. Entropy in Urban and Regional Modelling [In Russian. Russian edition's title: Entropy methods of modelling in complex systems. *Entropiynie metody modelirovaniya slozhnykh system*]. Moscow, Nauka publ., 1978, 247 p.

5. Popkov, Yu. S. Theory of macrosystems: Equilibrium models [*Teoriya makrosistem: Ravnovesnie modeli*]. Moscow, Editorial URSS, 1999, 320 p. ISBN 5-8360-0035-2.

6. Vasilyeva, E. V., Igudin, R. V., Livshits, V. N. Optimization of planning and management of transport systems [*Optimizatsiya planirovaniya i upravleniya transportnymi sistemami*]. Moscow, Transport publ., 1987, 207 p.

7. Vasilyeva, E. V., Levit, B. Yu., Livshits, V. N. Nonlinear transport problems on networks [*Nelineinie transportnie zadachi na setyakh*]. Moscow, Finansy i statistika publ., 1972, 103 p.

8. Shvetsov, V. I. Mathematical modelling of transport flows [*Matematicheskoe modelirovanie transportnykh potokov*]. *Avtomatika i telemekhanika*, 2003, Iss. 11, pp. 3–46. [Electronic resource]: http://www.mathnet.ru/php/getFT.phtml?jrnid=at&paperid=1966&what=fullt&option_lang=rus. Last accessed 19.11.2021.

9. Fedorov, V. A. Transport behaviour of individuals – the main source of urban transport problems [*Transportnoe povedenie individuumov – osnovnoi istochnik gorodskikh transportnykh problem*]. *Molodoi ucheniy*, 2015, Iss. 18 (98), pp. 309–316. [Electronic resource]: https://moluch.ru/archive/98/21292/. Last accessed 19.11.2021.

10. Muleev, E. Yu. «Transport behaviour» and «Mobility»: on the issue of conceptual terms *[«Transportnoe povedenie», «Podvizhnost*» *i* «*Mobilnost*»*: k voprosu o kontseptualnosti terminov*]. *Sotsiologicheskiy zhurnal*, 2015, Vol. 21, Iss. 3, pp. 8–28. DOI: https://doi.org/10.19181/socjour.2015.21.3.2375.

11. Savelyeva, V. Yu. Factors of formation of transport behaviour in the largest cities of Russia [*Faktory formirovaniya transportnogo povedeniya v krupneishikh gorodakh Rossii*]. *Urban Planning Journal. All-Union Scientific Research Institute for Problems of Scientific and Technological Progress of Information in Construction (Moscow)*, 2018, Iss. 5 (57), pp. 54–62. [Electronic resource]: https://www.elibrary.ru/item.asp?id=35781220. Last accessed 19.11.2021.

12. Muleev, E. Yu. Transport behaviour of the population of Russia: a brief report on a sociological study [*Transportnoe povedenie naseleniya Rossii: kratkiy otchet o sotsiologicheskom issledovanii*]. Moscow, Institute of Transport Economics and Transport Policy of National Research University Higher School of Economics, 2015, 37 p. [Electronic resource]: https://www.hse.ru/data/2015/03/10/1093862032/Транспортное%20поведение%20населения%20РФ_2014.pdf. Last accessed 19.11.2021.

13. Mobility in cities database. Synthesis report. UITP, 2015. [Electronic resource]: https://cms.uitp.org/wp/wp-content/uploads/2020/06/MCD_2015_synthesis_web_0.pdf. Last accessed 19.11.2021.

14. Quan Liang; Jiancheng Weng; Wei Zhou; Selene Baez, Santamaria; Jianming Ma; Jian Rong. Individual Travel Behavior Modeling of Public Transport Passenger Based on Graph Construction. *Hindawi Journal of Advanced Transportation*, Vol. 2018, Article ID 3859830. DOI: https://doi.org/10.1155/2018/3859830.

15. Agureev, I. E. Development of the Theory of Macrosystems as a Necessary Condition for Improving Quality of Transport Modelling. *World of Transport and Transportation*, 2020, Vol. 20, Iss. 2, pp. 6–20. DOI: https://doi.org/10.30932/1992-3252-2020-18-06-20.

16. Gasnikov, A. V., Gasnikova, E. V., Mendel, M. A., Chepurchenko, K. V. Evolutionary conclusions of the entropy model for calculating the correspondence matrix [*Evolyutsionnie vyvody entropiinoi modeli rascheta matrizy korrespondentsii*]. *Matematicheskoe modelirovanie*, 2016, Vol. 28, pp. 1–16. [Electronic resource]: http://www.mathnet.ru/php/getFT.phtml?jrnid=mm&paperid=3724&what=fullt&option_lang=rus. Last accessed 19.11.2021.

17. Imelbaev, Sh. S., Shmulyan, B. L. Modeling of stochastic communication systems [*Modelirovanie stokhasticheskikh kommunikatsionnykh sistem*]. *Entropiinie metody modelirovaniya slozhnykh sistem*. Moscow, Nauka publ., 1978, pp. 170–233. [Electronic resource]: http://www.mathnet.ru/php/getFT.phtml?jrnid=at&paperid=7358&what=fullt&option_lang=rus. Last accessed 19.11.2021.

18. Gasnikova, E. V. On the possible dynamics in the calculation model of the correspondence matrix [*O vozmozhnoi dinamike v modeli rascheta matrizy korrespondentsii*]. In: Introduction to mathematical modelling of transport flows. Ed. by Gasnikov A. V. [*et al*]. 2nd ed. Moscow, Moscow Centre for Continuous Mathematical Education, 2013, pp. 248–270. ISBN 9785443921501. [Electronic resource]: https://ru.pt1lib.org/dl/2899886/f9150c. Last accessed 19.11.2021.

19. Nesterov, Yu. E., Shpirko, S. V. Stochastic transport equilibrium [*Stokhasticheskoe transportnoe ravnovesie*]. In: Introduction to mathematical modelling of transport flows. Ed. by Gasnikov A. V. [*et al*]. 2nd ed. Moscow, Moscow Centre for Continuous Mathematical Education, 2013, pp. 314–324. ISBN 9785443921501. [Electronic resource]: https://ru.pt1lib.org/dl/2899886/f9150c. Last accessed 19.11.2021.

20. Wilson, A. G. Complex Spatial Systems: The Modelling Foundations of Urban and Regional Analysis. London, New York, 2000. DOI: https://doi.org/10.4324/9781315838045.

21. Dearden, J., Wilson, A. Exploring urban retail phase transition – 1: an analysis system. UCL, Working Papers Series, July 2008. Paper 140. [Electronic resource]: https://discovery.ucl.ac.uk/id/eprint/15193/1/15193.pdf. Last accessed 19.11.2021.

22. Wilson, A. G. The «Thermodynamics» of City: Evolution and Complexity Science in Urban Modelling. In: Complexity and Spatial Networks, Advances in Spatial Science. Springer-Verlag, Berlin, Heidlberg, 2009, pp. 11–31. DOI: 10.1007/978-3-642-01554-0_2.

23. Purvis, B., Mao, Yo., Robinson, D. Entropy and its Application to Urban Systems. *Entropy*, 2019, Vol. 21, 56 p. DOI: 10.3390/e21010056.

●

Information about the authors:

Agureev, Igor E., *D.Sc. (Eng), Head of the Department of Cars and Car Economy of Tula State University, Tula, Russia, agureev-igor@yandex.ru.*

Akhromeshin, Andrey V., *doctoral student at the Department of Cars and Car Economy of Tula State University, Tula, Russia, aakhromeshin@yandex.ru.*

Article received 07.11.2021, approved 20.12.2021, accepted 27.12.2021.