



Methods of Analysis and Synthesis of Switching Circuits of Photonic Switches Using the Example of Spanke Architecture



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ABSTRACT

The development of high-speed rail requires introduction of new telecommunications technology implemented in an integrated digital technological communication system (IDTC). Features of building such systems comprise provision of switching optical data channels using photonic switches (PS). Switching processes in PS occur at the photon (optical) level. A feature of construction of PS is the use of multi-tier topologies, performed using binary switches (BSs). BS is the simplest switching element with the number of input/output ports equal to one or two. The concepts for constructing PS are based on the technology of the well-known switching circuits using BS whose architecture and topology are assigned the names of their creators (Benes, Spanke, Spanke–Benes architecture, Clos network, etc).

With an increase in PS capacity, its structure becomes more complicated: the number of links in the switching circuit, the total number of BSs, the length of switching routes, and the redundancy factor increase. In addition, it becomes necessary to calculate the

probabilities of the occurrence of internal blocking in switching circuits, speed of switching optical signals, the value of attenuation of the optical signal in PS circuit, etc.

The objective of the study was to develop methods of analysis and synthesis of switching circuits of photonic switches using the example of a circuit of Spanke architecture of a given capacity with calculation of the probabilities of occurrence of internal blocking. The authors used general scientific and engineering methods of mathematical modelling, probability and queuing theory and an example of an algorithm for analysing the structures of Spanke topology with capacities from 4×4 to 128×128. Their topological and probabilistic characteristics (the number of links in the switching circuit, the total number of BS, the length of the switching routes, the probability of occurrence of internal blocking in PS circuits) have been determined. The results of calculations are presented in the form of tables. The developed methods of analysis and synthesis can be used in the study of similar switching circuits built using BS.

Keywords: photon switching, binary switch, Spanke architecture, structural characteristics, probability of internal blocking, high-speed transport.

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INTRODUCTION

The development of high-speed railway transport requires introduction of new telecommunications technology implemented in an integrated digital technological communication system (IDTC). One of the features of construction of such systems is provision of switching optical data transmission channels using photonic switches (PSs). Numerous research works have been dedicated to various aspects of the topic for many years [1–8].

PSs are one of the main elements of such systems and allow configuring their topology. Signal switching processes in PS occur at the photon (optical) level. The peculiarities of construction of PS with the number of inputs/outputs from 4×4 and more are associated with the use of multi-tier topologies performed using binary switches (BSs). BS is the simplest switching element with the number of inputs/outputs 2×2 , 2×1 or 1×2 .

In [9; 10], it was proposed to build PS schemes according to the architecture of classical networks which were assigned, as a rule, the names of their creators:

- Spanke network [R. A. Spanke].
- Matrix scheme.
- Delta scheme.
- Banyan network.
- Batcher–Banyan network [K. E. Batcher].
- Benes network [V. E. Beneš].
- Spanke–Benes network.
- Clos network [Ch. Clos].

With an increase in PS capacity, its structure becomes more complex: the number of links in the switching circuit, the total number of BSs, the length of the switching routes, and the redundancy factor [11] increase. In addition, it becomes necessary to calculate the probabilities of occurrence of internal blocking in switching circuits, the speed of switching optical signals, the values of attenuation of the optical signal in PS circuit, etc. Works [9–12] present the principles of PS operation, their technical characteristics, and features of constructing their

architecture. However, the authors have not considered the issues of analysis and synthesis of structural schemes of PS, presented methods for calculating their structural and probabilistic characteristics.

The *objective* of the research was to develop methods of analysis and synthesis of PS switching circuits using the example of Spanke networks with a capacity of $M = 2^N$, and methods for calculating their structural and probabilistic characteristics.

Research *methods* included general scientific and engineering methods of mathematical modelling, probability theory and queuing theory.

RESULTS

It is proposed to consider the structures of multi-link PS with the number of inputs/outputs $M \times M$, which are formed according to the modular principle. Namely, the switching circuit with a capacity of $M \times M$ is formed based on switching circuits with the number of inputs/outputs equal to $M/2 \times M/2$.

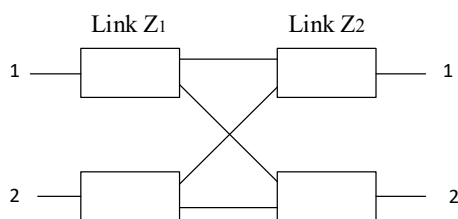
In accordance with the proposed modular principle for construction of Spanke switching networks, it is necessary to consider the features of the structure of circuits with a minimum capacity of 2×2 , followed by an increase in this capacity according to the law $M = 2^N$. Pic. 1 shows Spanke network with a minimum capacity of 2×2 [13].

Any Spanke switching scheme of capacity $M = 2^N$, where $N = 2, 3, 4 \dots$ can be interpreted as a three-link network. In this case, BSs of the first and third links are input and output BSs of 1×2 and 2×1 types, and BSs of the second link are blocks with a capacity of $M/4$.

For analysis and synthesis of structural diagrams of PS with a capacity of M , we introduce few definitions.

BS link is a set of BSs that belong to one switching stage. The links in Spanke network are subdivided into terminal and intermediate ones. There are no intermediate links in Spanke switching scheme with a capacity of 2×2 (Pic. 1). Terminal links are built on 1×2 and 2×1 BSs, the intermediate link consists of four blocks with a capacity of $M/2$.

BS block is a switching circuit with a capacity of $M/2$, based on which switching circuits with a capacity of M are built. At the same time, in Spanke schemes, the number of blocks is equal to four, regardless of the capacity of the scheme.



Pic. 1. 2×2 Spanke network [13].

Blocks are placed between the first and last links of the switch.

The first and third links of the switching circuit are subdivided into BS groups. Each group consists of $M/2$ core switches. The BS groups are assigned double numbering. The first digit indicates belonging to the link number (1 or 3), the second digit marks the sequence number of the group in the link (1 or 2). The total number of groups in Spanke schemes is always four.

Let us introduce the following designations for the structural characteristics of Spanke switch circuit:

M – number of inputs/outputs in the switch circuit ($M = 2^N$).

Z_k – number of the link in the switch circuit, $k = 1, 2, 3$.

B_k – number of BSs in the Z_k link of the switch circuit – for $k = 1, 3$, $B_k = M$; for $k = 2$, $B_k = 2 \cdot M \cdot (M - 2)$.

B – total number of BSs in the switch circuit.

G_{ij} – number of the group in the switch circuit, where $i = 1, 3$; $j = 1, 2$.

D_m – number of the block in the Z_2 link of the switch circuit, $m = 1, 2, 3, 4$.

As an example, let us analyse the structural characteristics of 4×4 Spanke network shown in Pic. 2.

The Z_1 link is embodied with 1×2 type BS, the Z_2 link is embodied with 2×2 type BS. The number of BSs in the links corresponds to the number of inputs (outputs) of the switch. Link Z_2 contains four blocks with a capacity of 2×2 . Lines of distinct types (solid and dotted) show the circuits connecting four blocks with a capacity of 2×2 to BS of links Z_1 and Z_3 . Link Z_1 consists of groups G_{11} and G_{12} , link Z_3 consists of groups G_{31} and G_{32} .

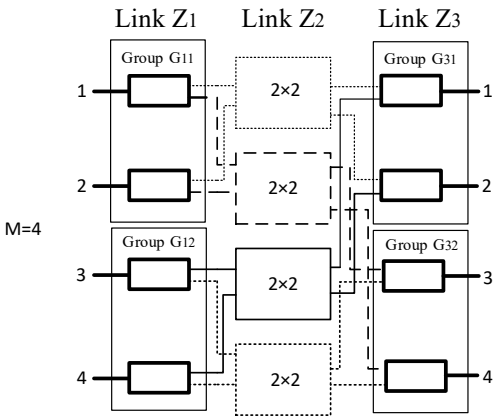
The authors of the article propose the following generalised algorithm for analysis and synthesis of Spanke switching schemes with a capacity of $M = 2^N$:

1. Initially determined are:

- Total number of BSs in the scheme according to the formula: $B = 2 \cdot M \cdot (M - 1)$ [10].
- Number of BSs in the first link B_1 and the last link B_3 in the switch circuit: $B_1 = B_3 = M$.
- Total number of BSs in the intermediate link Z_2 , calculated by the formula: $B_2 = 2 \cdot M \cdot (M - 2)$.

2. The D_m blocks of the Z_2 link are numbered from 1 to 4.

3. BS G_{ij} groups in the first (G_{11} and G_{12}) and in the third (G_{31} and G_{32}) links are numbered.



Pic.2. Spanke network (compiled by the authors).

4. The scheme of connecting the outputs of BS of the link Z_1 to the inputs of the blocks of the link Z_2 is as follows. The outputs of the first BS in the G_{11} group are connected to the first inputs of the D_1 and D_2 blocks. The outputs of the second BS of the G_{11} group are connected to the second inputs of the D_1 and D_2 blocks. The outputs of the first BS of the G_{12} group are connected to the first inputs of the D_3 and D_4 blocks. The outputs of the second BS of the G_{12} group are connected to the second inputs of the D_3 and D_4 blocks.

5. The scheme of connecting of the BS of the link Z_3 to the outputs of the blocks of the link Z_2 is as follows. The inputs of the first BS of the G_{31} group are connected to the first outputs of the D_1 and D_3 blocks. The inputs of the second BS of the G_{31} group are connected to the second outputs of the D_1 and D_3 blocks. The inputs of the first BS of the G_{32} group are connected to the first outputs of the D_2 and D_4 blocks. The inputs of the second BS of the G_{32} group are connected to the second outputs of the D_2 and D_4 blocks.

The algorithm developed above for analysis and synthesis of Spanke switching schemes allows to show in Pic. 3 a diagram of a switch with a capacity of 8×8 .

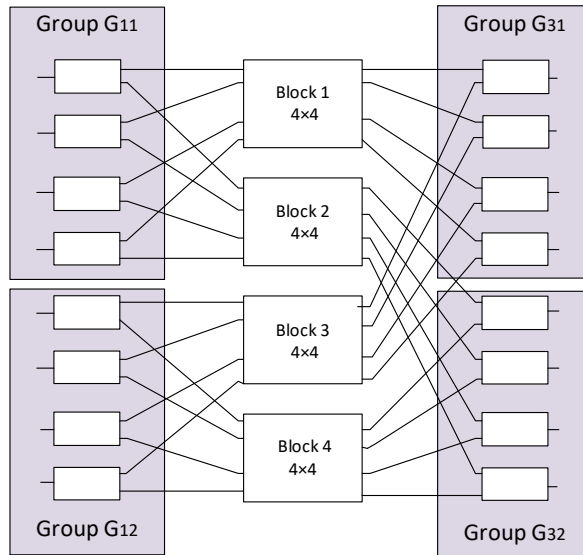
The authors propose the following method for calculating the probabilities of occurrence of internal blocking in Spanke switching schemes.

Let's consider an example of traffic transmission through 2×2 Spanke switching scheme (Pic. 1) from the first input to the first output.

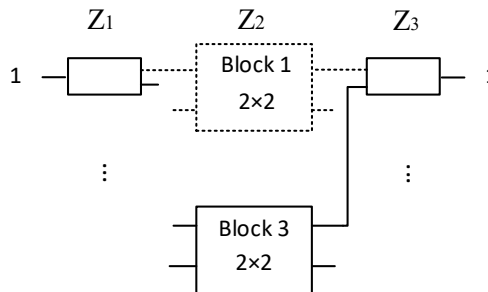
Let us introduce the following additional designations:

- q_{SM} – probability of occurrence of internal blocking in Spanke scheme with a capacity of M .





Pic. 3. An example of a synthesized 8x8 Spanke scheme (compiled by the authors).



Pic. 4. Fragment of 4x4 Spanke structure (compiled by the authors).

- ρ – average intensity of the incoming specific load at each of PS's inputs, E [Erlang].
- α – coefficient of gravitation of the incoming load within PS scheme.

Let's consider the procedure for servicing the load in the circuit of this switch. Internal blocking will occur when the observed serviced load and the competing load from the second PS input arrive at the inputs, for example, of the first BS of the link Z_1 . In this case, the probability of blocking the connection between the first input and the first output is determined by the formula:

$$q_{s2} = 1 - (1 - \alpha \cdot \rho) = \alpha \cdot \rho. \quad (1)$$

In an analogous way, let us determine the probability of occurrence of internal blocking for Spanke $4 \times 4 q_{s4}$ structure, representing each block in the link Z_2 by Spanke 2×2 scheme (Pic. 2).

Let us transform the scheme in Pic. 2 to the simplified one shown in Pic. 4. In the general scheme, we highlight the ways of establishing a

connection between the first input and the first output.

At the output 1 of the link Z_3 , the probability of blocking the transmitted traffic will depend on the amount of load coming from the output of the third 2×2 block.

Based on the foregoing, let us express the probability of occurrence of internal blocking in $4 \times 4 q_{s4}$ Spanke network through the probabilities of occurrence of internal blocking q_{s2} :

$$q_{s4} = 1 - (1 - q_{s2}) \cdot (\alpha \cdot \rho)^3 \cdot (1 - \alpha)^2. \quad (2)$$

In the classical Spanke networks [14–16], the serviced load is concentrated in 2×1 BS of the link Z_3 . This leads to significantly high probability of traffic blocking.

As an example, for $\rho = 0,001 E$, $\alpha = 0,5$, the probability of blocking in 4×4 Spanke structure will be $q_{s4} = 0,999$.

The authors propose to modernise the classic Spanke structure by replacing 2×1 BS with 2×2 BS in the link Z_3 . The diagram is shown in Pic. 5.

Table 1

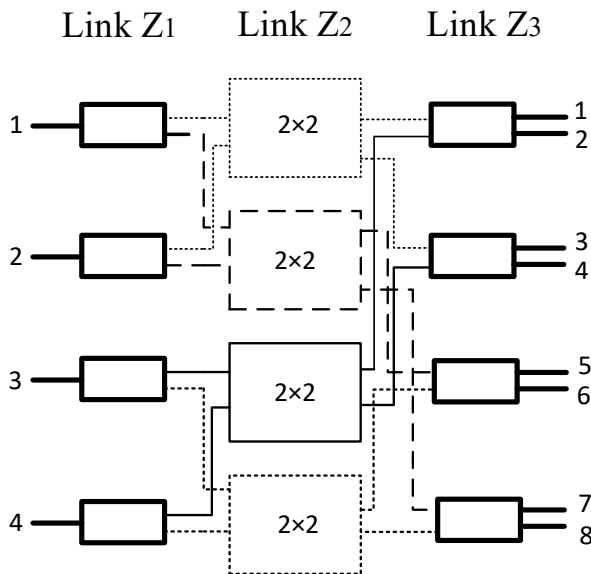
The probabilities of occurrence of internal blocking in the modernised Spanke networks of various capacity with the intensity of the incoming load $\rho = 0,001$ E

α	q_{S2}	q_{S4}	q_{S8}	q_{S16}	q_{S32}	q_{S64}	q_{S128}
0,1	$0,1 \cdot 10^{-3}$	$0,199 \cdot 10^{-3}$	$0,299 \cdot 10^{-3}$	$0,399 \cdot 10^{-3}$	$0,499 \cdot 10^{-3}$	$0,599 \cdot 10^{-3}$	$0,699 \cdot 10^{-3}$
0,5	$0,5 \cdot 10^{-3}$	$0,999 \cdot 10^{-3}$	$1,499 \cdot 10^{-3}$	$1,998 \cdot 10^{-3}$	$2,497 \cdot 10^{-3}$	$2,996 \cdot 10^{-3}$	$3,494 \cdot 10^{-3}$
0,9	$0,9 \cdot 10^{-3}$	$1,799 \cdot 10^{-3}$	$2,697 \cdot 10^{-3}$	$3,595 \cdot 10^{-3}$	$4,491 \cdot 10^{-3}$	$5,387 \cdot 10^{-3}$	$6,283 \cdot 10^{-3}$

Table 2

The probabilities of occurrence of internal blocking in the modernised Spanke networks of various capacity with the intensity of the incoming load $\rho = 0,01$ E

α	q_{S2}	q_{S4}	q_{S8}	q_{S16}	q_{S32}	q_{S64}	q_{S128}
0,1	$1 \cdot 10^{-3}$	$1,999 \cdot 10^{-3}$	$2,997 \cdot 10^{-3}$	$3,994 \cdot 10^{-3}$	$4,990 \cdot 10^{-3}$	$5,985 \cdot 10^{-3}$	$6,979 \cdot 10^{-3}$
0,5	$5 \cdot 10^{-3}$	$9,975 \cdot 10^{-3}$	$14,925 \cdot 10^{-3}$	$19,850 \cdot 10^{-3}$	$24,751 \cdot 10^{-3}$	$29,627 \cdot 10^{-3}$	$34,479 \cdot 10^{-3}$
0,9	$9 \cdot 10^{-3}$	$17,919 \cdot 10^{-3}$	$26,757 \cdot 10^{-3}$	$35,516 \cdot 10^{-3}$	$44,197 \cdot 10^{-3}$	$52,799 \cdot 10^{-3}$	$61,324 \cdot 10^{-3}$



Pic. 5. Modernised 4x4 Spanke structure (compiled by the authors).

In this case, the probability of internal blocking in the modernised 4x4 Spanke network will be:

$$q_{S4} = 1 - (1 - \alpha \cdot \rho) \cdot (1 - q_{S2}). \quad (3)$$

For an arbitrary value of the capacity M , the formula for calculating the probability of occurrence of internal blocking will take the form:

$$q_{SM} = 1 - (1 - \alpha \cdot \rho) \cdot (1 - q_{\frac{SM}{2}}). \quad (4)$$

The results of calculating a probability of occurrence of internal blocking in Spanke structures with 2x2 BS in the link Z_j are presented in Table 1 and Table 2.

CONCLUSIONS

The replacing of 2x1 output BS with 2x2 BS in Spanke structures proposed by the authors allows reducing probability of internal blocking by an average of three orders of magnitude, which will ensure high throughput and a possibility of using IDTC with photon switching in optical networks.

The developed method makes it possible to analyse and synthesise PS circuits using the example of a Spanke structure of any capacity and to identify a probability of internal blocking. A block approach to circuit synthesis is proposed, based on the results of analysis and synthesis of Spanke networks of smaller capacity.



The values of specific loads ρ and the coefficients of gravitation of the loads α affect the values of probability of occurrence of internal blocking, first, with an increase in PS capacity.

The probability of blocking in a PS of $M/2$ small capacity increases by about three times in circuits with M capacity.

With an increase in the coefficient of gravitation of the load, the probability of blocking can increase to up by nine times, regardless of the capacity of PS.

Further research is supposed to consider the results of mathematical modelling for the rest of the classical PS networks.

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