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Modelling of Mechanisms as a Methodological Tool (the Case of Designing a Cycloidal Pin Transmission)

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ABSTRACT

The suggested modern approach to modelling of objects and systems allows not only to create models but also to use them to study the main properties of the object (system) with a high degree of clarity and adequacy, as well as to develop most important skills of young engineers in creating and implementing digital models of engineering objects.

The objective of the study is to analyse capacity of one of the modern automated computational design systems as a methodological tool.

The functionality of an automated computational design system is considered for the case of constructing a model of a planetary cycloidal pinion transmission. The resulting model allows visualising the kinematics of the designed mechanism in the form of static or moving graphic images. The model built based on the described approach contains digital images of mechanism parts, which can be transferred without modification to specialised software systems for analysing strength characteristics or manufacturing material models of a product using rapid prototyping methods.

The proposed approach allows to perfect actions referring to the analysis of properties and synthesis of new structures using tools that correspond to the modern level of technology development and to get a visual idea of the process of developing a machine from a mathematical model to its material objectification.

The research methods are based on the fundamental principles of mathematical and simulation modelling, data analysis and processing using powerful automated computational design tools.

The tools used for modelling can be used for different forms of learning, i.e., without reference to specific premises and equipment.

Keywords: transport, computer-aided design systems, automated computational design systems, planetary gear, cycloidal pin transmission.

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INTRODUCTION

Developing skills in creating complex models of future products can be named among the most important tasks of the modern manufacturing process as well as of training technical specialists who will be involved in it in future. An integral part of these competencies is the ability to use various digital tools, such as systems of automation of mathematical calculations, automated computational design systems, software packages for development of illustrative material and software programs for equipment control.

The *objective* of the study is to assess and demonstrate the possibilities of using digital design tools in development of design of a mechanical engineering product, for example, a gearbox based on a cycloidal pin transmission. This type of mechanical transmission is well known and widely used, demonstrating its effectiveness. However, training, teaching and methodological sources offer relatively small volume of information on this type of transmission.

The structural diagram of the gearbox [1] is shown in Pic. 1. Circular eccentric 1 with axis *B* rotates around a fixed axis *A*. Washer 3, covering the eccentric 1, has teeth *b*, engaging with ring pins 4 rotating around fixed axes *E*. Washer 6 is rigidly connected to shaft 2 rotating around the axis *A,* and has pins 7 rotating around the axes *D* of the washer 6 and rolling along the inner side of the circular holes, centred at point *C* of the washer 3. The dimensions of the links of the mechanism satisfy the conditions *AB* = *DC* and *BC* = *AD*, i.e., the figure *ABCD* is a parallelogram.

When the eccentric 1 rotates around the axis *A*, the washer 3 engages with the pins 4 and thereby drives the shaft 2 into rotation.

Gear ratio of the mechanism:

$$u_{21} = \frac{Z_4 - Z_3}{Z_3},$$

where Z_4 is the number of pins 4;

Z_3 is the number of teeth of the washer 3.

With an internal cycloidal-pinned gearing [2], the profile of the teeth of the smaller wheel (satellite, Pic. 2) is determined as equidistant curve 1, located at a distance $D_{pin}/2$, to the shortened epicycloid 3, which is formed during rolling of a roller with a radius of r_r along a circle 2, the radius of which is equal to R_c (the distance from the centre of the roller to the point that forms the shortened epicycloid is equal to r_{rc}). The complete epicycloid 4 is shown in Pic. 2b. The parameters of the cycloidal profile are interconnected by dependencies:

$$r_r = \frac{R_c}{Z_w},$$

where Z_w is the number of teeth of the wheel:

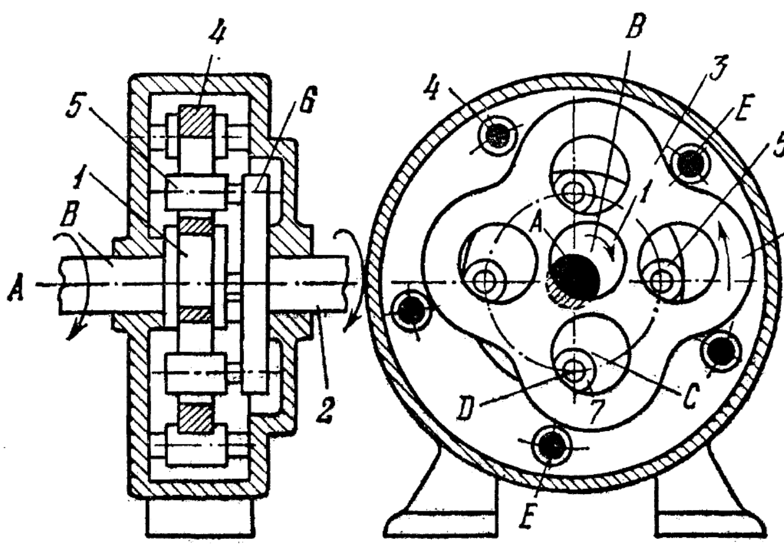
$$R_c = \frac{Z_w}{Z_{pin}} \cdot R_{rc},$$

where Z_{pin} – number of pins located along the circle of the radius R_{rc}.

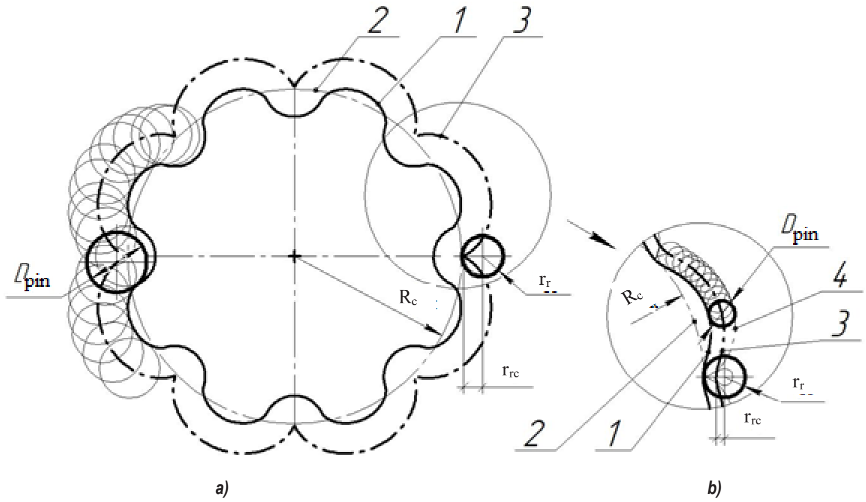
Let's introduce the notation:

$$a = \frac{r_{rc}}{r_r},$$

$$b = \frac{Z_{pin} - Z_w}{Z_w}.$$



Pic. 1. Cycloidal pin planetary gear with internal gearing [1].



Pic. 2. Diagram of formation of the profile of the cycloidal wheel (compiled by the authors).

Then the parametric equations of the shortened epicycloid, which forms the trajectory of displacement of the centre of the pin relative to the wheel, will take the form:

$$x(t) = R_c \cdot \left[(1+b)\cdot \cos bt - a\cdot b\cdot \cos(1+b)t\right];$$
$$y(t) = R_c \cdot \left[(1+b)\cdot \sin bt - a\cdot b\cdot \sin(1+b)t\right]. \quad (1)$$

The wheel profile is formed as an internal equidistant curve to the shortened epicycloid based on the formulas:

$$x_{eq}(t) = x(t) - \frac{\left(D_{pin}/2\right)\cdot y'(t)}{\sqrt{\left(x'(t)\right)^2 + \left(y'(t)\right)^2}};$$
$$y_{eq}(t) = y(t) + \frac{\left(D_{pin}/2\right)\cdot x'(t)}{\sqrt{\left(x'(t)\right)^2 + \left(y'(t)\right)^2}}. \quad (2)$$

derivatives $x'(t)$, $y'(t)$:

$$x'(t) = R_c \cdot \left[(1+b)\cdot b\cdot(-\sin bt) - a\cdot b\cdot(1+b)\cdot\left(-\sin(1+b)t\right)\right];$$
$$y'(t) = R_c \cdot \left[(1+b)\cdot b\cdot \cos bt - a\cdot b\cdot(1+b)\cdot \cos(1+b)t\right].$$

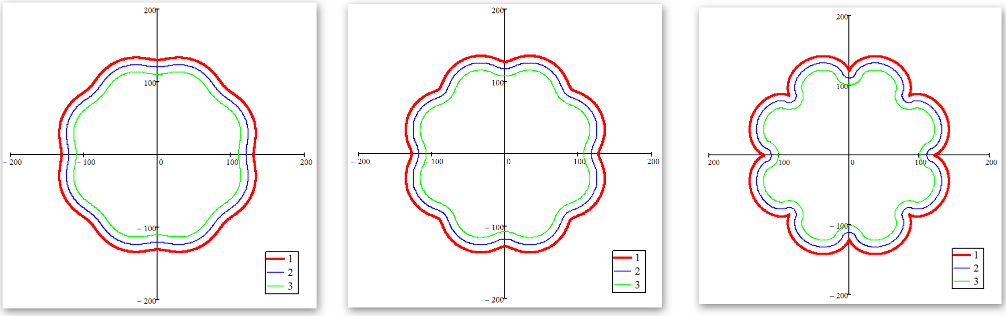
RESULTS

Modelling was carried out based on the above dependencies using the Mathcad system. The first stage provided for construction of graphs allowing preliminary assessment of the shape of the profile of the gear wheel (Pic. 3). The shortened epicycloid (trajectory of displacement of the centre of pins) corresponds to graph 1 in all pictures. A different form of this trajectory corresponds to different values of the parameter *a* in equations (1). Graphs 2 and 3, built according to equations (2), illustrate the position of the wheel profile with different values of the diameter of pins D_{pin}.

The functionality of the mathematical modelling system allows to quickly build a graphical model of a profile and, without long constructions, adjust the parameters and update the profile view. However, the automated system of mathematical calculations is not suitable to develop design documentation either to directly use the results of such a calculation for processing data that will form the basis of technological process.

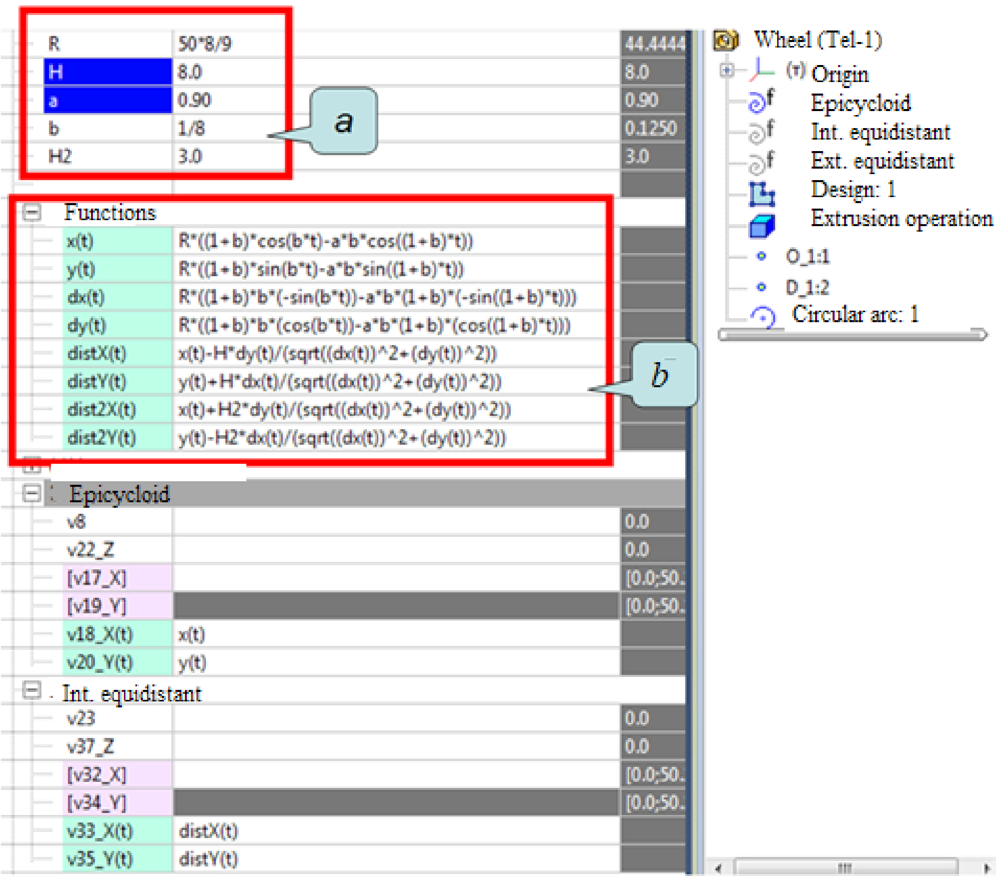
Methodological recommendations on construction of this profile [3–27] in most cases propose in fact to replicate the approach as when designing using conventional drawing tools, only converted to digital format. The curve is constructed by points using the kinematic method to determine the coordinates of nodes and the built-in system commands for constructing flat curves (NURBS, Bézier curve) from an array of points. Equidistant line construction is also performed using the built-in system command. This method is very time consuming; it requires repeated execution of the same type of operations, which can lead to errors and increase the design time. This is especially noticeable when it is necessary to adjust previously developed constructions.

Modern 3D modelling systems are capable to exclude manual steps from the process of developing a 3D model. For this, the so-called parametric capabilities of these systems are used. If, i. e., in the KOMPAS software package (ASCON, Russia) one specifies a group of model variables (Pic. 4a) corresponding to the parameters of formulas (1) and a group of user-



a) b) c)

Pic. 3. Results of modelling of the profile of a gear wheel in the automated system of mathematical calculations (compiled by the authors).

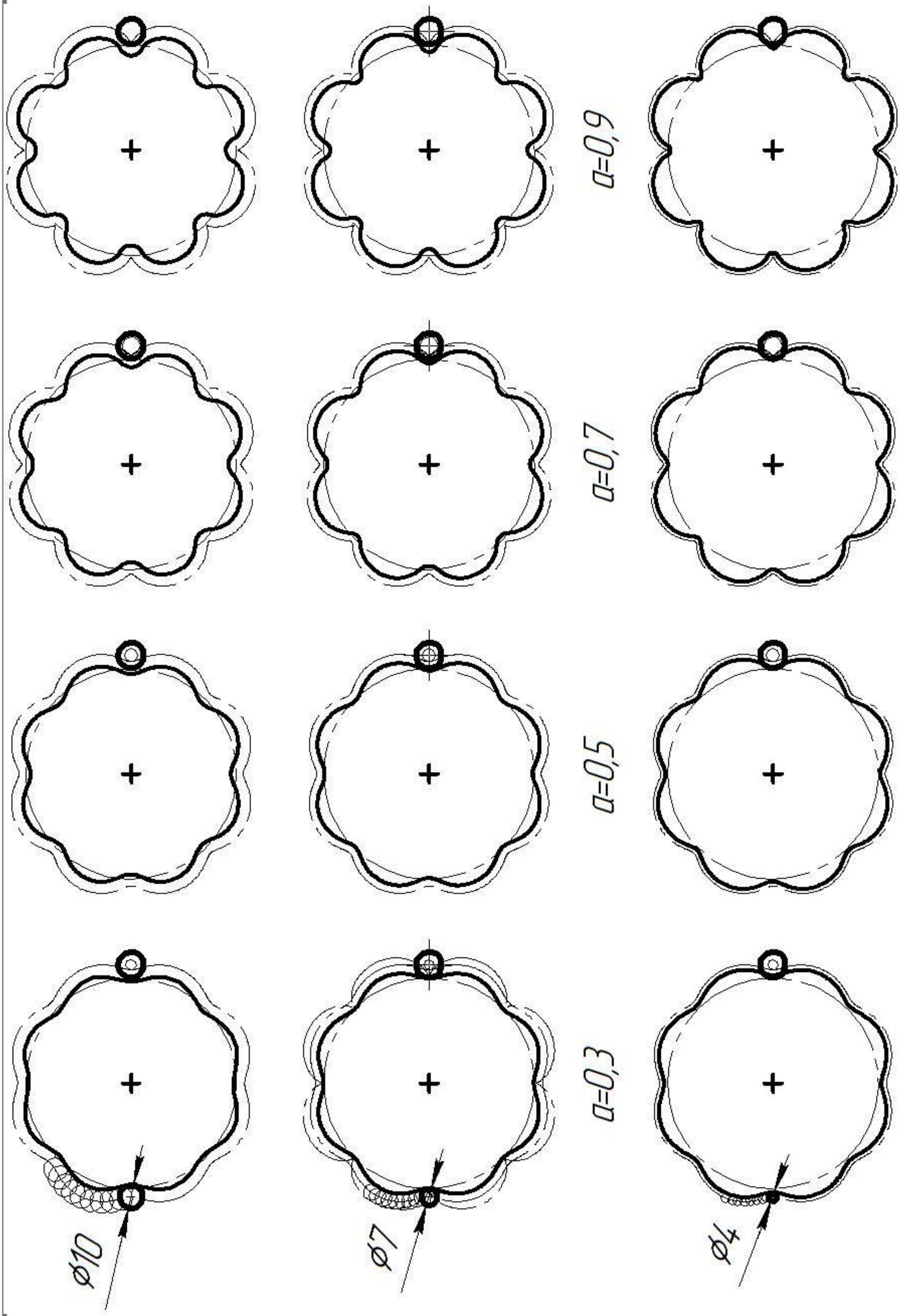


Pic. 4. The system of control variables of a 3D model of the cycloidal gear (compiled by the authors).

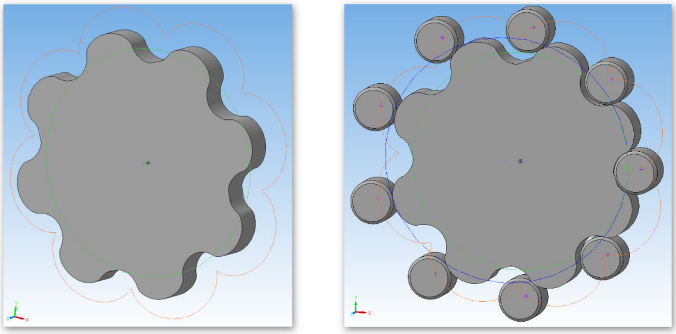
defined functions (Pic. 4b) that reproduce dependencies (2), a curve is formed in the model space that forms a closed loop to which the Extrude operation is applied. The result is a finished digital 3D model.

If it is necessary to adjust the parameters (diameters of pins, shortening factors, the radius of the circle on which the pins are located), you can limit yourself to changing the numerical values in the range of variables (Pic. 4a) and rebuilding of the contour, as well as of the entire 3D model, will be performed automatically.

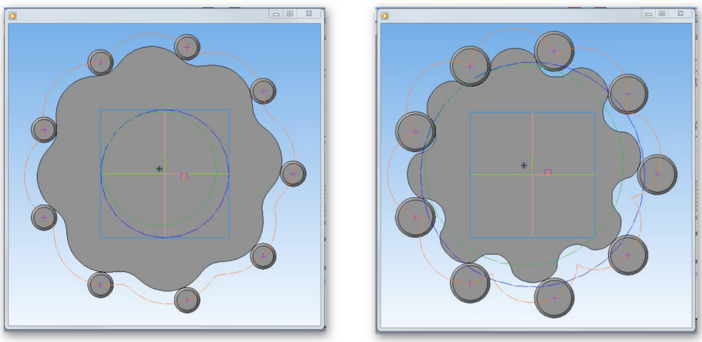
Using this approach can significantly increase flexibility and efficiency of the design process. Sketches of the working profiles of cycloidal wheels (Pic. 5) and a 3D model (Pic. 6) illustrate



Pic. 5. Working profiles of cycloidal wheels, for different values of the parameter 0,3 < a < 0,9 and the diameter of the pin d_c = 4, 7, 10 mm. (compiled by the authors).



a)



b)

Pic. 6. 3D model of the cycloidal pinned engagement (a) and freeze frames of the video file formed on its basis (b) (compiled by the authors).

the ability to quickly rebuild the model when changing transmission parameters, this can also be done in animation mode.

BRIEF CONCLUSION

The described results of modelling demonstrate within a specific case that automated computational design systems are tools allowing us to directly transform analytical (with formulae) descriptions of geometric objects into realistic digital models suitable for their direct use as objects of analysis for a number of physical features (regarding centre of mass, strength characteristics) and consumer properties (exterior, positioning in the interior of premises) properties.

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