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A Type of Transportation Problem to be Solved Following the Time Criterion and Considering Vehicle Features

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ABSTRACT

The formulation of classical minimax transport-type problems involves the search for an optimal transportation plan considering only time of delivery of resources. The inevitable additional costs of processing resources at the origin and destination are usually not considered. This approach is fully justified given incommensurability of delivery times of resources along available routes and times of preliminary/subsequent processing of resources. At the same time, in a number of practical problems, the time spent on loading/unloading (for example, when organising loading of packaged mineral fertilizers from port warehouses onto ships) can be of significant importance. In such cases, when searching for an optimal transportation plan, it is necessary to take into account not only travel times of vehicles used along the set routes, but also the costs of loading and unloading operations, considering the number of available vehicles and their characteristics, for example, payload.

In this regard, the objective of the study is not only to develop a method for calculating the optimal transportation plan, but also a method for distributing vehicles, considering their number and features.

At the same time, another no less important objective of the study is to substantiate the application of the method of successive reduction of residuals, considering the form of the objective function, which considers not only the main parameters of classical minimax transport-type problems, but also the quantitative characteristics of vehicles involved in the transport operation. It is fundamentally important that the use of the method of successive reduction of residuals determines the polynomial computational complexity of the algorithm, which makes it possible to use it in the operational solution of problems of practical dimension.

To solve the problem of distributing available vehicles according to the origin points, considering payload of vehicles, it is proposed to use the method of dynamic programming. An illustrative example of distribution of delivery vehicles, adapted for the use in MS Excel, is considered.

Keywords: *transportation problem, minimum time criterion, processing costs, carrying capacity.*

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INTRODUCTION

Minimax transportation-type problems [1–3] do not consider time for loading and unloading operations. The article proposes to consider time losses not only for delivery of resources, but also for losses during loading/unloading of resources at the origin point and at destination point. In this case, we will assume that time spent on processing resources has a linear dependence on the amount of resources forwarded along each of available routes. It is necessary to consider both the presence of vehicles involved and their characteristics, for example, payload capacity. This formulation of the problem is similar to the classical linear transport problem [4], to the problems with a minimax objective function [5; 6], and to problems with fixed surcharges [7–9], the solution of which during linearisation of the objective function leads to significant errors, and the search for an exact solution by combinatorial methods determines unacceptable (exponential) computational efficiency. The minimax nature of the control quality indicator of the problem under consideration makes it possible to solve the general problem as a set of subproblems on the maximum traffic flow [8; 10; 11]. Moreover, since the central place in the proposed algorithm is offered to the method of successive reduction of residuals with polynomial computational complexity [3], computational complexity of the proposed algorithm will have the same computational complexity [8; 12; 13]. Hence follows the conclusion about applicability of the proposed algorithm in problems with a sufficiently large number of variables. It should be noted that the approach based on a sequential dimension reduction of the problem [14] leads, as a rule, to solutions that are far from optimal ones. And the previously solved problems related to processing of resources at destinations [15; 16] did not consider either the number of available vehicles or the need for their optimal distribution along the operated routes. As another argument in favour of the relevance of the problem considered in the article, it should be pointed out that the study of transportation-type models, including time spent on loading and unloading operations, and, as a special case, time spent at intermediate processing points, still attracts close attention of researchers [17; 18].

The *objective* of the study is to develop a method for calculating the optimal transportation plan and a method for distributing vehicles considering their number and features.

RESULTS

Let there be *m* starting points with resources a_i ($i = 1 \dots m$) and *n* destinations with needs b_j ($j = 1 \dots n$). The condition for admissibility of the transportation plan $\|x_{ij}\|$ ($i = 1 \dots m, j = 1 \dots n$) is compliance with the usual constraints for a linear transportation problem:

$$\left.\begin{array}{l} \sum_{j=1}^{n} x_{ij} \le a_i,\ i = 1 \dots m; \\ \sum_{i=1}^{m} x_{ij} \le b_j,\ j = 1 \dots n; \\ x_{ij} \ge 0,\ i = 1 \dots m; j = 1 \dots n \end{array}\right\}. \qquad (1)$$

where a_i is resource volume in A_i;

b_j – needs of B_j;

x_{ij} – number of resource units on the route $A_i \rightarrow B_j$.

The solution will be optimal when the function $F(x_{ij})$ reaches the minimum:

$$F(x_{ij}) = \max f(x_{ij}),\ i = 1 \dots m, j = 1 \dots n, \qquad (2)$$

where:

$$f(x_{ij}) = \begin{cases} t_{ij} + (t'_i + t''_j)x_{ij}, & \text{if } x_{ij} > 0, \\ 0, & \text{if } x_{ij} = 0. \end{cases} \qquad (3)$$

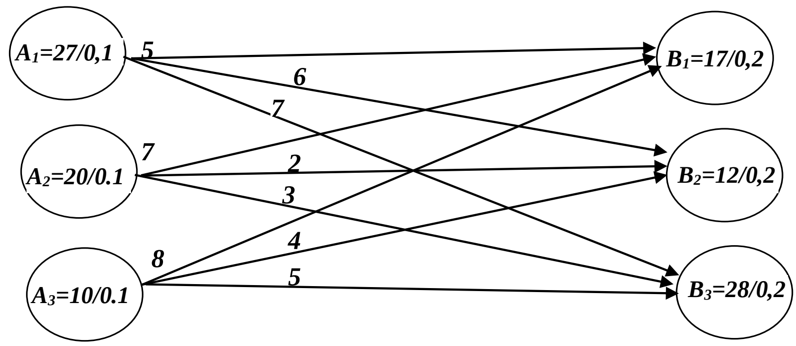
t_{ij} – time of travel from A_i to B_j;

t'_i – losses during loading (processing) of a resource unit in A_i;

t''_j – losses during unloading (processing) of a resource unit in B_j.

As a preliminary step, it is necessary to calculate the lower boundary of the objective function. Then, using the found value of the lower boundary F_n, it is necessary to calculate d_{ij} ($i = 1 \dots m, j = 1 \dots n$) and try to place nonzero x_{ij} only along «allowed» routes. For solving the problem, it is proposed to use the Egerváry (Hungarian) method [7; 8], since in this case there will be no requirement to balance resources and needs. In addition, the Hungarian method is not critical to the appearance of degeneracy, which can subsequently be eliminated at the stage of analysing the obtained optimal solution.

In case of successful placement of non-zero traffic along routes in which $F \le F_n$ (either the remainder of the resources at the starting points is equal to zero, or all the needs are satisfied), the optimal solution is found. If this is not the case, then it is necessary to slightly (minimally) increase the throughput of routes by increasing (however small) the value of the lower boundary. After completing this procedure, recalculation of the throughput and repetition of the procedure for increasing the flow in the network



Pic. 1. Network of an illustrative example [3].

Table 1

Routes' transit (throughput) capacity at F_n = 10,1 [3]

	b_1	b_2	b_3
a_1	5 17	6 12	7 9
a_2	7 10	2 12	3 10
a_3	8 7	4 10	5 10

Table 2

Original outline of the example [3]

	b_1	b_2	b_3
a_1	5 17	6 10	7
a_2	7	2 2	3 18
a_3	8	4	5 10

follow. In case of unsuccessful placement of non-zero traffic on available routes, it is necessary to re-increase the transit capacity of routes and recalculate the capacities, and so on. To illustrate the algorithm for calculating $F_{n,}$ we will use an illustrative example (Pic. 1 and Table 2). Above the edges of the graph, the values of times of delivery of resources between points are put. The data in the upper right corners of the cells of the Table 1 have the same meaning. The number of available resources in points A_i and the value of the needs of points B_j are indicated by numbers in the centres of the circles. Resource unit processing times are specified as denominators.

Based on the above constraints (2, 3), we proceed with the calculation of F_n:

$$F_n = max_i\{min_j\left[t_{ij} + min_j\left(a_i, b_j\right)\left(t^{'}_i + t"_j\right)\right]\}. \qquad (4)$$

In accordance with (4) $F_n = t_{11} + b_1(t'_1 + t''_1) =$ 10,1.

Based on the obtained value, we will calculate the throughput capacity and put them in the lower part of the cells in the Table 1.

To reduce the obviously unpromising steps of the algorithm, it makes sense to refine F_n. The reduction in the number of steps can be achieved by calculating the transit capacity according to the rule:

$$d_{ij} = \begin{cases} min_{i,j}\left[a_i, b_j, (F_n - t_{ij}) div\left(t^{'}_i + t"_j\right)\right], & if\ t_{ij} \le F_n; \\ 0, & if\ t_{ij} \le F_n, \end{cases} \qquad (5)$$

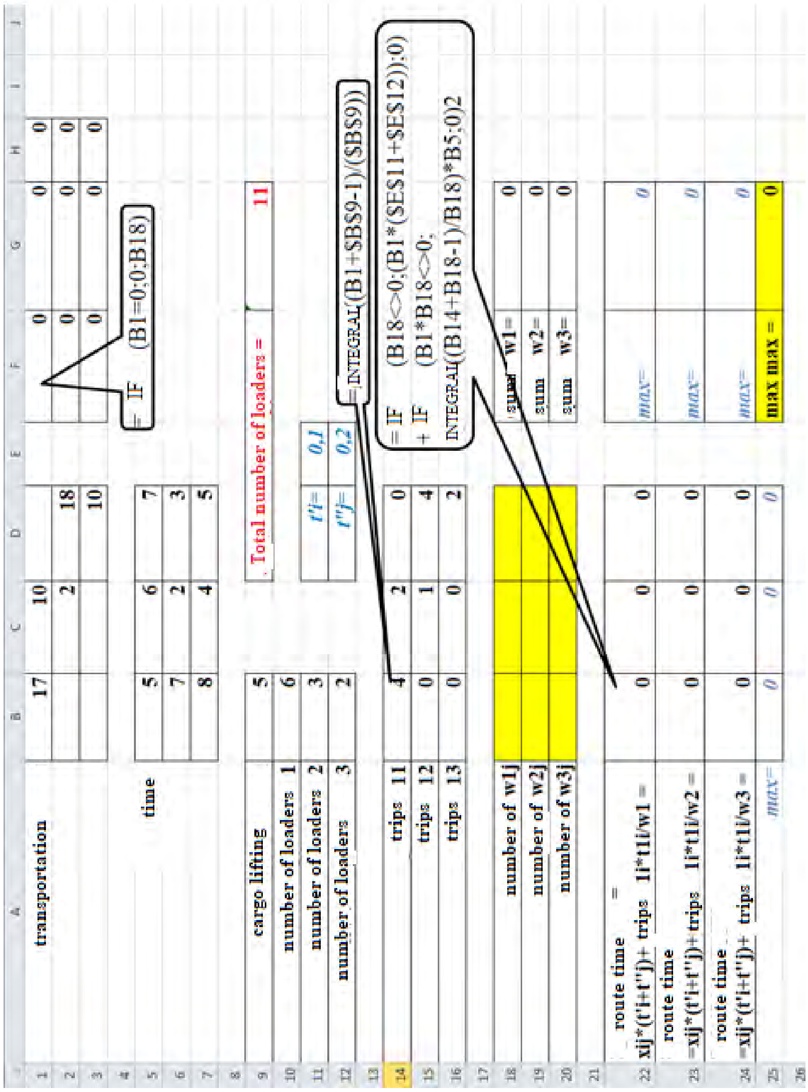
where *div* – division with discarding remainder.

The possibility of obtaining a feasible solution (satisfying all constraints) is ensured by validity of the conditions:

$$\left.\begin{array}{l} \sum_{j=1}^{n} d_{ij} \ge a_i,\ i = 1 \ldots m; \\ \sum_{i=1}^{m} d_{ij} \ge b_j,\ j = 1 \ldots n. \end{array}\right\}. \qquad (6)$$

If any of the inequalities (6) does not hold, then the conclusion follows either about the impossibility of exporting resources from some origin points, or about the impossibility of delivering the required amount of resources to one or several destinations. In such a situation, the procedure for increasing the lower boundary of the function follows. Based on the updated value of F_n. the route throughput capacity is calculated again in accordance with (5) and the validity of conditions (6) is again checked.

If the identified d_{ij} make it possible to export resources from all origin points and to deliver them to all destination points, then the transition to the next stage of the solution is carried out, that is to drawing up an initial plan, in which the values of transportation x_{ij} are determined by the formula:



Pic. 2. The optimal plan without considering the number and characteristics of available vehicles (compiled by the author).

$$x_{ij} = min_{ij}(a'_i, b'_j, d_{ij}), \qquad (7)$$

where a'_i, b'_j – resources and needs of the respective origin points and destinations, considering the already assigned transportation.

The transportation plan obtained in accordance with (7) is presented in Table 2, where the upper part of the cells contains times of movement along the corresponding routes, and below the actual values of x_{ij} are shown.

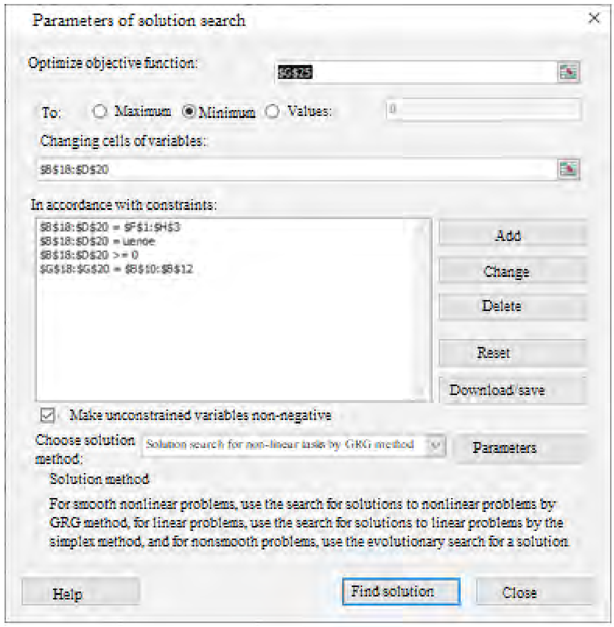
The resulting plan is a solution to the problem. If this statement is false, it is necessary to proceed to the procedure for increasing the traffic flow, increasing F_n if necessary, etc. [3; 7; 8].

Transportation plan in Table 2 ($F(x_{ij})$ = 10,1) is optimal under the assumption that the resources assigned for transportation along any of the considered routes are transported in one trip, that is, that both the number of vehicles at each starting point and the payload of these vehicles are not lower than existing needs. In this regard, the problem of implementing the previously developed optimal plan, considering the number of vehicles available at the origin points and their payloads, are of certain interest.

In this case, the number of trips on each route can be calculated using the formula:

$$l_{ij} = (x_{ij} + q_i - 1) div\ q_i, \qquad (8)$$

where q_i – payload (carrying capacity) of the i-th vehicle.



Pic. 3. Completed dialog box (compiled by the author).

In this case, time of travel through each route is calculated by the formula:

$$F'(w_i)=\begin{cases}(l_{ij}t_{ij}+w_i-1)/w_i+x_{ij}\left(t'_i+t''_j\right), & if\ l_{ij}\neq 0;\\ 0, & if\ l_{ij}=0,\end{cases} \qquad (9)$$

with a constraint $\sum_{i=1}^{m} w_i \le W$,

where w_i – number of vehicles in the *i*-th starting point;

W – total number of vehicles.

The task is to determine the values of the variables w_i, with which

$F(w_i) = max_i F'(w_i) \rightarrow min(i = 1 \ldots m)$. (10)

In view of the nature of the objective function (10), it is quite obvious that the solution can be found by combinatorial methods, for example, by the method of dynamic programming. At the same time, it should be noted that in problems of practical dimension, the procedures embedded in MS Excel are quite applicable.

At present, when solving optimisation problems, either programs in high-level programming languages, or common mathematical packages (for example, MathCad), or general-purpose programs (for example, spreadsheets) can be used.

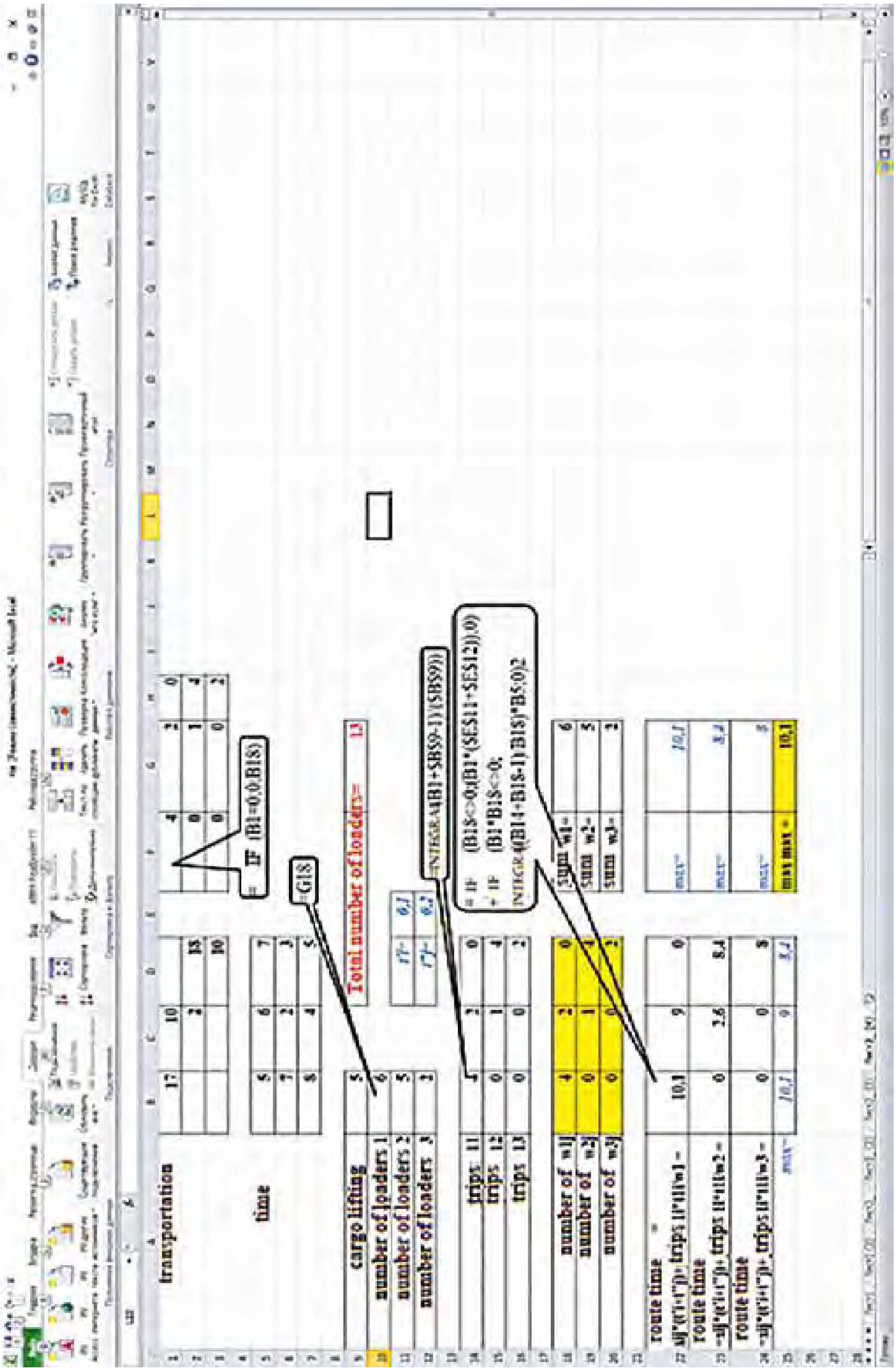
The main disadvantage of programs of the first type is that the developers of such programs are persons focused primarily on development of effective algorithms for the formulated problems [3]. Such programs are highly computationally efficient. When using them, the mathematical formulation of the problem is not required, since it has already been completed during development of the algorithm. The user only needs to enter the initial data. At the same time, the user's interface of such programs is not well-developed and is clear only to a narrow circle of specialists.

Common mathematical packages will require the task to be formalised, which requires a certain level of mathematical knowledge from the part of the user. In addition, the solution of optimisation problems in such packages can lead to the solution of a number of constituent subproblems [2; 7; 8].

General-purpose programs (such as spreadsheets) have a well-thought-out interface. A wide range of actions of the algorithms embedded in them leads to a serious deterioration in computational efficiency [3].

In this regard, the use of MS Excel is considered below to solve the formulated problem. A wider range of optimisation problems solved in the MS Excel environment (the main linear programming problem, the knapsack problem, the resource allocation problem, etc.) is considered in [2].

Pic. 2 shows the found optimal plan for a an illustrative example without considering the number and characteristics of available vehicles. To implement this plan, considering available



Pic. 4. Result of execution of the command «Search for a Solution» (compiled by the author).

vehicles and their characteristics, for example, payload, it is proposed to use the «solution search» command in MS Excel for small-scale problems.

Pic. 3 shows the completed dialog box of the «solution search» command, and Pic. 4 – the result of its execution.

CONCLUSIONS

Computational complexity of the proposed algorithm for finding the optimal transportation plan is not higher than polynomial, which makes it possible to use it in solving problems of practical dimension.

The procedure for optimal distribution of vehicles available to the decision-maker is formalised for solving small-scale problems using MS Excel.

Developed procedure for optimal allocation of vehicles can be programmatically implemented with one of combinatorial methods, for example, with the dynamic programming method.

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