



Optimisation of Inventory Levels as Logistics Challenge



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ABSTRACT

Inventory ensures stability of the production technical system. Production facilities have warehouses for workpieces to create final output parts, and warehouses for finished products to meet customer demand. Stocks of workpieces avoid manufacturing downtime. Stocks of finished products make it possible to meet demand in a timely manner and avoid penalties for late deliveries of finished products to consumers.

On the other hand, the objects that make up the inventory are «dead» material resources that do not bring profit and negatively affect economic efficiency and competitiveness. According to published data, in economically developed countries, such as the USA, Japan, Germany and others, up to 30–40 % of

production is in stocks. All this indicates the relevance of the issue of optimising the level of reserves, of solving the problem of establishing and maintaining the optimal level of the stock, its timely replenishment, considering time required for execution of applications of end users.

The objective of the work is to propose a solution to a stochastic logistic problem of inventory management with a random time delay and random demand, obtained by the authors using the methods of mathematical statistics, mathematical modelling, and production logistics for unknown and arbitrary form of distribution functions of demand and time delay. The work provides specific examples of approaches to implementation of suggested solution.

Keywords: logistics, spare parts, inventory management systems (IMS), management strategy, stochastic demand, stochastic time delay, technical and economic functions, targeted functionality, algorithms, dynamics of spare parts inventory levels.

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INTRODUCTION

There is a need to improve the efficiency of management systems governing spare parts inventory for transport and special vehicles by building models and optimising them. The problem of inventory management has arisen for a long time. In transport mechanical engineering, inventories play the role of a shock absorber in conditions of a mismatch between the rates of production and the rate of receipt of raw materials. The presence of reserves at an industrial enterprise is an objective reality and an economic necessity. In this case, material resources concentrated in stocks are diverted from the production sphere. The largest share in the working capital of industrial enterprises is made up of funds invested in inventories. The structure of stocks consists mainly of raw materials, basic materials, components and purchased semi-finished products.

Scientific theories of inventory management emerged in the late 19th century. The first mathematical models of inventory management systems (IMS) were deterministic in nature. The economic and mathematical functions used and the criteria for effectiveness of inventory management systems built on their basis were linear and often unimodal. This, in turn, dictated the choice of the optimisation method for the proposed IMS models. Attempts have been made to solve the problem analytically. Later, methods of linear and dynamic programming were used. Improvement of industrial production technologies, complication and development of relations and cooperation of enterprises, including the structures of the material and technical supply, has resulted in the understanding of the need to consider the influence of many factors on operation of IMS. Deterministic models had to be abandoned as inadequate to real conditions. Time has come for stochastic IMS models. The development of industrial logistics as a system for managing flows of material resources and finances led to sophistication of IMS, required development of qualitatively new mathematical models of IMS, an in-depth study of their mathematical properties. For example, in the USA, the share of reserves in gross domestic product for almost 35 years is about 28 %. By abandoning the traditional approaches to managing supply, manufacturing, and sales processes since each of these processes was not managed comprehensively, but separately and independently of each other, American firms have

achieved a significant reduction in the costs of maintaining and servicing inventories and their share in the annual volume of costs. The transition to logistics approaches of IMS in American firms has reduced the share of working capital invested in inventories to 18 % in the US gross domestic product [1; 2].

The overwhelming majority of recent publications seem to give preference to logistics and stochastic models as to the most adequate description of real conditions [3; 4].

Hence, the *objective* of the study is to propose a solution to a stochastic logistic problem of inventory management with a random time delay and random demand, obtained using the *methods* of mathematical statistics, mathematical modelling, and production logistics for unknown values and arbitrary form of distribution functions of demand and time delay.

RESULTS

Let us consider the mathematical model of IMS corresponding to the most common conditions of functioning of an industrial enterprise. Demand is usually random, and applications are processed with a random delay. The distribution functions of demand and time delay are a priori arbitrary and unknown, and often not unimodal. In this setting, the problem has not been solved. Under these conditions, to achieve the goal, it is natural to use modern methods of production logistics, including methods of mathematical statistics, mathematical modelling, information technology and, in particular, identification algorithms.

Let the inventory management system be characterised by the cost function Z for stock storage, delivery, shortage, etc. Inventory management is carried out by choosing a management strategy. One of the sufficiently effective and flexible strategies is (s, S) -strategy according to which an order of size $S - y$ is formed by reducing the level of stock y below the level s , $s < S$. For random demand and delay $Z = Z(y(s, S, x))$, where, according to Yu. I. Ryzhikov [5], $y(s, S, x)$ is current stock level, x is random demand, s, S are levels in (s, S) -strategy. Then the criterion of optimality of the system J is some averaged function of the costs Z :

$$J(s, S) = Mx \{F[y(s, S, x)]\} \rightarrow \min s, S. \quad (1)$$

Applying to (1) the method of stochastic approximation, which is a method of information technology, we obtain a self-consistent system of adaptive algorithms [6–8]:





$$\begin{aligned} S[p+1] &= S[p] - \gamma_{\alpha}(S[p+1]) \cdot f_s \cdot \\ &\cdot F(Z(y(s[p], S[p], x[p+1]))); \\ s[p+1] &= s[p] - \gamma_{\alpha}(s[p+1]) \cdot f_s \cdot \\ &\cdot F(Z(y(s[p], S[p], x[p+1]))), \end{aligned} \quad (2)$$

where gradient in α is a derivative in the direction;

$\alpha \cdot \gamma_{\alpha} \cdot [p+1]$ are coefficients;

f_s, f_s are functions that consider constraints for s and S .

It is assumed that the functions F and Z are differentiable, otherwise search algorithms can be applied. Sufficient conditions for convergence (2) are given in numerous monographs on methods of stochastic approximation. In the process of calculating in (2), it is necessary to know the explicit form of the function $y(s, S, x)$ to find the derivatives with respect to s and S . Let us write an equation that simulates the dynamics of inventory and uses the (s, S) -strategy for the control. It is known that within the framework of (s, S) -strategy for $y \geq s$, the system does not require replenishment of stocks, and for $y < s$, an order for replenishment of stock is sent in the amount of $(S - y)$. Unsatisfied applications do not leave the system but wait until the stock shortage is eliminated by the next order. All research is done for discrete demand.

Let's renumber applications as they enter the system. We can assume that their number is infinite. The obvious relationship holds:

$$\begin{aligned} y_k(x[i]) &= y_{k-1}(x[\eta_k - 1]) + \\ &+ (S - y_k(x[i]) \operatorname{sgn}(s - y_k(x[i - 1]))) - x[i], \end{aligned} \quad (3)$$

where $i = 1, 2, 3, \dots$;

$y_0(x[\eta_0])$ is initial stock level;

k is period number by order;

η_k is number of the last application in the k -th period, determined from the condition:

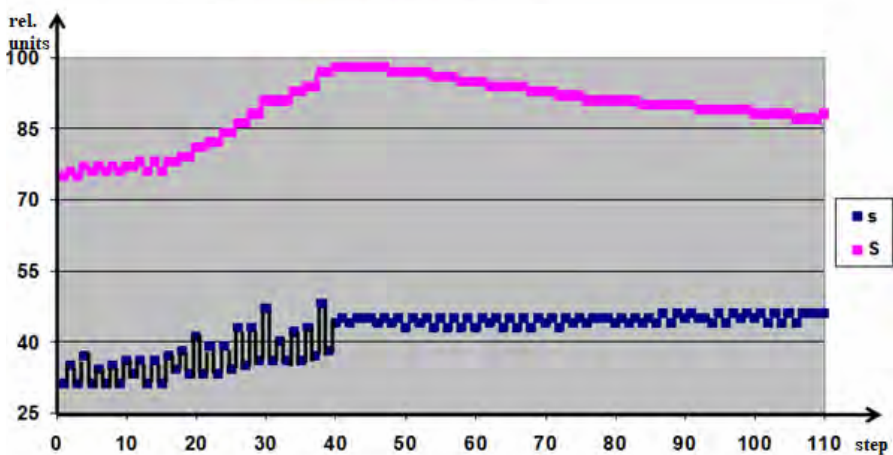
$$\operatorname{sgn}(s - y_k(x[i - 1]) - x[i]) = 1 \rightarrow i = \eta_k. \quad (4)$$

It should be noted that in (1)–(4) time does not occur explicitly. The time dependence is essential for objects, the cost functions of which change over the considered period. As a rule, the type of the cost function for mechanical engineering objects does not change during the optimisation period.

Nevertheless, the problem of optimisation of stocks of spare tools and accessories (spare parts) for road, railway transport and for special vehicles is of considerable practical interest.

Special equipment is a large class of special vehicles in construction, industry, cargo transportation, and military field. Road special vehicles is subdivided according to its functional purpose into civil construction, roadbuilding, agricultural, fire-fighting vehicles, railway special vehicles, etc., which require optimal number of spare parts for good operation.

Time factor in IMS often plays an auxiliary role. In fact, the level of stock in the warehouse is important at the moment of receipt of the next order, while the time itself of receipt of orders is not so important. A deficit is possible only for $s < 0$. If there is a delay in the system, then a deficit may appear for $s > 0$ as well. In this case, when obtaining the initial information and with an equation similar to (3), it is possible to apply the algorithms without using time in an explicit form, despite the fact that delays have the



Pic. 1. Graphs of changes in stock levels of spare part (plate 1) for the automotive industry enterprises (compiled by the authors).

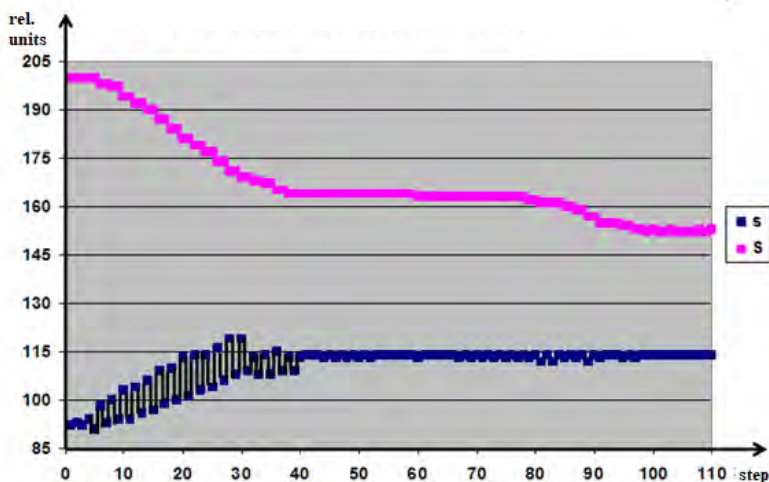
meaning of a time interval. When modelling the dynamics of the stock level, we will consider a random delay through a random number of applications arriving during this time interval. With this approach, it is possible to abandon timing of the change in the level y , which significantly reduces complexity of obtaining the initial information. All that is needed is data on the size and sequence order of individual applications, not on their temporal distribution. Information about the size and sequence order of applications in the examples under consideration was established by accounting documents.

Since the methods using the first derivative (gradient) are a necessary condition for an extremum, we will compare the values of the criterion in the process of calculating $s[p]$, $S[p]$

for different p . For functional (1), we use an algorithm of the type (2):

$$\begin{aligned} J_e[p] &= J_e - l[p] - f_c[p] \{F(Z(y(s[p], \\ S[p], x[c]))) - J_e - l[p]\}, \\ \text{where } J_e[p] &= J_e(s[p], S[p]). \end{aligned} \quad (5)$$

As a possible conditional example, a mathematical model of the problem of optimising the level of stocks of plates 1 and 2 (Pics. 1, 2) is considered for the automotive industry with random demand and random time delay. The authors checked efficiency of the computation program developed by them according to the system of algorithms (1)–(7) on the given model of stocks movement. To avoid discussion that is not related to the content of the work, the authors provide data in relative units, while the mode of operation



Pic. 2. Graphs of changes in stock levels of spare parts (plate 2) for the automotive industry enterprises (compiled by the authors).

of a real warehouse was simulated. Initial input information about movement of material objects can be obtained from the reporting data directly at the warehouse. Technical and economic functions can be obtained from financial reports with subsequent approximation of the initial data using orthogonal Chebyshev polynomials according to well-known techniques [9–11].

The results of calculations carried out by the authors using relations (1)–(5) for conditional examples are shown in Figs. 1, 2. Along the horizontal axis, the numbers of demand applications are plotted in the sequent order of reception by the control system, along the vertical axis, the changes in the levels s and S are plotted as compared with the initial values established by experts. The converged values of s and S are taken as optimal, as corresponding to the minimum of the objective functional.

CONCLUSIONS

The paper presents a mathematical model of a stochastic IMS developed by the authors based on production logistics methods and proposes an implicit method for accounting for delay in ordered delivery aimed at replenishment of stock in a warehouse through accounting for orders received during the time of delay of applications (demand), which was the objective of this publication. Recurrence relations (2) were obtained by methods of identification theory [6]. Thus, the use of modern methods of production logistics made it possible to construct a scheme for optimising the level of material resources in the warehouse, which can increase the efficiency and competitiveness of the activities of specific enterprises.

It should be noted that digital technologies, which are also classified as IT technologies, are currently widely used in management of industrial production and technological processes. In particular, research on application of digital technology is currently being carried out in the Russian The Central research and development

automobile and engine institute NAMI, in many other Russian transport organisations and associations.

The model proposed by the authors can be useful for adoption and implementation in inventory management.

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