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Mathematical Modelling of Tank Wagon Vibrations Considering Partially Filling of the Tank with Liquid Cargo

Pavel S. GRIGORIEV

Sergey N. KORZHIN

Sherzod R. IBODULLOEV

Phu Thuan TRAN

Pavel S. Grigoriev[1], Sergey N. Korzhin[2], Sherzod R. Ibodulloev[3], Phu Thuan Tran[4]

[1, 2] Russian University of Transport, Moscow, Russia.
[3] National University of Uzbekistan named after Mirzo Ulugbek, Tashkent, Uzbekistan.
[4] Society of Vietnamese Citizens in the Russian Federation, Ho Chi Minh City, Vietnam.
✉ *[1] grigorev.p@gmail.com.*

ABSTRACT

Mathematical modelling of processes of motion makes it possible to assess the dynamic characteristics of a wagon at the stage of its design. However, it is necessary to consider the type of cargo transported, the movement of which affects the values of these features.

The paper considers a mathematical model of an eight-axle railway tank wagon developed using the Lagrange's equation of the second kind. The considered mathematical model suggests an approach based on the consideration of the influence of the energy of a liquid cargo in a steady state of motion. This influence was considered by evaluating the kinetic and potential energies of vibrations of the transported liquid cargo.

Differential equations of vibration compiled for the model under consideration represent the liquid cargo as a solid. The approach for considering the effect of liquid cargo during vibrations of a tank wagon assumes that the total volume of the displaced liquid approximately corresponds to the volume of the layer of the fluid determined by displacement of bouncing, or in the case of galloping, with an angular displacement of one end section of the tank wagon, the second section rises by the same value, in other words, we observe the system of communicating vessels. Based on these assumptions, energy additions are obtained that consider movement of a liquid cargo under steady-state modes of motion.

According to the proposed approach, preliminary calculations were performed, and the results obtained were assessed. The results obtained showed satisfactory convergence with the calculations carried out using other approaches to modelling of the processes of movement of railway tank wagons.

Keywords: *transport, tank wagon, dynamic properties, liquid cargo vibrations, eight-axle tank wagon, loading, barrel.*

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INTRODUCTION

While modelling the processes of rolling stock movement along a railway track, researchers are interested in indicators of dynamic properties (dynamic coefficients, dynamic forces of various directions, acceleration, movement of structural elements) [1; 2], the values of which are ultimately determined by the values of kinetic and potential energies and by their ratios during operation of the system.

Given this circumstance, we can say that if studying the processes of movement of a wagon with a liquid cargo with a mathematical model, it becomes possible to reflect with relative success the energy indicators of moving masses of liquid, then as a result, it will be possible to create an optimised simplified model reflecting the effect of a liquid cargo on the indicators of smoothness at steady state of motion [3]. The solution to this problem would allow to obtain assessment of the ride performance and loading of the cargo tank for any transport units associated with transportation of liquid cargo. The *objective* of the work is to solve this problem by creating an optimised simplified model reflecting the effect of a liquid cargo on the indicators of smoothness at steady state of motion regarding railway tank wagons.

MATERIALS AND METHODS

It should be noted that, as a rule, when developing mathematical models of oscillations of tank wagons, the transported cargo is considered as a solid body [4; 5]. However, as noted above, the transported cargo affects the indicators of the dynamic properties of the considered models of tank wagons. Thus, it is necessary to use computational approaches that consider the energy contribution of the liquid cargo to the overall system.

The works [6–9] estimate the influence of a liquid cargo during transient modes of operation, where the description of flow of fluid is carried out by equations of continuum mechanics. Consideration of the influence of liquid cargo in modern software systems is carried out by solving the problems of hydrodynamics [10; 11] or using pendulum analogy [12–14].

In the proposed work, the technique is based on the assumptions that the tank shell is absolutely rigid, and the structure is symmetrical with respect to the longitudinal and transverse planes. The tank has a circular cylindrical shape. The origin of coordinates coincides with the centre of the middle cross section. The longitudinal *X* axis is directed along the cylinder axis, the *Y* transverse axis coincides with the horizontal diameter of the middle cross section, and the vertical *Z* is directed along the vertical diameter of the middle cross section.

The generated models are fundamentally based on the law of conservation of energy, i.e., transformation of kinetic energy, associated with the speed of the wave, into potential energy, specified by the height of the wave.

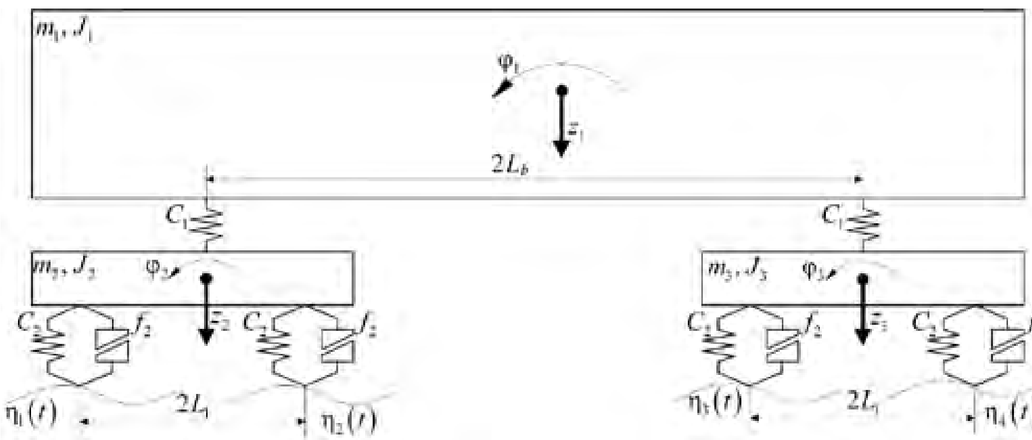
The development of mathematical models is carried out using the Lagrange's equation of the second kind [15], which for the problem under consideration will get the form:

$$\frac{d}{dt}\frac{\partial T_\Sigma}{\partial \dot{q}_i}+\frac{\partial P_\Sigma}{\partial q_i}=0, \qquad (1)$$

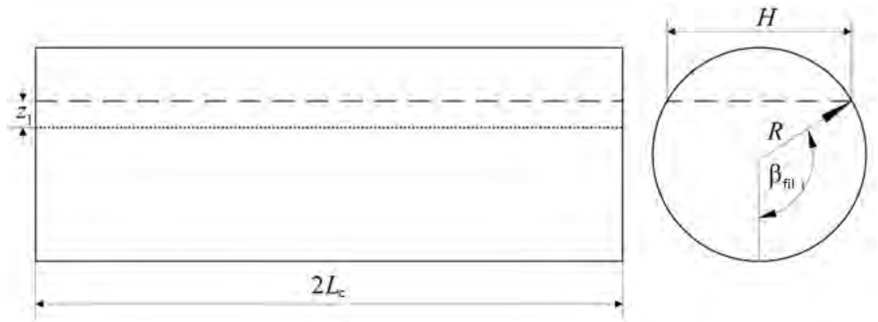
where q_i is generalised coordinate,

T_Σ – kinetic energy of the system,

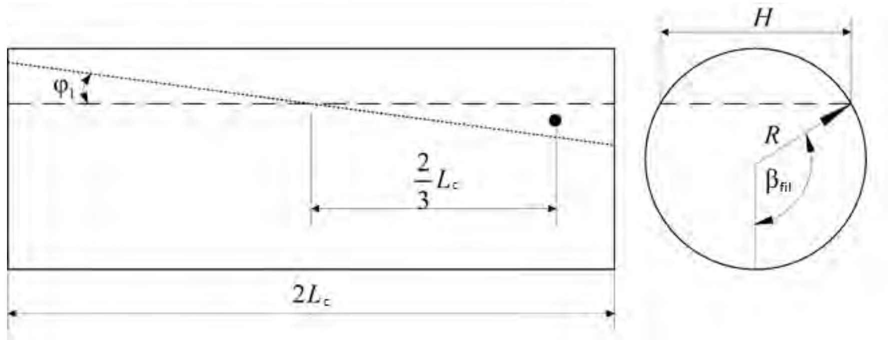
P_Σ – potential energy of the system.



Pic. 1. Design scheme for determining oscillations of a tank wagon (developed by the authors).



Pic. 2. Design scheme for determining the energy additions due to vibrations of the liquid load during oscillation of bouncing (developed by the authors).



Pic. 3. Design scheme for determining the energy additions due to vibrations of the liquid cargo during galloping oscillations (developed by the authors).

RESULTS

System of Differential Equations

As a design model, we will consider a conditional model of an eight-axle railway tank wagon.

In this work, we use the following designations:

m_1 – gross mass of the tank;

J_1 – moment of inertia of the tank;

$m_{2,3}$ – mass of the connection beam;

$J_{2,3}$ – moment of inertia of the connection beam;

C_1 – vertical stiffness of the connection beam;

C_2 – vertical stiffness of the spring suspension set;

f_2 – coefficient of dry friction of the friction vibration damper;

$2L_b$ – tank base;

$2L_c$ – length of the cylindrical part of the tank;

$2L_1$ – bogie base;

$\eta_{1,2,3,4(t)}$ – function of unevenness of the track under the corresponding bogie;

z_1 – vertical displacement of the centre of mass of the tank;

φ_1 – angle of rotation of the tank relative to the transverse axis Y;

$z_{2,3}$ – vertical displacement of the centre of mass of the connection beam;

φ_2 – angle of rotation of the connection beam relative to the transverse axis Y;

ρ_l – density of the mass of the transported liquid cargo;

g – acceleration of gravity;

β_{fil} – angle of filling with the liquid cargo;

R – radius of the tank;

H – linear area of the free surface of the liquid in the transverse direction, defined as $H = 2\sin\beta_{fil}$.

For the scheme shown in Pic. 1, the system of differential equations will look like this:

$$\begin{cases} m_1\ddot{z}_1 + C_1\left(2z_1 - z_2 - z_3\right) = 0 \\ J_1\ddot{\phi}_1 + C_1 L_b\left(2\phi_1 L_b - z_2 + z_3\right) = 0 \\ m_2\ddot{z}_2 + C_1\left(-z_1 - \phi_1 L_b + z_2\right) + \\ +C_2\left(1 + f_{tr}\operatorname{sign}\left(\Delta_3, \dot{\Delta}_3\right)\right)\left(2z_2 - \eta_1(t) - \eta_2(t)\right) = 0 \\ J_2\ddot{\phi}_2 + C_2\left(1 + f_{tr}\operatorname{sign}(\Delta_4, \dot{\Delta}_4)\right)\left(2\phi_2 L_1 - \eta_1(t) - \eta_2(t)\right) = 0 \\ m_3\ddot{z}_3 + C_1\left(-z_1 + \phi_1 L_b + z_3\right) + \\ +C_2\left(1 + f_{tr}\operatorname{sign}(\Delta_5, \dot{\Delta}_5)\right)\left(2z_3 - \eta_3(t) - \eta_4(t)\right) = 0 \\ J_2\ddot{\phi}_3 + C_2\left(1 + f_{tr}\operatorname{sign}(\Delta_6, \dot{\Delta}_6)\right)\left(2\phi_3 L_1 - \eta_3(t) - \eta_4(t)\right) = 0, \end{cases} \quad (2)$$

where $\Delta_{3,5} = 2z_{2,3} - \eta_{1,3}(t) - \eta_{2,3}(t)$,

$\dot{\Delta}_{3,5} = 2\dot{z}_{2,3} - \dot{\eta}_{1,3}(t) - \dot{\eta}_{2,3}(t)$,

$\Delta_{4,6} = 2\phi_{2,3}L_1 - \eta_{1,3}(t) - \eta_{2,4}(t)$,

$\dot{\Delta}_{4,6} = 2\dot{\phi}_{2,3}L_1 - \dot{\eta}_{1,3}(t) - \dot{\eta}_{2,4}(t)$,

$\mathrm{sign}(x) = \begin{cases} 1 \text{ when } x \geq 0 \\ -1 \text{ when } x < 0 \end{cases}$ – discontinuous function denoting sign change.

In the given system of differential equations (2), the transported liquid cargo is perceived as a solid. Naturally, the values of the kinetic and potential energies of the wave motion of the fluid surface depend on the height of the waves arising on the free surface of the liquid cargo. When simulating oscillations of the free surface of a liquid in problems, it is possible to introduce the assumption that the total volume of the moving liquid approximately corresponds to the volume of the layer of fluid determined by the bouncing displacement, i.e., by the value z_1 (Pic. 2). This assumption can be substantiated by the fact that the volume of the wave lifted above the layer in state of rest is equal to the volume of the wave under this layer. Thus, the presented model additionally introduces the contributions of the kinetic T_1 and potential P_1 energies of the liquid cargo moving due to the vibrations of the tank.

Let's determine the volume and mass of the liquid cargo captured because of bouncing oscillations. We will assume that this level corresponds to the level of displacement of the tank z_1. Then the volume of this mass will be equal to:

$$V_1 = 2R\sin\beta_{\text{fil}} 2L_c z_1.$$

Then the kinetic T_{lvert} and potential P_{lvert} energies will be determined as:

$$T_{\text{lvert}} = \frac{\rho_1 2R\sin\beta_{\text{fil}} 2L_c z_1}{2}\dot{z}_1^2; \qquad P_{\text{lvert}} = \rho_1 2R\sin\beta_{\text{fil}} 2L_c z_1 g z_1. \tag{3}$$

Substituting expressions (3) into equation (1), we obtain expressions for substitution into the initial system of differential equations (2):

$$\frac{d}{dt}\frac{\partial^2 T_{\text{lvert}}}{\partial \dot{z}_1^2} = 4\rho_1 R\sin\beta_{\text{fil}} L_c\left(\dot{z}_1^2 + z_1\ddot{z}_1\right); \qquad \frac{\partial P_{\text{lvert}}}{\partial z_1} = 8\rho_1 R\sin\beta_{\text{fil}} g L_c z_1. \tag{4}$$

Let's consider angular oscillations. With angular vibrations, we observe the model of communicating vessels, i.e., with an angular displacement of one end section of the tank (for example, downward), the second section rises by the same value. And at the same time, there is obviously a displacement of the volume of the liquid. With angular vibrations, the volume of the liquid situated to the left of the middle section should be equal to the volume of the liquid situated to the right.

Let's find the volume of liquid V_{lang} and speed of movement of the centre of mass v_{ct}, changing due to angular oscillations of the tank

$$V_{\text{lang}} = L_c^2 R\sin\beta_{\text{fil}}\phi_1; \; v_{\text{ct}} = \frac{2}{3}L_{\text{ct}}\dot{\phi}_1. \tag{5}$$

Taking this into account, the kinetic T_{lang} and potential P_{lang} energies of the flowing liquid will be determined as:

$$T_{\text{lang}} = \frac{4}{9}\rho_1 L_c^4 R\sin\beta_{\text{fil}}\phi_1\dot{\phi}_1; \qquad P_{\text{lang}} = \frac{4}{3}L_c^3\rho_1 R\sin\beta_{\text{fil}} g\phi_1^2. \tag{6}$$

Substituting expressions (6) into equation (1), we obtain expressions for substitution into the initial system of differential equations (2):

$$\frac{d}{dt}\frac{\partial^2 T_1}{\partial \dot{\phi}_1^2} = \frac{8}{9}\rho_1 L_c^4 R\sin\beta_{\text{fil}}\left(\dot{\phi}_1^2 + \phi_1\ddot{\phi}_1\right); \qquad \frac{\partial P_1}{\partial \phi_1} = \frac{8}{3}L_c^3\rho_1 R\sin\beta_{\text{fil}} g\phi_1^2. \tag{7}$$

Besides vertical displacement of the centre of mass of the flowing liquid, it is necessary to consider its horizontal displacement. Using the reasoning described above, we obtain the values of the kinetic T_{lhor} and potential P_{lhor} energies of the horizontal displacement of the centre of the mass of the flowing liquid cargo:

$$T_{\text{lhor}} = 2\frac{\rho_1 L_c^2 R\sin\beta_{\text{fil}}\phi_1}{2}\left(\frac{2}{3}R\cos\beta_{\text{fil}}\dot{\phi}_1\right)^2; \qquad P_{\text{lhor}} = 2\rho_1 L_c^2 R\sin\beta_{\text{fil}}\phi_1 g\frac{2}{3}R\cos\beta_{\text{fil}}\phi_1. \tag{8}$$

Substituting expressions (8) into equation (1), we obtain expressions for inclusion in the initial system of differential equations (2):

$$\frac{d}{dt}\frac{\partial^2 T_1}{\partial \dot{\phi}_1^2} = \frac{8}{9}\rho_1 L_c^2 R^3\sin\beta_{\text{fil}}\cos^2\beta_{\text{fil}}\left(\dot{\phi}_1^2 + \phi_1\ddot{\phi}_1\right); \qquad \frac{\partial P_1}{\partial \phi_1} = \frac{8}{3}L_c^2\rho_1 R^2\sin\beta_{\text{fil}}\cos\beta_{\text{fil}} g\phi_1. \tag{9}$$

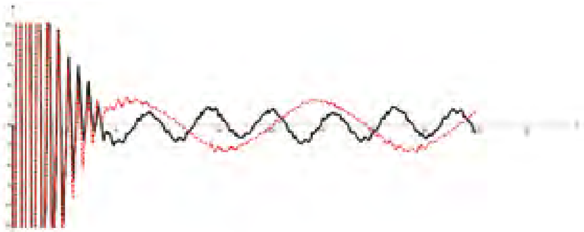
Thus, substituting expressions (4), (7) and (9) into the expression, we obtain a general system of differential equations for the oscillations of the shell of the tank considering the dynamics of the liquid cargo (10).

Table 1

Coefficients of vertical dynamics (authors' calculations)

Speed, km/h	Rate of loading of the tank with the liquid cargo							
	95 %		80 %		70 %		50 %	
	k_{d1}	k_{d2}	k_{d1}	k_{d2}	k_{d1}	k_{d2}	k_{d1}	k_{d2}
20	0,003	0,004	0,006	0,004	0,012	0,004	0,022	0,004
40	0,014	0,017	0,01	0,017	0,015	0,017	0,017	0,016
60	0,041	0,045	0,033	0,041	0,033	0,046	0,029	0,042
80	0,124	0,132	0,1	0,124	0,087	0,126	0,079	0,131
100	0,26	0,287	0,217	0,279	0,22	0,27	0,197	0,264

k_{d1} – coefficient of vertical dynamics considering oscillations of the liquid cargo.
k_{d2} – coefficient of vertical dynamics without considering oscillations of the liquid cargo.



Pic. 4. An example of a graph of changes in the forces acting on the seats of the tank: black lines denote changes in forces without considering movement of the liquid cargo, red lines (italics) show changes in the forces on the seats of the tank considering movement of the liquid cargo (developed by the authors).

$$\begin{cases} m_1\ddot{z}_1 + C_1\left(2z_1 - z_2 - z_3\right) + 4\rho_1 R\sin\beta_{fil} L_c\left(\dot{z}_1^2 + z_1\ddot{z}_1\right) + \\ +8\rho_1 R\sin\beta_{fil} g L_c z_1 = 0 \\ J_1\ddot{\phi}_1 + C_1 L_b\left(2\phi_1 L_b - z_2 + z_3\right) + \\ +\frac{8}{9}\rho_1 L_c^4 R\sin\beta_{fil}\left(\dot{\phi}_1^2 + \phi_1\ddot{\phi}_1\right) + \\ +\frac{8}{3}L_c^3\rho_1 R\sin\beta_{fil} g\phi_1^2 + \\ +\frac{8}{9}\rho_1 L_c^2 R^3\sin\beta_{fil}\cos^2\beta_{fil}\left(\dot{\phi}_1^2 + \phi_1\ddot{\phi}_1\right) + \\ +\frac{8}{3}L_c^2\rho_1 R^2\sin\beta_{fil}\cos\beta_{fil} g\phi_1 = 0 \\ m_2\ddot{z}_2 + C_1\left(-z_1 - \phi_1 L_b + z_2\right) + \\ +C_2\left(1 + f_{tr}\operatorname{sign}\left(\Delta_3, \dot{\Delta}_3\right)\right)\left(2z_2 - \eta_1(t) - \eta_2(t)\right) = 0 \\ J_2\ddot{\phi}_2 + C_2\left(1 + f_{tr}\operatorname{sign}(\Delta_4, \dot{\Delta}_4)\right)\cdot \\ \cdot\left(2\phi_2 L_1 - \eta_1(t) - \eta_2(t)\right) = 0 \\ m_3\ddot{z}_3 + C_1\left(-z_1 + \phi_1 L_b + z_3\right) + \\ +C_2\left(1 + f_{tr}\operatorname{sign}(\Delta_5, \dot{\Delta}_5)\right)\left(2z_3 - \eta_3(t) - \eta_4(t)\right) = 0 \\ J_2\ddot{\phi}_3 + C_2\left(1 + f_{tr}\operatorname{sign}(\Delta_6, \dot{\Delta}_6)\right)\cdot \\ \cdot\left(2\phi_3 L_1 - \eta_3(t) - \eta_4(t)\right) = 0. \end{cases} \quad (10)$$

SOLUTION

The system of differential equations (10) was solved by the numerical Runge–Kutta method in the MathCad software environment [16]. The roughness approximating function proposed in the literature [4] served to denote the track roughness:

$$\eta(t) = b_1\sin\left(\frac{\pi v}{L_r}t\right) + b_2\sin\left(\frac{3\pi v}{L_r}t\right).$$

Numerical integration of the system of differential equations allowed to obtain the values of the forces on the tank seats.

Pic. 4 shows an example of a graph constructed for the case of a car moving at a speed of 20 km/h with a tank filled at 80 % of its capacity. The graph shows that under steady-state modes of motion, the amplitude of forces, considering movement of the liquid cargo, has increased by about 60 %, and the period of oscillations has also increased threefold.

DISCUSSION

Some results of the numerical implementation of the proposed approach are shown in Pic. 4 and Table 1. The results obtained are in good agreement with the results obtained by the method of mathematical modelling using the Universal Mechanism software package [12; 13]. It should be noted that at low speeds of movement and, consequently, with low frequencies of disturbing influence, the impact of the liquid cargo is maximal. This is quite well traced from the results presented in Table 1. Particularly, at the speed of 20 km/h and with the tank filled at 50 % of its capacity, a significant impact of liquid cargo is noticeable.

Verification of the obtained solutions regarding various initial data showed satisfactory convergence of the results with the results obtained using other approaches.

CONCLUSIONS

The proposed approach has allowed to develop the system of differential equations. The system of equations is non-linear. Due to the complexity of obtaining an analytical solution of this system, a numerical method was applied to solve the Runge–Kutta differential equations of the 4th order.

This approach makes it possible to assess the impact of liquid cargo transported in railway tank wagons, e. g., to determine the effect of the level of tank loading with a liquid cargo on the dynamic properties for different parameters of the tank and the elastic elements of the entire wagon.

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Information about the authors:

Grigoriev, Pavel S.*, Ph.D. (Eng), Associate Professor at the Department of Electric Trains and Locomotives of Russian University of Transport, Moscow, Russia, grigorev.p@gmail.com.*

Korzhin, Sergey N.*, Ph. D. (Eng), Associate Professor at the Department of Wagons and Wagon Operations of Russian University of Transport, Moscow, Russia, korjin@miit.ru.*

Ibodulloev, Sherzod R.*, Ph.D. student at the Department of Mechanics and Mathematical Modelling of National University of Uzbekistan named after Mirzo Ulugbek, Tashkent, Uzbekistan, sherzod_uzmu@inbox.ru.*

Tran, Phu Thuan*, D.Sc. (Eng), First Deputy Chairman of the Society of Vietnamese Citizens in the Russian Federation, Ho Chi Minh City, Vietnam, tranphuthuan@yahoo.com.*

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