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Heavy Train Starting Model

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ABSTRACT

The starting mode for a ground vehicle is the most difficult since the static friction force is much greater than the dynamic friction force. For trains, this mode is such a serious problem that sometimes it is necessary to take special measures, such as the use of sand in the contact area of the wheel tire with the rail or an auxiliary locomotive. An effective way of starting a train is to select coupling clearances. In this case, cars are set in motion sequentially and the inert mass, as well as the static friction force at the immediate moment of starting are minimal.

This method, however, has two significant drawbacks: a small set value of clearances in couplings, which limits the effectiveness of the method, and the shock nature of the impulse transmission negatively affecting the state of train structural elements. These disadvantages can be avoided by using elastically deformable couplings.

The objective of the work is to show the advantage of starting a train with elastic couplings in comparison with the traditional mode using its mathematical description and analysis. Starting a train with elastic couplings is much easier than that of a non-deformable one. Moreover, the greater is the number of wagons, the greater is the advantage of the former over the latter. The softening of the mode of starting the train from rest is essentially due to replacement of the simultaneous start-off of the sections by alternate. This process is relevant for inertial forces. Regarding the static friction force, the mechanism will be similar, i. e., not all the static friction force is overcome at the same time, but its small parts are overcome one by one. To exclude longitudinal vibrations of the train, after reaching the maximum tension of the coupling, it is necessary to mechanically block the possibility of its harmonic compression with subsequent sampling of elastic deformation, for example, using damping devices.

The elimination of the transmission of shock loads to the locomotive engines can be considered as an additional positive effect from the use of elastic couplings.

Keywords: *transport, railway, train, starting, couplings, friction, movement, speed.*

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INTRODUCTION

The static friction force significantly exceeds the motion friction force. This leads to the fact that the starting mode for a ground vehicle is the most difficult [1–4]. For trains, this mode is such a serious problem that sometimes it is necessary to take special measures, such as the use of sand in the contact zone of the wheel tire with the rail or an auxiliary locomotive [5–7]. This problem is exacerbated by the steady trend of increasing train mass and length.

An effective way of starting a train is to select coupling clearances. In this case, wagons are set in motion alternately and the inert mass, as well as the static friction force immediately at the moment of starting, are minimal [8; 9].

This method, however, has two significant drawbacks: a small fixed value of clearances in couplings, which limits the effectiveness of the method and the shock nature of the impulse transmission negatively affecting the state of train structural elements.

These disadvantages can be avoided by using elastically deformable couplings [10].

The literature presents the results of theoretical studies of structural elements using flexible and elastic components [11–13].

In this regard, the dynamics of the entire train in the starting mode is of interest.

The *objective* of the work is a *mathematical description* of the «easy» starting of a train with elastic couplings.

RESULTS

Calculation of the mechanical system containing massive locomotives, wagons and elastic couplings is rather cumbersome [14; 15]. To minimise it, the following assumptions are made: the force F developed by the locomotive is a constant value; masses of the locomotive and wagons are equal to each other and are m.

Locomotive and a Single Wagon

The equation of the forces applied to the locomotive has the form:

$$F = m\frac{d^2x_1}{dt^2} + k(x_1 - x_2), \qquad (1)$$

where x_1, x_2 are movements of, respectively, the locomotive and the wagon;

k is elasticity coefficient of the coupling [16].

The forces applied to the wagon satisfy the equation:

$$0 = m\frac{d^2x_2}{dt^2} - k(x_1 - x_2).$$

From the latter equation it follows that:

$$x_1 = \frac{m}{k}\frac{d^2x_2}{dt^2} + x_2. \qquad (2)$$

Substitution of this equation in (1) gives:

$$F = \frac{m^2}{k}\frac{d^4x_2}{dt^2} + m\frac{d^2x_2}{dt^2} + m\frac{d^2x_2}{dt^2} + kx_2 - kx_2 = \\ = \frac{m^2}{k}\frac{d^4x_2}{dt^2} + 2m\frac{d^2x_2}{dt^2}. \qquad (3)$$

Then let:

$$\frac{d^2x_2}{dt^2} = z. \qquad (4)$$

And then (3) is written as:

$$z'' + 2\frac{k}{m}z = \frac{kF}{m^2}. \qquad (5)$$

Characteristic equation is:

$$r^2 + 2\frac{k}{m} = 0.$$

Its roots are equal to:

$$r_{1,2} = \pm i\sqrt{2\frac{k}{m}}.$$

General solution of the corresponding homogeneous equation is:

$$z_1 = C_1\cos\sqrt{2\frac{k}{m}}t + C_2\sin\sqrt{2\frac{k}{m}}t.$$

Particular solution in accordance with (5) has the form:

$$z_2 = A.$$

Its substitution in (5) gives:

$$2\frac{k}{m}A = \frac{kF}{m^2}, \quad A = \frac{F}{2m}.$$

General solution of the equation (5) is found as:

$$z = z_1 + z_2 = C_1\cos\sqrt{2\frac{k}{m}}t + C_2\sin\sqrt{2\frac{k}{m}}t + \frac{F}{2m}.$$

At the moment of time $t = 0$ the coupling is not deformed, therefore, the force does not act on the wagon and the value (4) is equal to zero. Therefore, for $t = 0$, the latter expression will take the form:

$$z(0) = 0 = C_1\cos\sqrt{2\frac{k}{m}}0 + C_2\sin\sqrt{2\frac{k}{m}}0 + \frac{F}{2m}, \quad C_1 = -\frac{F}{2m}.$$

Taking this into account:

$$z = -\frac{F}{2m}\cos\sqrt{2\frac{k}{m}}t + C_2\sin\sqrt{2\frac{k}{m}}t + \frac{F}{2m}. \qquad (6)$$

In accordance with (4):

$$v_2 = \int z dt = -\frac{F}{2m}\sqrt{\frac{m}{2k}}\sin\sqrt{2\frac{k}{m}}t - \\ -C_2\sqrt{\frac{m}{2k}}\cos\sqrt{2\frac{k}{m}}t + \frac{F}{2m}t + C_3;$$

$$x_2 = \int v_2 dt = \frac{F}{4k}\cos\sqrt{2\frac{k}{m}}t - \\ -C_2\frac{m}{2k}\sin\sqrt{2\frac{k}{m}}t + \frac{F}{4m}t^2 + C_3t + C_4. \qquad (7)$$

Taking into account (2), (4), (6) and (7):

$$x_1=-\frac{F}{2k}\cos\sqrt{2\frac{k}{m}}t+C_2\frac{m}{k}\sin\sqrt{2\frac{k}{m}}t+$$
$$+\frac{F}{2k}+\frac{F}{4k}\cos\sqrt{2\frac{k}{m}}t-$$
$$-C_2\frac{m}{2k}\sin\sqrt{2\frac{k}{m}}t+\frac{F}{4m}t^2+C_3t+C_4;$$
$$v_1=\frac{dx_1}{dt}=\frac{F}{2k}\sqrt{2\frac{k}{m}}\sin\sqrt{2\frac{k}{m}}t+C_2\sqrt{2\frac{k}{m}}\frac{m}{k}\cos\sqrt{2\frac{k}{m}}t-$$
$$-\frac{F}{4k}\sqrt{2\frac{k}{m}}\sin\sqrt{2\frac{k}{m}}t-C_2\sqrt{2\frac{k}{m}}\frac{m}{2k}\cos\sqrt{2\frac{k}{m}}t+\frac{F}{2m}t+C_3;$$
$$a_1=\frac{dv_1}{dt}=\frac{F}{2k}2\frac{k}{m}\cos\sqrt{2\frac{k}{m}}t-$$
$$-C_2 2\frac{k}{m}\frac{m}{k}\sin\sqrt{2\frac{k}{m}}t-\frac{F}{4k}2\frac{k}{m}\cos\sqrt{2\frac{k}{m}}t+$$
$$+C_2 2\frac{k}{m}\frac{m}{2k}\sin\sqrt{2\frac{k}{m}}t+\frac{F}{2m};$$
$$x_2(0)=0=\frac{F}{4k}\cos\sqrt{2\frac{k}{m}}0-C_2\frac{m}{2k}\sin\sqrt{2\frac{k}{m}}0+$$
$$+\frac{F}{4m}0^2+C_3 0+C_4;$$
$$\frac{F}{4k}+C_4=0;\ C_4=-\frac{F}{4k};$$
$$v_2(0)=0=-C_2\sqrt{\frac{m}{2k}}+C_3;$$
$$v_1(0)=0=C_2\sqrt{2\frac{k}{m}}\frac{m}{k}-C_2\sqrt{2\frac{k}{m}}\frac{m}{2k}+C_3=$$
$$=C_2\sqrt{2\frac{k}{m}}\frac{m}{2k}+C_3;$$
$$\begin{cases}-C_2\sqrt{\dfrac{m}{2k}}+C_3=0\\ C_2\sqrt{\dfrac{m}{2k}}+C_3=0\end{cases};\ C_2=0,\quad C_3=0.$$

The final solution is:

$$x_1=-\frac{F}{4k}\cos\sqrt{\frac{2k}{m}}t+\frac{F}{4m}t^2+\frac{F}{4k};$$
$$x_2=\frac{F}{4k}\cos\sqrt{\frac{2k}{m}}t+\frac{F}{4m}t^2-\frac{F}{4k};$$
$$v_1=\frac{F}{2\sqrt{2km}}\sin\sqrt{\frac{2k}{m}}t+\frac{F}{2m}t;$$
$$v_2=-\frac{F}{2\sqrt{2km}}\sin\sqrt{\frac{2k}{m}}t+\frac{F}{2m}t;$$
$$a_1=\frac{F}{2m}\cos\sqrt{\frac{2k}{m}}t+\frac{F}{2m};$$
$$a_2=-\frac{F}{2m}\cos\sqrt{\frac{2k}{m}}t+\frac{F}{2m}.$$

The characteristic time interval τ_2 (the index «$_2$» means the number of components of the train) for the case under consideration is determined from the condition of maximum stretching of the elastic coupling. Where in:

$$a_1(\tau_2)-\frac{F}{2m}=0\ \text{ or }\ \frac{F}{2m}\cos\sqrt{\frac{2k}{m}}\tau_2=0;$$
$$\sqrt{2\frac{k}{m}}\tau_2=\frac{\pi}{2};\ \tau_2=\frac{\pi}{2}\sqrt{\frac{m}{2k}}.$$

For time τ_2 the locomotive will cover the distance:

$$x_1(\tau_2)=-\frac{F}{4k}\cos\sqrt{\frac{2k}{m}}\frac{\pi}{2}\sqrt{\frac{m}{2k}}+$$
$$+\frac{F}{4m}\frac{\pi^2}{4}\frac{m}{2k}+\frac{F}{4k}=\frac{F\pi^2}{32k}+\frac{F}{4k}$$

and will develop the speed:

$$v_1(\tau_2)=\frac{F}{2\sqrt{2km}}\sin\sqrt{\frac{2k}{m}}\frac{\pi}{2}\sqrt{\frac{m}{2k}}+$$
$$+\frac{F}{2m}\frac{\pi}{2}\sqrt{\frac{m}{2k}}=\frac{F}{2\sqrt{2km}}+\frac{F\pi}{4\sqrt{2km}}.$$

It is appropriate to compare these indicators with the corresponding values for a non-deformable train:

$$a=\frac{F}{2m};\ v=\frac{F}{2m}t;\ x=\frac{F}{4m}t^2;$$
$$x(\tau_2)=\frac{F}{4m}\frac{\pi^2}{4}\frac{m}{2k}=\frac{F\pi^2}{32k};$$
$$v(\tau_2)=\frac{F}{2m}\frac{\pi}{2}\sqrt{\frac{m}{2k}}=\frac{F\pi}{4\sqrt{2km}};$$
$$\frac{x_1(\tau_2)}{x(\tau_2)}=\frac{F\pi^2/(32k)+F/(4k)}{F\pi^2/(32k)}=1+\frac{32}{4\pi^2}\approx 1{,}81;$$
$$\frac{v_1(\tau_2)}{v(\tau_2)}=\frac{F/(2\sqrt{2km})+F\pi/(4\sqrt{2km})}{F\pi/(4\sqrt{2km})}=1+\frac{2}{\pi}\approx 1{,}64.$$

The ratio of kinetic energies of the locomotive [17] is:

$$\frac{E_1(\tau_2)}{E(\tau_2)}=2{,}69.$$

The obtained ratios clearly demonstrate that the starting-off of a train with elastic couplings is much easier than that of a non-deformable one.

Locomotive and Two Wagons

The equations of forces applied, respectively, to the locomotive and the cars are:

$$F=m\frac{d^2x_1}{dt^2}+k(x_1-x_2); \tag{8}$$
$$k(x_1-x_2)=m\frac{d^2x_2}{dt^2}+k(x_2-x_3); \tag{9}$$
$$k(x_2-x_3)=m\frac{d^2x_3}{dt^2}.$$

From the latter equation it follows that:

$$x_2=\frac{m}{k}\frac{d^2x_3}{dt^2}+x_3. \tag{10}$$

The derivative of this expression is:

$$\frac{d^2x_2}{dt^2}=\frac{m}{k}\frac{d^4x_3}{dt^4}+\frac{d^2x_3}{dt^2}.$$

Substitution of two latter expressions in (9) gives:

$$x_1 = \frac{m}{k}\frac{d^2x_2}{dt^2} + 2x_2 - x_3 =$$
$$\frac{m^2}{k^2}\frac{d^4x_3}{dt^4} + \frac{m}{k}\frac{d^2x_3}{dt^2} + 2\frac{m}{k}\frac{d^2x_3}{dt^2} + 2x_3 - x_3 =$$
$$= \frac{m^2}{k^2}\frac{d^4x_3}{dt^4} + 3\frac{m}{k}\frac{d^2x_3}{dt^2} + x_3 . \qquad (11)$$

The derivative of this expression is equal to:

$$\frac{d^2x_1}{dt^2} = \frac{m^2}{k^2}\frac{d^6x_3}{dt^6} + 3\frac{m}{k}\frac{d^4x_3}{dt^4} + \frac{d^2x_3}{dt^2} .$$

Substitution of the obtained expressions in (8) gives:

$$\frac{F}{k} = \frac{m^3}{k^3}\frac{d^6x_3}{dt^6} + 3\frac{m^2}{k^2}\frac{d^4x_3}{dt^4} + \frac{m}{k}\frac{d^2x_3}{dt^2} +$$
$$+ \frac{m^2}{k^2}\frac{d^4x_3}{dt^4} + 3\frac{m}{k}\frac{d^2x_3}{dt^2} + x_3 - \frac{m}{k}\frac{d^2x_3}{dt^2} - x_3 =$$
$$= \frac{m^3}{k^3}\frac{d^6x_3}{dt^6} + 4\frac{m^2}{k^2}\frac{d^4x_3}{dt^4} + 3\frac{m}{k}\frac{d^2x_3}{dt^2} ,$$
$$\frac{d^6x_3}{dt^6} + 4\frac{k}{m}\frac{d^4x_3}{dt^4} + 3\frac{k^2}{m^2}\frac{d^2x_3}{dt^2} = \frac{k^2F}{m^3} . \qquad (12)$$

Let:

$$\frac{d^2x_3}{dt^2} = z . \qquad (13)$$

Then (12) is written as:

$$z'''' + 4\frac{k}{m}z'' + 3\frac{k^2}{m^2}z = \frac{k^2F}{m^3} . \qquad (14)$$

Characteristic equation:

$$r^4 + 4\frac{k}{m}r^2 + 3\frac{k^2}{m^2} = 0 ;$$
$$r_{1,2}^2 = -2\frac{k}{m} \pm \frac{k}{m} = ; \; r_1^2 = -3\frac{k}{m} ;$$
$$r_2^2 = -\frac{k}{m} ; \; r_{1,2} = \pm i\sqrt{3\frac{k}{m}} ; \; r_{3,4} = \pm i\sqrt{\frac{k}{m}} .$$

General solution of the corresponding homogeneous equation is:

$$z_1 = C_1\cos\sqrt{3\frac{k}{m}}t + C_2\sin\sqrt{3\frac{k}{m}}t +$$
$$+ C_3\cos\sqrt{\frac{k}{m}}t + C_4\sin\sqrt{\frac{k}{m}}t.$$

Particular solution has the form:

$$z_2 = A .$$

Its substitution in (14) gives:

$$3\frac{k^2}{m^2}A = \frac{k^2F}{m^3} ; \; A = \frac{F}{3m} .$$

General solution is found as:

$$z = z_1 + z_2 = C_1\cos\sqrt{\frac{3k}{m}}t + C_2\sin\sqrt{\frac{3k}{m}}t +$$
$$+ C_3\cos\sqrt{\frac{k}{m}}t + C_4\sin\sqrt{\frac{k}{m}}t + \frac{F}{3m}. \qquad (15)$$

In accordance with (13):

$$v_3 = \int zdt = C_1\sqrt{\frac{m}{3k}}\sin\sqrt{\frac{3k}{m}}t - C_2\sqrt{\frac{m}{3k}}\cos\sqrt{\frac{3k}{m}}t +$$
$$+ C_3\sqrt{\frac{m}{k}}\sin\sqrt{\frac{k}{m}}t - C_4\sqrt{\frac{m}{k}}\cos\sqrt{\frac{k}{m}}t + \frac{F}{3m}t + C_5 ; \qquad (16)$$

$$x_3 = \int v_3dt = -C_1\frac{m}{3k}\cos\sqrt{\frac{3k}{m}}t - C_2\frac{m}{3k}\sin\sqrt{\frac{3k}{m}}t -$$
$$- C_3\frac{m}{k}\cos\sqrt{\frac{k}{m}}t - C_4\frac{m}{k}\sin\sqrt{\frac{k}{m}}t + \frac{F}{6m}t^2 + C_5t + C_6 . \qquad (17)$$

Taking into account (10), (13), (15) and (17):

$$x_2 = \frac{m}{k}C_1\cos\sqrt{\frac{3k}{m}}t + \frac{m}{k}C_2\sin\sqrt{\frac{3k}{m}}t +$$
$$+ \frac{m}{k}C_3\cos\sqrt{\frac{k}{m}}t + \frac{m}{k}C_4\sin\sqrt{\frac{k}{m}}t + \frac{m}{k}\frac{F}{3m} -$$
$$- C_1\frac{m}{3k}\cos\sqrt{\frac{3k}{m}}t - C_2\frac{m}{3k}\sin\sqrt{\frac{3k}{m}}t - C_3\frac{m}{k}\cos\sqrt{\frac{k}{m}}t -$$
$$- C_4\frac{m}{k}\sin\sqrt{\frac{k}{m}}t + \frac{F}{6m}t^2 + C_5t + C_6 =$$
$$= \frac{2m}{3k}C_1\cos\sqrt{\frac{3k}{m}}t + \frac{2m}{3k}C_2\sin\sqrt{\frac{3k}{m}}t +$$
$$+ \frac{F}{3k} + \frac{F}{6m}t^2 + C_5t + C_6; \qquad (18)$$

$$v_2 = \frac{dx_2}{dt} = -\frac{2m}{3k}\sqrt{\frac{3k}{m}}C_1\sin\sqrt{\frac{3k}{m}}t +$$
$$+ \frac{2m}{3k}\sqrt{\frac{3k}{m}}C_2\cos\sqrt{\frac{3k}{m}}t + \frac{F}{3m}t + C_5 =$$
$$= -\frac{2}{3}\sqrt{\frac{3m}{k}}C_1\sin\sqrt{\frac{3k}{m}}t +$$
$$+ \frac{2}{3}\sqrt{\frac{3m}{k}}C_2\cos\sqrt{\frac{3k}{m}}t + \frac{F}{3m}t + C_5; \qquad (19)$$

$$a_2 = \frac{dv_2}{dt} = -2C_1\cos\sqrt{\frac{3k}{m}}t - 2C_2\sin\sqrt{\frac{3k}{m}}t + \frac{F}{3m} . \qquad (20)$$

Taking into account (11), (17), (18) and (20):

$$x_1 = -2C_1\frac{m}{k}\cos\sqrt{\frac{3k}{m}}t - 2C_2\frac{m}{k}\sin\sqrt{\frac{3k}{m}}t + \frac{F}{3m}\frac{m}{k} +$$
$$+ 2\frac{2m}{3k}C_1\cos\sqrt{\frac{3k}{m}}t + 2\frac{2m}{3k}C_2\sin\sqrt{\frac{3k}{m}}t +$$
$$+ \frac{2F}{3k} + \frac{2F}{6m}t^2 + 2C_5t + 2C_6 +$$
$$+ C_1\frac{m}{3k}\cos\sqrt{\frac{3k}{m}}t + C_2\frac{m}{3k}\sin\sqrt{\frac{3k}{m}}t + C_3\frac{m}{k}\cos\sqrt{\frac{k}{m}}t +$$
$$+ C_4\frac{m}{k}\sin\sqrt{\frac{k}{m}}t - \frac{F}{6m}t^2 - C_5t - C_6 =$$
$$= -C_1\frac{m}{3k}\cos\sqrt{\frac{3k}{m}}t - C_2\frac{m}{3k}\sin\sqrt{\frac{3k}{m}}t + C_3\frac{m}{k}\cos\sqrt{\frac{k}{m}}t +$$
$$+ C_4\frac{m}{k}\sin\sqrt{\frac{k}{m}}t + \frac{F}{k} + \frac{F}{6m}t^2 + C_5t + C_6 ;$$
$$v_1 = \frac{dx_1}{dt} = C_1\sqrt{\frac{m}{3k}}\sin\sqrt{\frac{3k}{m}}t -$$
$$- C_2\sqrt{\frac{m}{3k}}\cos\sqrt{\frac{3k}{m}}t - C_3\sqrt{\frac{m}{k}}\sin\sqrt{\frac{k}{m}}t +$$
$$+ C_4\sqrt{\frac{m}{k}}\cos\sqrt{\frac{k}{m}}t + \frac{F}{3m}t + C_5 ; \qquad (21)$$
$$a_1 = C_1\cos\sqrt{\frac{3k}{m}}t - C_3\cos\sqrt{\frac{k}{m}}t + \frac{F}{3m} .$$

In accordance with (20):

$$a_2(0) = -2C_1 + \frac{F}{3m} = 0 ; \; C_1 = \frac{F}{6m} .$$

Table 1

Number of train sections	$\frac{x_1(\tau)}{x(\tau)}$	$\frac{v_1(\tau)}{v(\tau)}$	$\frac{E_1(\tau)}{E(\tau)}$
2	1,81	1,64	2,69
3	2,6	2,22	4,93

In accordance with (15):

$$z(0)=0=\frac{F}{6m}+C_3+\frac{F}{3m};\ C_3=-\frac{F}{2m}.$$

In accordance with (18):

$$x_2(0)=\frac{2m}{3k}C_1+\frac{F}{3k}+C_6=0;$$

$$\frac{F}{9k}+\frac{F}{3k}+C_6=0;\ C_6=-\frac{4F}{9k}.$$

In accordance with (16), (19) and (21):

$$v_1(0)=-C_2\sqrt{\frac{m}{3k}}+C_4\sqrt{\frac{m}{k}}+C_5=0;$$

$$v_3(0)=-C_2\sqrt{\frac{m}{3k}}-C_4\sqrt{\frac{m}{k}}+C_5=0,\ C_4=0;$$

$$v_2(0)=\frac{2}{3}\sqrt{\frac{3m}{k}}C_2+C_5=0;\ C_2=0;\ C_5=0.$$

Final solution is:

$$x_1=-\frac{F}{18k}\cos\sqrt{\frac{3k}{m}}t-\frac{F}{2k}\cos\sqrt{\frac{k}{m}}t+\frac{F}{6m}t^2+\frac{5F}{9k};$$

$$x_2=\frac{F}{9k}\cos\sqrt{\frac{3k}{m}}t+\frac{F}{6m}t^2-\frac{F}{9k};$$

$$x_3=-\frac{F}{18k}\cos\sqrt{\frac{3k}{m}}t+\frac{F}{2k}\cos\sqrt{\frac{k}{m}}t+\frac{F}{6m}t^2-\frac{4F}{9k};$$

$$v_1=\frac{F}{6\sqrt{3km}}\sin\sqrt{\frac{3k}{m}}t+\frac{F}{2\sqrt{km}}\sin\sqrt{\frac{k}{m}}t+\frac{F}{3m}t;$$

$$v_2=-\frac{F}{3\sqrt{3km}}\sin\sqrt{\frac{3k}{m}}t+\frac{F}{3m}t;$$

$$v_3=\frac{F}{6\sqrt{3km}}\sin\sqrt{\frac{3k}{m}}t-\frac{F}{2\sqrt{km}}\sin\sqrt{\frac{k}{m}}t+\frac{F}{3m}t;$$

$$a_1=\frac{F}{6m}\cos\sqrt{\frac{3k}{m}}t+\frac{F}{2m}\cos\sqrt{\frac{k}{m}}t+\frac{F}{3m};$$

$$a_2=-\frac{F}{3m}\cos\sqrt{\frac{3k}{m}}t+\frac{F}{3m};$$

$$a_3=\frac{F}{6m}\cos\sqrt{\frac{3k}{m}}t-\frac{F}{2m}\cos\sqrt{\frac{k}{m}}t+\frac{F}{3m}.$$

The characteristic time interval τ_3 for the case under consideration is determined from the condition of maximum tension of the elastic coupling.

Thus:

$$a_1(\tau_3)-\frac{F}{3m}=0 \text{ or } \frac{F}{6m}\cos\sqrt{\frac{3k}{m}}\tau_3+\frac{F}{2m}\cos\sqrt{\frac{k}{m}}\tau_3=0;$$

$$\frac{1}{3}\cos\sqrt{3}\sqrt{\frac{k}{m}}\tau_3+\cos\sqrt{\frac{k}{m}}\tau_3=0.$$

Solution of the latter equation has the form:

$$\sqrt{\frac{k}{m}}\tau_3=0,427\pi;\ \tau_3=0,427\pi\sqrt{\frac{m}{k}}.$$

For time τ_3 the locomotive will cover the distance:

$$x_1(\tau_3)=-\frac{F}{18k}\cos\sqrt{\frac{3k}{m}}\bullet 0,427\pi\sqrt{\frac{m}{k}}-$$
$$-\frac{F}{2k}\cos\sqrt{\frac{k}{m}}\bullet 0,427\pi\sqrt{\frac{m}{k}}+$$
$$+\frac{F}{6m}\left(0,427\pi\sqrt{\frac{m}{k}}\right)^2+\frac{5F}{9k}=$$

$$=\frac{F}{k}\begin{bmatrix}-\frac{1}{18}\cos\sqrt{3}\bullet 0,427\pi-\\ -\frac{1}{2}\cos 0,427\pi+\frac{1}{6}(0,427\pi)^2+\frac{5}{9}\end{bmatrix}=$$

$$=\frac{F}{k}\begin{bmatrix}-\frac{1}{18}\cos\sqrt{3}\bullet 0,427\pi-\\ -\frac{1}{2}\cos 0,427\pi+\frac{1}{6}(0,427\pi)^2+\frac{5}{9}\end{bmatrix}=0,78\frac{F}{k}$$

and will develop speed:

$$v_1(\tau_3)=\frac{F}{6\sqrt{3km}}\sin\sqrt{\frac{3k}{m}}\bullet 0,427\pi\sqrt{\frac{m}{k}}+$$
$$+\frac{F}{2\sqrt{km}}\sin\sqrt{\frac{k}{m}}\bullet 0,427\pi\sqrt{\frac{m}{k}}+\frac{F}{3m}\bullet 0,427\pi\sqrt{\frac{m}{k}}=$$

$$=\frac{F}{\sqrt{km}}\begin{pmatrix}\frac{1}{6\sqrt{3}}\sin\sqrt{3}\bullet 0,427\pi+\\ +\frac{1}{2}\sin 0,427\pi+\frac{1}{3}0,427\pi\end{pmatrix}=\frac{F}{\sqrt{km}}.$$

It is appropriate to compare these indicators with the corresponding values for a non-deformable train:

$$a=\frac{F}{3m};\ v=\frac{F}{3m}t;\ x=\frac{F}{6m}t^2;$$

$$x(\tau_3)=\frac{F}{6m}\left(0,427\pi\sqrt{\frac{m}{k}}\right)^2=0,3\frac{F}{k};$$

$$v(\tau_3)=\frac{F}{3m}\bullet 0,427\pi\sqrt{\frac{m}{k}}=0,45\frac{F}{\sqrt{mk}};$$

$$\frac{x_1(\tau_3)}{x(\tau_3)}=2,6;$$

$$\frac{v_1(\tau_3)}{v(\tau_3)}=2,22.$$

The ratio of kinetic energies for the locomotive is:

$$\frac{E_1(\tau_3)}{E(\tau_3)}=4,93.$$

CONCLUSIONS

The use of elastically deformable couplings solves the problem of starting a heavy train.

Table 1 summarises displacements, speeds and kinetic energies of the locomotive for the moments of maximum tension of the elastic coupling, referred to corresponding parameters of the non-deformable train.

The obtained ratios clearly demonstrate that the starting-off of a train with elastic couplings is much easier than that of a non-deformable one. Moreover, the greater is the number of wagons, the greater is the advantage of the former over the latter.

The softening of the train start-off mode is essentially due to replacement of simultaneous start-off of sections by alternate start-off. This process is described above for inertial forces. Regarding the static friction force, the mechanism will be similar, i. e., not all static friction force is overcome at the same time, but its small parts are overcome one by one.

The expressions obtained for displacements, speeds and accelerations of the locomotive and wagons have harmonic components. To exclude longitudinal vibrations of the train, after reaching the maximum tension of the coupling, it is necessary to mechanically block the possibility of its harmonic compression with subsequent sampling of elastic deformation, for example, using damping devices.

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