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# On the Issue of Determining Relative Rail Rolling Contact Fatigue Damageability

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## ABSTRACT

*Adoption of heavy haul traffic on many railroads, comprising Russian railways, has highlighted the relevance of assessing the effect of increased axial loads on the contact fatigue life of rails.*

*The article describes a set of theoretical studies carried out to create a scientifically substantiated method for predicting the contact fatigue life of rails depending on the values of axial loads. The stress-strain state of the contact area has been determined using the finite element model of wheel rolling on a rail. It has been found that the wheel-rail rolling contact area undergoes complex multiaxial loading with the simultaneous action of normal and shear strains. Based on the analysis of models describing multiaxial fatigue damage, the Brown–Miller model was chosen, which considers the simultaneous action of normal strains at the contact area and of maximum shear strains, which most fully describes the stress-strain state of the wheel-rail rolling contact area. To apply the Brown–Miller model, fatigue stress-strain curves for rail steel have been identified. Based on the analysis of methods for determining the parameters of stress-strain curves carried out by V. A. Troschenko, a modified Roessle–Fatemi hardness method has been applied. Based on the experimentally determined values of hardness on the rolling surface, the parameters of the curves of elastic and plastic fatigue have been revealed by calculation and experiment. To establish the damaging effect of the load from wheel rolling on a rail, the concept of relative damage per rolling cycle had been assumed which is the value inverse to the number of cycles preceding formation of a contact-fatigue crack at a given value of the axial load.*

*Calculations of the relative damage rate of the rolling surface of rails caused by contact fatigue defects were carried out with the Fatigue software package considering mean values of the indicators of the degree of fatigue strength and plasticity of rail steel and the calculated stresses in the wheel-rail contact area, as well as the plasticity correction using Neuber method. The polynomial dependence of relative damageability of the rolling surface of rails is obtained. The established functional dependence of relative damageability of the rolling surface of rails on the values of vertical forces can be used as the basis for the developed methodology for predicting the contact fatigue life of rails.*

*Keywords: railway, stress-strain state, wheel-rail contact area, contact fatigue life, multiaxial fatigue, parameters of elastic and plastic fatigue, relative damageability.*

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**For the original Russian text of the article please see p. 6.**

## BACKGROUND

The adoption of heavy haul traffic highlights the relevance of the issue of assessing the effect of increased axial loads on the contact fatigue life of rails. The *objective* of the research has been to create a scientifically substantiated method for predicting the contact fatigue life of rails depending on the values of axial loads.

A feature of the rolling surface of the rail head is that it works in the elastoplastic area, as evidenced by formation of a cold-worked layer on the rolling surface and flattening of the rail head on inner rails in curved track sections and in the areas of welded joints [1–5]. By simulating the process of wheel rolling on a rail surface with a finite element model, it was found that the components of the stress tensor on the rail surface are in a disproportionate multiaxial stress state. To describe that state, and to predict rail fracture, it is necessary to use the theory of multiaxial fatigue [6–9]. The accumulation of fatigue damage to the rail head material occurs due to repetitive loading caused by a rolling wheel.

Another feature of the contact of the rolling wheel with the rail determines the need to thicken the mesh of the finite element model used to calculate the stress-strain state (SSS) [9]. In this case, it is commonly practiced applying the deformation approach to establish contact fatigue damage. Several stress-strain fatigue failure models are known, which generalise the classical Coffin–Manson relation. For multiaxial fatigue fracture, the Brown–Miller, Fatemi–Socie, and Smith–Watson–Topper models are used [10–13]. Fracture mechanisms corresponding to these models [14] are shown in Pic. 1.

## RESULTS

Based on the results of a comparison of the fracture mechanisms corresponding to the above models and considering the features of operation of the rail head surface in the elastoplastic area under multiaxial cyclic loading, including tensile-compression deformations normal to the area of maximum shear strains in the wheel-rail contact area, the Brown–Miller model [11] was chosen to calculate the contact-fatigue durability of rails.

It describes the case of multiaxial fatigue failure by the equation:

$$\varepsilon_a \cong \frac{\Delta\gamma_{max}}{2} + \alpha_{bm}\Delta\varepsilon_{\perp} = \\ = \beta_1 \frac{\sigma'_f - 2\sigma_{\perp mean}}{E}(2N_p)^b + \beta_2\varepsilon'_f(2N_p)^c, \quad (1)$$

where $\varepsilon_a$ is total deformation amplitude;

$\frac{\Delta\gamma_{max}}{2}$ is amplitude of the maximum shear strain achieved at a certain site in the contact area;

$\alpha_{bm}$, $\beta_1$, $\beta_2$ are coefficients equal to:

$\alpha_{bm} = 0{,}3$; $\beta_1 = (1 + \nu) + (1 - \nu)\,\alpha_{bm}$; $\beta_2 = 1{,}5 + 0{,}5\alpha_{bm}$,

where ν is Poisson's ratio;

$\Delta\varepsilon_{\perp}$ is the range of normal strains at this site;

$\sigma'_f$, $\varepsilon'_f$ are coefficients of elastic and plastic fatigue strength;

$\sigma_{\perp\, mean}$ is average (mean) normal stress per cycle at this site;

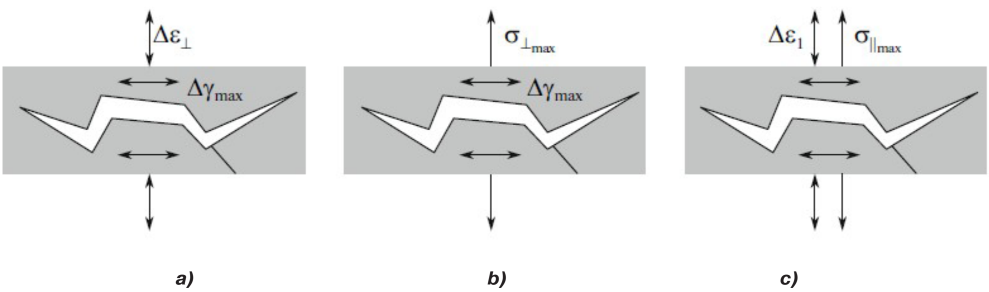
*E* is Young's modulus for rail steel;

$N_p$ is number of cycles before failure;

*b* is exponent of fatigue strength (Basquin stress–life relation);

*c* is exponent of fatigue plasticity (Coffin–Manson).

Deformation approaches based on considering both elastic and plastic components of deformation are suitable for studying material fatigue on the rolling surface of rails. In this case, the process of interaction of a wheel with a rail can be conditionally divided into two components. The initial period is




***Pic. 1. Fracture mechanisms corresponding to the Brown–Miller (a), Fatemi–Socie (b), Smith–Watson–Topper models (c) [14].***

*Pic. 2. The template of the rail with indents from the indenters (balls) following Brinell test for determining hardness along the cross-section of the rail [19].*

characterised by a plastic flow of metal, which leads it to the area of adaptability. After reaching the area of adaptability, plastic deformation practically stops, and the material works in the elastic area.

The relative damageability of the rail head was determined based on the calculated components of the stress tensor, the Brown–Miller model, and multiaxial plastic correction according to Neuber method for the cyclic deformation diagram [15]. The calculations were carried out with a probability of non-destruction of 95 %.

To determine the contact fatigue damageability for the case of elastoplastic deformation, it is proposed to use the deformation approach. The fatigue stress-strain curve is the dependence of the total deformation amplitude on the number of half-cycles before failure. The dependence of the deformation amplitude on the number of cycles until the initiation of a fatigue crack (fatigue failure) has the form [10]:

$$\varepsilon_a = \frac{\sigma'_f}{E}(N_p)^b + \varepsilon'_f(N_p)^c. \quad (2)$$

Equation (2) is called the Basquin–Coffin–Manson equation [15].

In expression (2) under cyclic loading, the amplitude of the total deformation $\varepsilon_a$ can be represented as the sum of the amplitudes of the elastic ($\varepsilon_{ae}$) and plastic ($\varepsilon_{ap}$) components of the deformation:

$$\varepsilon_{ae} = \frac{\sigma'_f}{E}(N_p)^b; \quad (3)$$

$$\varepsilon_{ap} = \varepsilon'_f(N_p)^c. \quad (4)$$

An important characteristic of material properties under cyclic loading is the cyclic deformation curve describing the dependence of the stress amplitude on the deformation amplitude under cyclic loading, the so-called Ramberg–Osgood diagram [17]:

$$\varepsilon_a = \varepsilon_{ae} + \varepsilon_{ap} = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{K'}\right)^{1/n'}, \quad (5)$$

where *K'* is cyclic strength coefficient;

*n'* is coefficient of cyclic strain hardening.

As the data from sources [16; 17] show, the following dependences exist between the parameters of equations (3)–(5), which, despite a good qualitative agreement with the experimental results, can find practical use only in the case of the correct choice of the parameters included in them:

$$\sigma'_f = K'(\varepsilon'_f)^{n'}; \quad (6)$$

$$n' = b / c. \quad (7)$$

The empirical parameters $\sigma'_f$, $\varepsilon'_f$, *b*, *c* in equation (1) are the basis for plotting elastic and plastic fatigue curves in the coordinates lg $\varepsilon_a$ – lg $N_p$ and can be found using the characteristics of the mechanical properties of metals and alloys. Besides, it is important to identify the correlation between the specified parameters and the characteristics of strength and ductility of rail steel under cyclic loading.

The research papers have proposed many different methods for estimating the parameters of the Basquin–Coffin–Manson equation (2) [17; 18]. The work of V. A. Troschenko [18] analysed the methods for determining the estimated values of the parameters $\sigma'_f$, $\varepsilon'_f$, *b*, *c* of the Basquin–Coffin–Manson equation, and it was recommended to use the modified Roessle–Fatemi hardness method for steels. Let us determine the values of the coefficients of fatigue plasticity $\varepsilon'_f$ and elastic fatigue $\sigma'_f$ [18]:

$$\sigma'_f = 4,5HB + 225; \quad (8)$$

$$\varepsilon'_f = (0,32HB^2 - 487HB + 191000) / E, \quad (9)$$

where HB is hardness of rails on the rolling surface.

The hardness of templates of DT350 rails manufactured by ZSMK CJSC [EVRAZ Western Siberian Steel Mill Close Joint-Stock Company] and ChMK PJSC [Chelyabinsk Metallurgical Plant Public Joint-Stock Company], volume-hardened T1 rails manufactured by CJSC ZSMK [19] was experimentally determined in accordance with GOST [State Standard] R 51685-2013[1] (Pic. 2) At the site of JSC VNIKTI.

[1] GOST R 51685-2013. Railway rails. General technical conditions. Moscow, Standartinform publ., 2014, 121 p.

**Table 1**

**Results of hardness measurement of T1 volume-hardened rails and of DT350 differentially heat-treated rails**

| Element | Rail category | | |
|---|---|---|---|
| | T1 | DT350 (PJSC ChMK) | DT350 (JSC EVRAZ ZSMK) |
| Rail head | 363; 363; 363; 363; 363; 363; 363; 352; 352; 352; 352; 363; 363; 363; 363; 341; 341; 363; 363; 363; 363; 363 | 363; 363; 363; 363; 363; 363; 363; 352; 352; 352; 352; 363; 363; 352; 352; 341; 341; 352; 352; 341; 341; 341 | 373; 371; 369; 369; 359; 369; 363; 362; 363; 353; 353; 353; 352; 352; 362; 347; 345; 348; 341; 341; 342 |
| Rail web | 352; 341 | 302; 302 | 307; 362 |
| Rail foot | 341; 341; 341 | 302; 302; 302 | 306; 306; 306 |

**Table 2**

**Calculated values of elastic, plastic, and total deformations of curves for rail steel (HB = 360) (compiled by the authors)**

| No. | $N_p$ | $\varepsilon_{ac}=\frac{\sigma_f}{E}(N_p)^b$ | $\varepsilon_{ap}=\varepsilon'_f(N_p)^c$ | $\varepsilon_a=\varepsilon_{ac}+\varepsilon_{ap}$ |
|---|---|---|---|---|
| 1 | 0 | 0,083600 | 0,272000 | 0,355600 |
| 2 | 10 | 0,068000 | 0,075000 | 0,143000 |
| 3 | $10^2$ | 0,059000 | 0,020600 | 0,079600 |
| 4 | $10^3$ | 0,048000 | 0,005600 | 0,053600 |
| 5 | $10^4$ | 0,039000 | 0,001560 | 0,040560 |
| 6 | $10^5$ | 0,032000 | 0,000430 | 0,032430 |
| 7 | $10^6$ | 0,026000 | 0,000118 | 0,026118 |
| 8 | $10^7$ | 0,020000 | 0,000033 | 0,020033 |
| 9 | $10^8$ | 0,016000 | 0,000009 | 0,016009 |

**Table 3**

**Values of parameters of stress-strain fatigue curves of rail steel HB = 360 in logarithmic coordinates lg $\varepsilon_a$ – lg $N_p$ (compiled by the authors)**

| No. | lg $N_p$ | lg $\varepsilon_{ac}$ | lg $\varepsilon_{ap}$ | lg $\varepsilon_a$ |
|---|---|---|---|---|
| 1 | 0 | -1,0700 | -0,5650 | -0,4490 |
| 2 | 1 | -1,1600 | -1,1250 | -0,8400 |
| 3 | 2 | -1,2300 | -1,6860 | -1,0240 |
| 4 | 3 | -1,3700 | -2,2500 | -1,2680 |
| 5 | 4 | -1,4100 | -2,8100 | -1,3979 |
| 6 | 5 | -1,4900 | -3,3700 | -1,4890 |
| 7 | 6 | -1,5800 | -3,9300 | -1,5830 |
| 8 | 7 | -1,7000 | -4,4300 | -1,6980 |
| 9 | 8 | -1,7900 | -5,0400 | -1,7950 |

The results of measuring hardness of T1 volume-hardened rails, DT350 differentially heat-strengthened (heat-treated) rails are presented in Table 1.

According to the data on hardness of rail steels obtained by the specialists of JSC VNIKTI (Table 1), the values of the parameters $\sigma'_f$, $\varepsilon'_f$, *b*, *c* were calculated using the Basquin–Coffin–Manson equation.

In the modified Roessle–Fatemi hardness method [18], the slope coefficients of stress-strain fatigue curves *b* and *c* are considered constant, and the values $b$ = -0,09, $c$ = -0,56 are recommended for steels.

The values of elastic $\varepsilon_{ac}$, plastic $\varepsilon_{ap}$, and total $\varepsilon_a$ deformations were calculated from the number of cycles until fracture $N_p$ under cyclic loading.

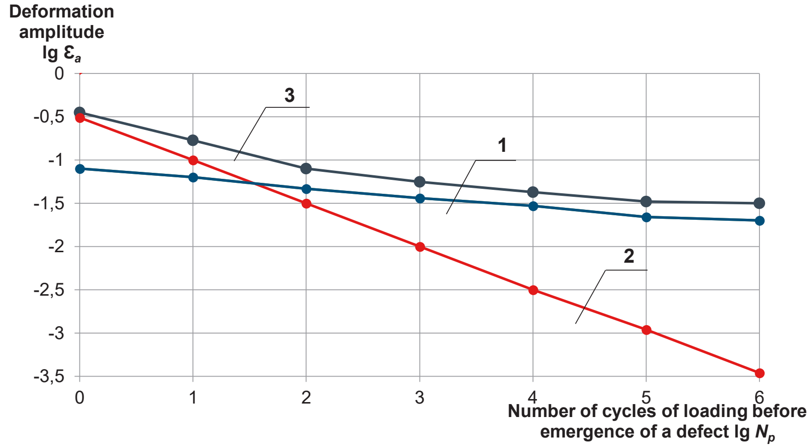
The amplitude of elastic deformation $\varepsilon_{ac}$ from the number of cycles until fracture was determined according to expression (3), the amplitude of plastic deformation $\varepsilon_{ap}$ depending on the number of cycles was determined in accordance with expression (4).

The total deformation from the number of cycles to the initiation of a fatigue crack was determined as the sum of elastic and plastic deformations. The results of calculating the elastic, plastic, and total deformations depending on the number of cycles $N_p$ are presented in Table 2.

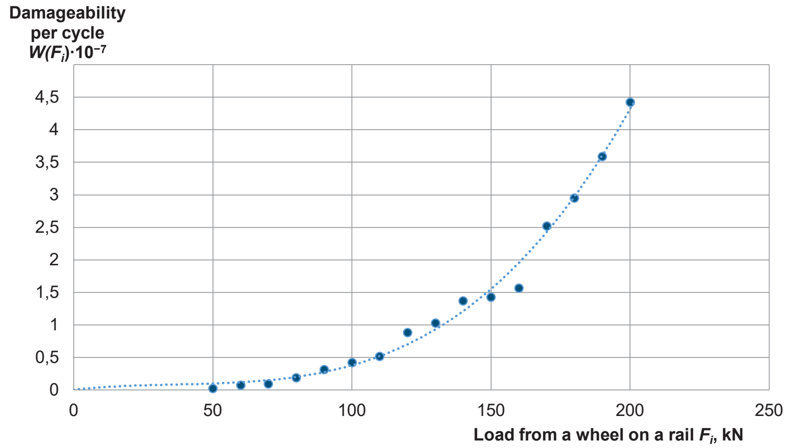
For the convenience of representing stress-strain fatigue curves for rail steels, the calculated data $\varepsilon_{ap}$, $\varepsilon_{ac}$, $\varepsilon_a$ for different $N_p$ are recalculated for the coordinate system lg $\varepsilon_a$ – lg $N_p$. The calculation results are presented in Table 3.

Pic. 3 schematically shows stress-strain fatigue curves in logarithmic coordinates in accordance with expressions (2)–(4). The empirical parameters $\sigma'_f$, $\varepsilon'_f$, *b*, c in equation (2), which are the basis for constructing fatigue curves in coordinates lg $\varepsilon_a$ – lg $N_p$, have been found using the characteristics of the mechanical properties of rail steel. Depending on the technology of manufacturing and processing of the rail, these parameters may differ from those obtained.

To determine the damaging effect of the load from wheel rolling on a rail, the concept of relative damage per rolling cycle is assumed. Relative damage is the reciprocal of the number




***Pic. 3. Deformation fatigue curves for rail steel: 1 – deformation curve of elastic fatigue; 2 – deformation curve of plastic fatigue; 3 – total deformation curve of fatigue (compiled by the authors).***




***Pic. 4. Relative damageability per a cycle of wheel rolling over the examined area at different loads produced by the wheel (compiled by the authors).***

of cycles before formation of a contact fatigue crack at a given value of the axial load. The maximum relative damage in the rail at different values of the vertical load on the wheel is shown in Pic. 4.

The performed approximation of the points of relative damageability to rails caused by wheel rolling with different load values has made it possible to obtain a theoretical dependence on the vertical force in the form of a polynomial:

$$W(F_i) = 9{,}15 \cdot 10^{-7} F_i^3 - 7{,}65 \cdot 10^{-5} F_i^2 + 0{,}00289 F_i, \quad (10)$$

where $F_i$ is vertical force acting on the rail from wagons' wheel rolling with different wheel loads, kN.

The obtained results of relative damageability caused by wheel rolling over the rolling surface of a rail, depending on the values of vertical forces, have been the basis for calculating the contact fatigue life of rails.

## CONCLUSIONS

1. Considering the features of the operation of the rail rolling surface in the elastoplastic area under multiaxial cyclic loading, the Brown–Miller fatigue fracture model was selected to calculate the contact fatigue life of rails.

2. The calculation of the parameters of stress-strain fatigue curves for rail steels using the modified method of hardness of Roessle–Fatemi has been made. The empirical parameters of the elastoplastic fatigue curves have been determined: $\sigma'_f = 1845$, $\varepsilon'_f = 0{,}272$, $b = -0{,}09$, $c = -0{,}56$.

3. Elastic, plastic, and total fatigue curves for rail steel from the number of cycles until failure have been plotted. Considering the averaged characteristics of the indicators of the degree of fatigue strength and plasticity of rail steel and the calculated stresses in the wheel–rail contact area, and the Neuber plastic correction, calculations of relative damageability of the rolling surface of rails by contact fatigue defects have been carried out using the Fatigue software package. The polynomial dependence of relative damageability of the rolling surface of rails has been obtained. The found functional dependence of relative damageability of the rolling surface of rails on the values of vertical forces is considered as the basis for calculating the contact fatigue life of rails.

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