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History of Mathematical Olympiads

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ABSTRACT

Any Olympiad is one of the most significant forms of development of human cognitive activity. Mathematical Olympiads for schoolchildren have been held in our country for several decades. Such a «long life» of the Olympiad movement speaks of importance of this form. The article discusses the main stages of formation and development of mathematical Olympiads. A brief overview of emergence of the olympic movement in Russia and other countries is given. A special place is given to the experience of holding such Olympiads within the walls of Russian University of Transport, where mathematical Olympiads have been held since 2000. Therefore, the current year can be considered an anniversary year. The article presents some forms of work with schoolchildren that preceded the emergence of mathematical Olympiads within the University. The importance of such work, which is aimed at developing interest in engineering education and a deeper study of mathematics, is discussed.

Keywords: *mathematical Olympiads, engineering education, transport education.*

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As Mikhail Lomonosov said: «*And then you should learn mathematics, that it puts your mind in order*» [1]. Therefore, the emergence of mathematical Olympiads was a necessary, useful and natural phenomenon. If we speak in the dry language of definitions, then «*a mathematical Olympiad is a subject Olympiad between students of schools or universities in solving non-standard mathematical problems*». When organizing the Olympiad, the task is not only to identify the most gifted students, but also to create a general atmosphere of a holiday of mathematics, to develop interest in solving problems and independent thinking.

Competitiveness and interest in mathematics have always been inherent in a man. And mathematical competitions themselves appeared as long ago as the queen of the sciences. Historically, they also decided to call them Olympiads. The very first mentions of this kind of competition are found in the history of Ancient India. Much later, Archimedes sent curious mathematical problems to Alexandria [2] to his rivals and colleagues.

Mathematical tournaments, for example, were held in the Kingdom of Naples by Frederick II von Hohenstaufen. Associated with them is the name (known to us, thanks to Dan Brown's novel The Da Vinci Code) of the mathematician Leonardo Fibonacci (Leonardo Pisano, 1170–1250).

The formula for solving quadratic equations was found in Ancient Babylon. And all the outstanding mathematical minds for two millennia tried in vain to find formulas for solving equations of the third and fourth order.

Archimedes' profile at the Fields Medal – the most prestigious award in mathematics. [Electronic resource]: https://elementy.ru/nauchno-populyarnaya_biblioteka/434859/Arkhimed. Last accessed 18.09.2020.

In this story full of dramas and secrets, not the last place was given to «mathematical competitions». True, then they were more personal in nature. Competitions in which Johannes of Palermo and Leonardo Pisano (13th century) or Niccolo Tartaglia and Anton Del Fiore (16th century) took part can rather be called «mathematical duels». It is curious that the intelligent public of that time was quite actively interested in these duels. During one such duel, for example, Niccolo Tartaglia found a formula for solving a cubic equation. And this was the greatest discovery in mathematics [3].

In 18th century, the correspondence competition became popular. Johann Bernoulli, Gottfried Leibniz, Isaac Newton, Leonhard Euler were noticed in such correspondence... The results of the epistolary genre, as a rule, were published in scientific journals of that time. For example, in the German *Acta Eruditorum*.

Thus, in June 1696, the Swiss mathematician Johann Bernoulli sent out to his most distinguished colleagues a curious problem on the brachistochrone. It required «*to find a curve, moving along which solely under the action of gravity, the body will travel from point A to point B in the least amount of time*». In May 1697, Leibniz's decision was published. In total, four solutions were obtained, the authors of which were Leibniz, the marquis de L'Hôpital, Jacob Bernoulli and the author of the problem, Johann Bernoulli. But there was also a decision by an unknown author, which was published in the journal «Philosophical Notes» in January 1697, that is, six months later. As everyone already knows, this unknown author was Isaac Newton. Seeing a surprisingly simple 77-word solution, Johann Bernoulli, of course, guessed who the author was, and said the famous: «*Tanquam ex ungue leonem*» – «*They recognize a lion by the claws*» [4].

Later there were competitions for the prize of the French Academy of Sciences. Sofya Kovalevskaya once became its winner, and no less brilliant Bernhard Riemann, without whom there would be no theory of relativity, was an ordinary participant in these competitions.

In 19th and 20th centuries, France hosted the Concours General (general competition) and the Concours of the French grandes ecoles (Big Schools competition). Everyone knows about the famous Cambridge Mathematical Tripos,

where the goal was not to win, but to get the highest mark. Outstanding records of such exams will be remembered forever. Until now, the result of a 18-year-old French schoolboy Jacques Solomon Hadamard (1865–1963), who hated arithmetic problems as a child, but became a world famous mathematician, has not been beaten. At the exams at the Ecole Polytechnique, he set a record – 1834 points out of 2000 possible, but chose Ecole Normal to study. Nobody ever showed such a result [5].

Obviously, the above competitions had «higher» goals than simple school math Olympiads, but, as they say, «it's a poor soldier that never wants to become a general».

The prototype of modern schoolchildren's competitions can be considered the Eotvos competition in Hungary in 1896, which became the first mathematical Olympiad [6]. The format of the assignments has remained practically unchanged since that time. At the qualifying stage, participants must solve problems from the school curriculum (from elementary to problems of medium complexity). As a rule, the problems proposed for solving are based on interesting ideas, guessing which it is possible to make an oral solution or a solution with minimal calculations. Complex tasks are not offered at the first stage. The second (final) stage of the Olympiad should already include problems of increased complexity (from medium to very difficult). These can be tasks that require serious thought and painstaking calculations. But the assignments are still compiled taking into account the fact that solutions should not require knowledge outside the school curriculum. And to win, the participants also need a certain «mathematical culture».

At the end of 19th century, the Astronomical Society of the Russian Empire became the first founder of the Olympiad for Student Youth. Unfortunately, there is no documentary evidence with tasks, participants, winners about this. A little later, in 1884, the publication of the journal «Bulletin of Experimental Physics and Elementary Mathematics» was launched, in which competition assignments for schoolchildren were published. This was the first correspondence Olympiad. And that was the start.

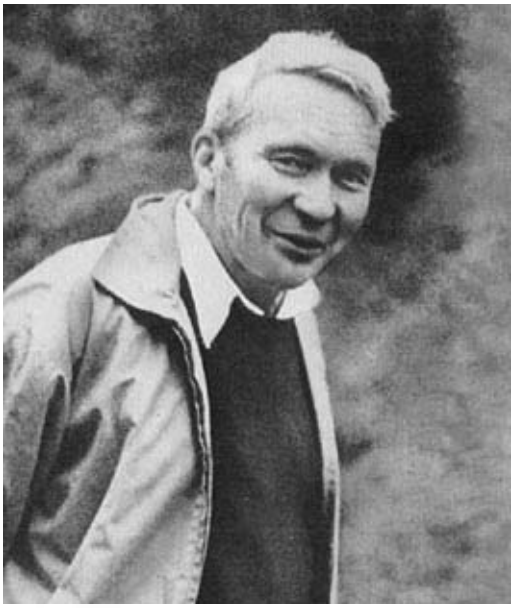
In December 1911, the First All-Russian Congress of Mathematics Teachers was held in St. Petersburg, which brought together teachers

Niccolo Fontana Tartaglia (1499/1500–1557), Italian mathematician. He translated Euclid's «Elements» into Italian and commented on them. [Electronic resource]: https://elementy.ru/nauchno-populyarnaya_biblioteka/432699/Skandal_davno_minuvshikh_dney. Last accessed 18.09.2020.

from all provinces and professors from major universities (1217 participants). The problems of teaching mathematics and the idea of competition in it were vividly discussed at the congress [7].

In 1922, a student at Moscow State University, an aspiring mathematician, 19-year-old Andrey Kolmogorov began working as a teacher at Potylikh Experimental School of the People's Commissariat for Education in Moscow. A very curious fact is that this experimental school partly repeated the famous New York school of Dalton (it was «immortalized» by Woody Allen in the film «Manhattan»). In the Dalton plan, it is declared: «*for successful learning you need freedom, independence, cooperation*» [8].

The Dalton plan, according to which they worked at the school where student Kolmogorov taught physics and mathematics, provided for an individual work plan for each student. The child independently drew up a training program for a month. «*Each student spent most of his school time at his table, went to ... libraries to take out the book he needed, wrote something*», Kolmogorov recalled in his last interview. «*And the teacher was sitting in the corner, reading, and the students came up one by one and showed what they had done*». This picture – a teacher sitting silently in the corner – decades later can be seen in the classroom of mathematical circles [9].



Andrey Nikolaevich Kolmogorov (nee Kataev) (1903–1987) – Russian and Soviet mathematician, one of the greatest mathematicians of XX century. One of the founders of modern probability theory. [Electronic resource]: https://interesnyefakty.org/andrej-kolmogorov/. Last accessed 18.09.2020.

However, a school with such a relaxed creative atmosphere simply could not exist in the USSR for a long time. And in the 1930s, the Dalton Plan was condemned and banned.

Mathematics, classical music, poetry, playing sports and exchanging ideas with friends – this is the image of an ideal person and an ideal school according to Kolmogorov. At about forty years of age, he drew up «*A Concrete Plan of How to Become a Great Man, if There is Enough Desire and Diligence*». By the time he was 60, Kolmogorov had to stop scientific work and devote his life entirely to teaching in high school. And he lived according to this plan [9].

The decisions of that very congress of mathematics teachers were only implemented in the Soviet Union, when in 1933 the famous St. Petersburg mathematician Boris Delone proposed a new system of working with children. This system was based on math circles at schools and houses of pioneers. At the end of each academic year, final Olympiads were to be held, which determined the winners.

It is curious that these first Olympiads were not divided by age and all schoolchildren solved the same tasks. The winners of the first St. Petersburg Mathematical Olympiad received leather briefcases with the inscription in gold letters: «*There is no royal road to science, and only those who do not dread the fatiguing climb of its steep paths have a chance of gaining its lumin*» (K. Marx). Also, the winners were given the right to be enrolled without exams for the chosen physics and mathematics specialty of Leningrad University [10].

The special appeal of the mathematical Olympiads is that even primary school students can take part in them, and the organizers can find «their student» much earlier.

The author of the book «Intricate Problems» Boris Kordemsky writes: «*Any task of quick wits is fraught with zest, its own uniqueness, even if it belongs to a certain type of problem. It is a so-called «tough nut», which is not so easy to bite, but all the more tempting*» [11, p. 6].

In 1935, the mathematical Olympiads also reached the capital, which «by chance» coincided with the move to the capital of Boris Delone. Already in the spring the great Kolmogorov and Aleksandrov organized in Moscow the first mathematical Olympiad for children, which was held with the assistance of Moscow Mathematical Society. The experience of our Leningrad colleagues turned out to be very useful. At the end of February, an announcement about the Olympiad and samples of tasks for preparation were distributed to schools. The competition was traditionally held in two rounds. More than 300 applications were submitted for the first round, and not only from schoolchildren. In the Olympiad, both workers' faculty and students of evening schools decided to try themselves. The youngest mathematician was 14, the oldest was 29. The assignments, as was customary everywhere, did not go beyond the school curriculum.

The second round was planned for the summer and 131 people were allowed to participate. Moreover, all the guys who coped with the first round were invited to lectures by the most prominent Soviet mathematicians: Pavel Sergeevich Alexandrov, Nil Alexandrovich Glagolev, Andrey Nikolaevich Kolmogorov, Alexander Gennadievich Kurosh. Igor Zverev, Nikolay Korobov and Anna Myshkis became the winners of that first Olympiad. Subsequently, they entered the Faculty of Mechanics and Mathematics at Moscow State University, from there they were drafted to war: Anna died in 1943, Igor and Nikolay returned and taught at the Faculty of Mechanics and Mathematics [12].

Railway seven-year school № 2. Pupils at work in a carpentry workshop. Late 1920s–early 1930s. [Electronic resource]: russiainphoto.ru/. Last accessed 18.09.2020.

The Moscow Olympiads were not held from 1942 to 1944. But this does not mean that they were not held at all – the competitions, like people, were also «evacuated»: to Ashgabat and Kazan. But immediately after the end of the war, they returned to Moscow. Around this time, the Olympiads were held in Tbilisi and Kiev. Since 1947 – in Vologda, Ivanov, Irkutsk, Smolensk; from 1949 – in Saratov; since 1950 – in Belarus and a number of other republics of the USSR [12, pp. 5–17; 16, pp. 3–20].

In 1961, the first All-Russian Olympiad in Mathematics was held. In 1964, the Joint Olympiad Committee began its work, chaired by Academician Pyotr Kapitsa. A year later, this post was taken by the experimental physicist Isaak Konstantinovich Kikoin. It was he, together with Kolmogorov, who founded the famous mathematics boarding school at Moscow State University (now the Kolmogorov specialised training and research centre) and became the first editor-in-chief of the famous «Quantum», which was used by all children keen on mathematics, it was he who achieved benefits for the winners of the Olympiads regarding admission to universities.

In 1967, the Ministry of Education and Science of the RSFSR was transformed into a union ministry and the Olympiads received the status of all-union (since 1992 – again all-Russian). Accordingly, from this year, All-Union Olympiads in mathematics, physics and chemistry began to be held. The first All-Union Olympiad was held in Tbilisi. The system of intellectual competitions at this moment developed very rapidly: more and more correspondence competitions appeared, schools with in-depth study of subjects were opened, summer schools were organized (including the school in the famous «Orlyonok» youth summer camp).

In the 1980s, an evening mathematical school appeared at Moscow State University of Railway Engineering (MIIT). It is known that already in the 1970s, the teachers of Moscow Institute of Transport Engineers took an active part in conducting mathematical

Boris Nikolaevich Delone (1890–1980). [Electronic resource]: https://risk.ru/u/img/154/153981.jpg/. Last accessed 18.09.2020.

Olympiads and creating Olympiad problems. Over time, mathematical Olympiads began to appear, conducted by individual institutes. This is how MPTI came up with their own system of physics and mathematics Olympiads. Students, graduate students and teachers, leaving on vacation, held the Olympiad in their hometowns. Then all the work was taken to Moscow and checked.

Obviously, not all prize-winners of mathematical Olympiads have become or are becoming major mathematicians and choose the «queen of sciences» as the work of their entire life, but most famous scientists made their first steps into science at the Olympiads. Among them, for example, three Soviet mathematicians of different generations, each of whom became famous for solving one of the «Hilbert problems» posed at the turn of 19–20th centuries – V. I. Arnold, Yu. I. Matiyasevich, V. M. Kharlamov.

Soviet schoolchildren began to participate in the international mathematical Olympiads starting from the very first one held in 1959 in Romania, wehere they got only sixth place in the team classification, and then were suspended for the next two years . During the second attempt in Prague in 1962 the Soviet Union ranked first. Until 1991, our team dropped below third place only three times.

Since 1967, Great Britain, Sweden, Italy, France began to participate in the International Mathematical Olympiad, and the number of participating countries was growing and has attained more than 100 states.

As in any Olympic movement, the main thing here is not victory, but participation «*for the sake of strengthening friendship and future cooperation in the international scientific community*». Although medals of various denominations are awarded to the winners who obtain best personal scores. But the most important are the unofficial results of the team championship. Nobody canceled it. These totals are always tacitly calculated and serve as the best indicator of scientific potential in «big three» of sciences: mathematics, physics and chemistry.

In the middle of the last century, mathematics was so popular that it practically became a mass profession. And the Olympiads of all levels played an important role in this process. At the 15th Mathematical Congress (Moscow, 1966), it was announced about «*solving the problem of describing all convex three-dimensional polyhedra, whose faces are arbitrary regular polygons*». This was the work of 12 schoolchildren of the 11th grade of Leningrad school No. 239 [13]. At the end of the 1970s, one of the greatest mathematicians of 20th century, Grigory Perelman, taught at that school. Pupils of this school continued to win at city, all-Union and international Olympiads.

From 1962 to 1991 the Soviet Union permanently lead at the Olympiads in mathematics (14 first places, 5 second and 3 third). The United States made the strong competence (participating from 1974 the USA had got three first places, four second places, and four third places until 1991). With the appearance of schoolchildren from China in 1985 on the Olympic horizon, everything changed. China started from 32nd place in the team competition, but three years later, in 1988, it became the first. In the last 35 Olympiads, the Chinese team has become the first 21 times, eight times it came at the second place, and twice at the third place.

Since 1991 Russia only twice became the first, eight times the second and five times the

third. That is, for the past thirty years, the PRC has been the undisputed leader, followed by the United States with a large margin, and the Russian Federation is behind the Americans by an even greater margin.

The last years have been the most disastrous for us. In 2015 – eighth place, in 2016 – seventh, in 2017 – eleventh, in 2019 – sixth place, ahead of us were teams from China, the United States, North Korea, South Korea and Thailand. The last, 61st International Mathematical Olympiad was held in St. Petersburg in September this year. This was the first Olympiad to be held remotely. The team from Russia took second place. The national team of China was the first, and the national team of the United States got the third place.

In international physics Olympiads, the situation is similar: during 20 Olympiads of the Soviet period, we took the first team place 11 times, the second 3 times and the third place 4 times. But after China has appeared in stage, in 1990, we shared the first place, and later on, Chinese schoolchildren became winners in the team classification 21 times. Russia during these years was the first three times. Also, the unofficial championship was won twice by Taiwan, twice by Iran and once by the United States.

The picture at the chemistry Olympiad is about the same, with only one difference: the whole podium is occupied by the representatives of the Southeast Asian region.

The Russian team has the consolation status of «one of the favorites». We have more than three dozen gold medals in the individual standings. The usual «collective» three winners are China, Korea and Taiwan. Sometimes they change places [14].

The experience of conducting mathematical Olympiads in the USA is of interest also. Most of the competitions are paid (about $50 for participation). The Olympiads themselves can be divided into three types: «test» Olympiads, where you have to solve about 20 problems with a choice of answer options. These include:

- Washington State Math Championship;
- Michigan Mathematics Prize Competition;
- New York City interscholastic mathematics league;
- Snow College Mathematics Contest;
- Knights of Pi Math Tournament (Grades 5–8);
- Math is Cool.

«Classic» Olympiads – several difficult problems and a lot of time to solve. These include:

- Utah Math Olympiad;
- United States of America Mathematical Olympiad (USAMO).

We should also mention Canadian Mathematical Olympiad (CMO).

And then there is the annual USA Mathematical Talent Search (USAMTS) Olympiad. It goes through several stages throughout the year. A month is given to solve problems of a stage (as a rule, rather complex, creative and research ones).

Successful participants in this Olympiad have no problems with admission (and receiving an impressive discount) to the most prestigious universities in the USA and Canada. In all American Olympiads, there is no strict rule that complexity of tasks must exactly match the school curriculum.

The USA also hosts the Math Prize for Girls, an annual math Olympiad for girls only. It has been held since 2009. Future «Sofya Kovalevskaya» is offered 20 problems and 2,5 hours to solve. The problems are not easy – to solve them, you have to love mathematics very much. Naturally, the winners are awarded expensive jewelry and large cash prizes.

There is also a math Olympiad for girls in China, which has been held since 2002. This Olympiad is distinguished by a very difficult level of problems, quite at the level of the International Olympiad.

There are a lot of mathematical schools in China and there is a clear selection system. The famous Chinese hard work also plays an important role in development of any science. It is customary for Chinese students and schoolchildren to study a lot and hard.

The main mathematics competitions in Germany are the National Olympiad and the Federal Mathematics Competition. Moreover, it started in the GDR in 1961.

The so-called «Games of Archimedes» are held in Italy. The winner of these games will be eligible for admission to Roma Tre University. Holders of the second and third places are entitled to a 50 % discount on tuition at that university. Besides that National Olympiad in Mathematics (the winners of which then take part in the International Mathematical Olympiad), the Italian National Mathematical Marathon is held as well. This marathon

(Maratona Nazionale di Matematica) is held strictly among the students of the terza media [15].

Leading universities in our country have always looked for ways to attract the brightest students to their walls. Russian University of Transport (RUT) was no exception [16, pp. 142–147; 5, pp. 163–165; 17, pp. 120–124]. Regarding popularization of engineering professions, search for talents for transport education, mathematical competence is an essential requirement. It is almost impossible to delineate the role of mathematics in practical engineering fields.

Surprisingly, mathematics in modern realities is met so often that, one might say, it has become familiar. In the professions in which it is used, logic, precision and calculation are important. Any field of activity, no matter how precise, creative or abstract it is, always in its essence contains the simplest thing – the mechanism of action, that which can be decomposed into mathematical formulas. Therefore, mathematics is the skeleton of any process. Mathematics surrounds us everywhere. Soloing or in symbiosis with other sciences, it forms the foundation for everything new. It can help solve the most complex technical problems in development of a vehicle based on mathematical models that allow you to simulate engines, locomotives, reactors or airplanes. Predictions of, for example, the behavior of the vehicle body on impact or the effects of air turbulence on aircraft wings can be computed. Or, for example, in order to develop efficient control systems for railway vehicles,the mathematicians build the electric drive components of the locomotive on a smaller scale. In addition to such electromechanical models, a mathematical model can be created that describes the dynamics of the system through equations. This allows us to simulate the entire system, for example, to predict how changes in parameters will affect that system.

RUT Regional Mathematical Olympiad celebrates its 20th anniversary this year. Since 2009, it has become known as the «Sails of Hope». The geography of this Olympiad is impressive. Over the years, it was held in Belgorod, Vorkuta, Vladivostok, Yekaterinburg, Yelets, Zheleznogorsk, Irkutsk, Kazan, Karasuk, Kursk, Lgov, Labytnang, Lipetsk, Nizhny Novgorod, Omsk, Rostov-on-Don, Michurinsk, Samara, St. Petersburg, Sochi, Khabarovsk.

In Moscow, the Olympiad was always held in two days, since it was impossible to accommodate everyone during a single day. In Russia, the number of participants in the final stage reached two to three thousand.

Over the years, a large team of authors worked on creation of the problems of the Mathematical Olympiad: M. N. Arshinov, V. S. Antonenko, A. I. Gusev, V. N. Desnyansky, V. B. Minasyan, A. S. Milevsky, V. M. Safro, V. Kh. Khakhanyan.

There are special requirements for the Olympiad problems. They should be beautiful, bright, diverse both in content and in the ideas underlying their solution. Moreover, their decisions, as usual, should not go beyond the school curriculum.

Among the proposed problems, of course, there were problems, the solution of which required fluency in school knowledge of mathematics. But there were also problems, the solution of which required a certain amount of ingenuity and the ability to look at the problem from an unexpected angle.

Let me give you an example of such a task.

Four sheets of paper were taken. Some of these sheets were cut into 4 pieces. Then some of the quarters were cut into 4 pieces again, and so on. When counting all the pieces received, it turned out 2016. Prove that the counting was done incorrectly.

To solve this problem, it is enough to notice that when one sheet is cut, the total number of parts increases by 3. If the counting was done correctly, then the difference between the total number of pieces and the four initial pieces would be a multiple of three. But 2016–4 = 2012, and is not divisible by three.

After reading the solution to the problem, you might think that the problem is too easy. I dare to assure you that a very small number of participants in the Olympiad coped with its solution.

Since 2009, a joint interuniversity Olympiad in mathematics has been held. RUT from the very beginning of this Olympiad has been one of the venues for its holding. University teachers actively cooperate with the organizing committee of the Olympiad, participate in development of Olympiad problems.

In 2016, RUT decided to create another mathematical Olympiad in honour of the outstanding Soviet mathematician, author of

textbooks, prose writer, D.Sc. (Eng), professor Elena Sergeevna Ventzel.

Elena Ventzel is a prominent scientist who has worked at our university for many years, the author of fundamental works on probability theory and operations research. Elena Sergeevna is also known to many under the pseudonym of Irina Grekova, the author of the sensational novels of the time: «The Chair», «The Ship of Widows», «The Turning Point», etc.

Of course, now there are not so many participants in the Mathematical Olympiads. But interest in Russian University of Transport is great. And now in many cities of Russia schoolchildren take part in the mathematical Olympiad «Sails of Hope» and link their future with Russian University of Transport. And the communication of teachers working in schools with leading university teachers has always been in demand and is in demand now [17, pp. 254–258].

I would like to once again recall the organizer of the first St. Petersburg Olympiad for schoolchildren, Corresponding Member of the USSR Academy of Sciences Boris Delone. He was a very gifted person in completely different and, it seemed, rather incompatible fields: a mathematician, one of the first aircraft designers in Russia and a glider pilot, a famous mountaineer in the country... According to the memoirs of students during his lectures at Leningrad State University, Boris Nikolaevich often «demonstrated» a headstand at the table, then jumped down and said: *«Well, let's continue»*...

Any Olympiad in the life of each participant is just a bright but short episode. After this «headstand» there will be a «normal adult» life, like everyone else has, or almost like everyone else. But Olympiads are very important, especially important for future researchers and engineers. Today in Russia there are about eight hundred Olympiads. Therefore, every student can find something important and interesting for himself, regardless of age, talents and hobbies.

About half of the participants in the «high» Olympiads become «pure» mathematicians. And this happens not only with the Russian participants. Roughly the same indicators are in other countries where the Olympic movement is developed. The rest go to «related» professions: someone becomes a programmer, someone becomes an analyst in a bank, someone masters the profession of an engineer... In these areas, you also need to think well. And mathematics is very conducive to this!

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