

# OPTIMIZATION OF LOCOMOTIVES' TURNOVER WITH A «LABYRINTH» SYSTEM

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## ABSTRACT

The problem of rational use of locomotives is quite acute. Here it is necessary to find the best compromise. In case of excess of locomotives there are less delays of trains, but locomotive costs are higher. In case of insufficient number of locomotives the situation is reversed. A model for calculating the optimal modes

of turnover of train locomotives serving train flow is offered. The model promotes the further development of the dynamic transport task. The parameters of trains movement and the use of locomotives are provided, work schedules for each of them are built. The impact of the number of locomotives on the train's mobility is estimated.

**Keywords:** railway, locomotive, train flow, model, optimization, transport task.

**Background.** It is extremely difficult to calculate the optimum number of required locomotives basing on the given cost parameters. Usually the calculations are accompanied by hand-held schedules of turnover for a very limited period and under certain specific assumptions. But if we consider a network with several hundred locomotives and period of several days, the options of locomotives' turnover will be numerous. An optimization model is necessary.

**Objective.** The objective of the authors is to consider the issue of optimization of locomotives' turnover with the use of «Labyrinth» system.

**Methods.** The authors use general scientific and mathematical methods, graph construction, comparative analysis.

**Results.** The system of optimization locomotives' turnover «Labyrinth» is described in the article. The model is based on a dynamic transport task [1]. When solving, this task reduces to the static task by the method of reproduction over time [2]. The problem has been discussed by the authors in previous publications [3]. The material of this article is a further development of the proposed approach. A previously described model has become a system of optimization of locomotives' turnover, which can become an optimizing unit in the corresponding automated control systems.

The station scheme is represented in the model as divided into three parts – input sector (index  $i$ ), output sector (index  $j$ ) and point of origin of gathered sets of railway wagons (index  $q$ ).

The input sector englobes trains (sets of railway wagons with locomotives) or locomotives in reserve. The output sector is the point of departure of trains

(locomotives with sets of railway wagons) or locomotives in reserve. At the point of origin sets of railway cars appear in accordance with train departure timetable. The coupling of a locomotive with a set of railway wagons occurs at the same point. Pic. 1 shows a process of coupling of a locomotive with a set of railway wagons. The locomotive arrived in this case as a reserve unit from the previous station (variable  $y_{ab}(t - \tau_{ab})$ ). Then the locomotive goes to point  $q$ . If here there is a set of wagons (variable  $x_{aq}(t) \neq 0$ ), then the locomotive is coupled with the set, making up a new train. The train goes to the departure sector (variables  $y_{aq}(t - \tau_{aq})$  and  $x_{aq}(t - \tau_{aq})$ ) and can then depart to the next station  $c$  (variables  $y_{bc}(t)$  and  $x_{bc}(t)$ ).

If the locomotive is not needed, it is waiting in the input sector (variable  $y_{ia}(t)$ ).

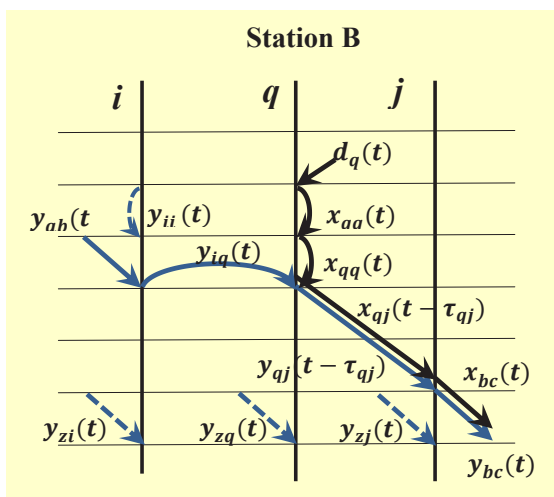
The model introduces the concept of the pool of free locomotives (index  $z$ ). Locomotives may appear there in the early calculation period or as required. In the second case, the experiments can be carried out to determine the number of locomotives required for the given parameters of handling of train flow. Locomotives may come from a pool to the station to all the sectors (variables  $y_{zi}(t)$ ,  $y_{zq}(t)$ , and  $y_{zj}(t)$ ).

The time of movement from point to point is denoted by  $\tau$ . For example,  $y_{ab}(t - \tau_{ab})$  will denote the number of locomotives arrived at the station B at time  $t$ , but departed from the station A earlier providing for the travel time  $\tau_{ab}$ .

Movement of a train from a station in transit is displayed as the joint movement of the locomotive and the set of wagons (Pic. 2).

Here the set of wagons (variable  $x_{bc}(t - \tau_{bc})$ ) and locomotive (variable  $y_{bc}(t - \tau_{bc})$ ) arrive together. They

**Pic. 1.**  
**Scheme of**  
**connection of**  
**a locomotive**  
**with a train:**  
 $i$  – input  
sector,  
 $j$  – output  
sector,  
 $q$  – point  
of origin of  
compositions.



move from the input sector to the output sector together (variables  $x_{ij}(t-\tau_{ij})$  and  $y_{ij}(t-\tau_{ij})$ ). Due to impossibility to send the train delays occur (variables  $x_{ij}(t)$  and  $y_{ij}(t)$ ).

For each train arrival time without delays is set. Arrival is fixed by a variable  $d_i(t^*)$  (Pic. 3).

If the train arrived later, the variable of delay  $\Delta x_i(t)$  appears. In this case, locomotive either is left in reserve (variables  $y_{ij}(t)$  and  $y_{de}(t)$ ), or is waiting in the input or output sector (variables  $y_{ij}(t)$  or  $y_{ij}(t)$ , see Pic. 1).

In the dynamic transport task for the flow source and drain should be specified. For flows, which to the time  $T$  did not come to their drains, artificial drains (variables  $S_i(T)$ ) are introduced.

### Basic equations

Dynamics of compositions in the input sector

$$x_{ab}(t-\tau_{ab}) = x_{ij}(t) + \Delta x_i(t) + d_i(t) + S_i(T), \text{ at } 0 \leq t \leq T.$$

Dynamics of set of wagons in the output sector of the station

$$x_{ij}(t) = x_{ij}(t-1) + x_{qj}(t-\tau_{qj}) + x_{ij}(t-\tau_{ij}) - x_{bc}(t) - S_j(T).$$

Display of train delay

$$\Delta x_i(t) = x_{ab}(t-\tau_{ab}) - x_{ij}(t) - d_i(t),$$

$$\Delta x_i(t-1) = \Delta x_i(t) - d_i(t).$$

Here  $\Delta x_i(t)$  – number of delaying trains at the step  $t$ .

Display of sets of wagons waiting for a locomotive

$$x_{qj}(t) = x_{qj}(t-1) + d_j(t) - x_{qj}(t) - S_j(T).$$

Balance of locomotives in the input sector

$$y_{ij}(t) = y_{ij}(t-1) + y_{ij}(t) - y_{ij}(t) - y_{qj}(t) + y_{ab}(t-\tau_{ab}).$$

A new locomotive either arrives from the previous station, or is taken from the pool.

Dynamics of locomotives in the input sector

$$y_{ij}(t) = y_{ij}(t-1) + y_{ij}(t-\tau_{ij}) + y_{qj}(t) + y_{qj}(t-\tau_{qj}) - y_{bc}(t).$$

Balance of locomotives at the point of origin of sets of wagons

$$y_{qj}(t) + y_{qj}(t) = y_{qj}(t).$$

### Additional restrictions

Impossibility for sets of wagons to exit to the output sector without locomotive exit

$$x_{qj}(t) = y_{qj}(t).$$

The set of wagons in the output sector can be only with the locomotive

$$x_{ij}(t) \leq y_{ij}(t).$$

Impossibility of train departure without the locomotive

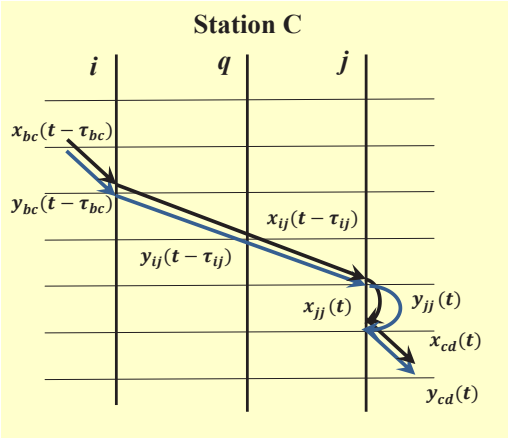
$$x_{ab}(t) \leq y_{ab}(t).$$

### Functional

$$\sum_{i=0}^T (c^c \sum_k (x_{ij}(t) + x_{qj}(t)) + c^o \sum_k \Delta x_i^k(t) + c_\beta^c \sum_k (y_{ii}^k(t) + y_{ij}^k(t) + y_{jj}^k(t)) + \sum_\eta c_\eta (y_\eta(t) - x_\eta(t)) \rightarrow \min.$$

Here:

–  $\eta$  – number of a section,



**Pic. 2. Scheme of train movement in transit.**

- $k$  – number of a station,
  - $c^c$  – cost of one hour of lay-over of the set of wagons,
  - $c^o$  – cost of one hour of train waiting at the station,
  - $c_\beta^c$  – cost of one hour of locomotive use.
- Costs  $c^c$ ,  $c^o$ ,  $c_\beta^c$  are reduced to one simulation time-step.
- $c_\eta$  – cost of one locomotive-kilometer, multiplied by the length of the section  $\eta$ .

### Conceptual meaning of terms of the functional

$c \sum_k (x_{ij}(t) + x_{qj}(t))$  – cost of lay-over of sets of wagons at the stations (all costs are measures for one time-step).

$c^o \sum_k \Delta x_i^k(t)$  – penalty for train late arrival.

$c_\beta^c \sum_k (y_{ii}^k(t) + y_{ij}^k(t) + y_{jj}^k(t))$  – cost of locomotive use.

Technical operations at the station are represented by the variable  $y_{ij}^k(t)$ .

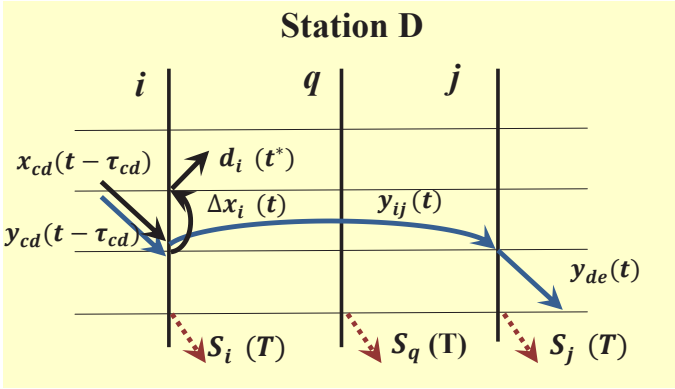
$\sum_\eta c_\eta (y_\eta(t) - x_\eta(t))$  – cost of reserve run of locomotive.

Here, the cost of set of wagons movement is deducted from general train expenses.

### Model of the section of locomotives' handling

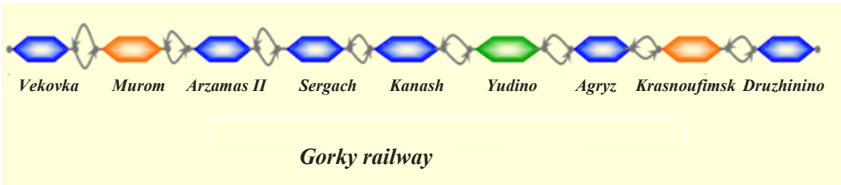
Testing was carried out at the section of the Gorky Railway Druzhinino–Vekovka (Pic. 4).

A number of calculations were carried out. The following are some of the results of one of them.

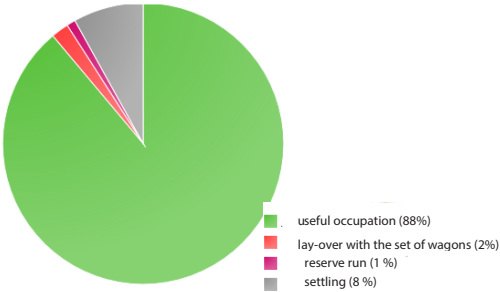


**Pic. 3. Scheme of train arrival at the terminal station. Backward arc  $\Delta x_i(t)$  – delay.**

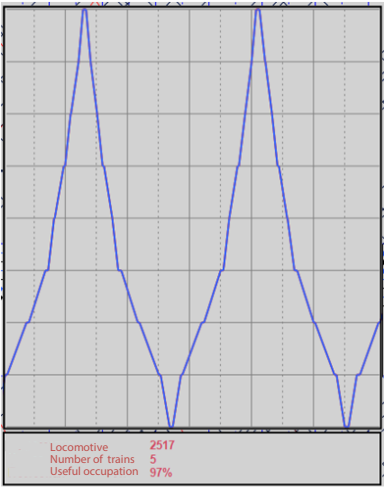




**Pic. 4. The scheme of circulation of locomotives.**



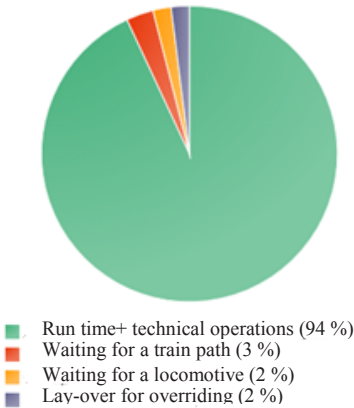
**Pic. 5. Operation of locomotives.**



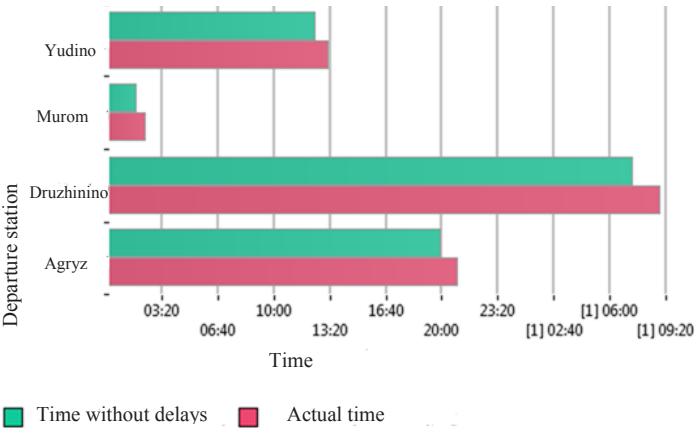
**Pic. 6. The graph of locomotive turnover without reserve runs.**

**Table 1**  
**Parameters of locomotives' work**

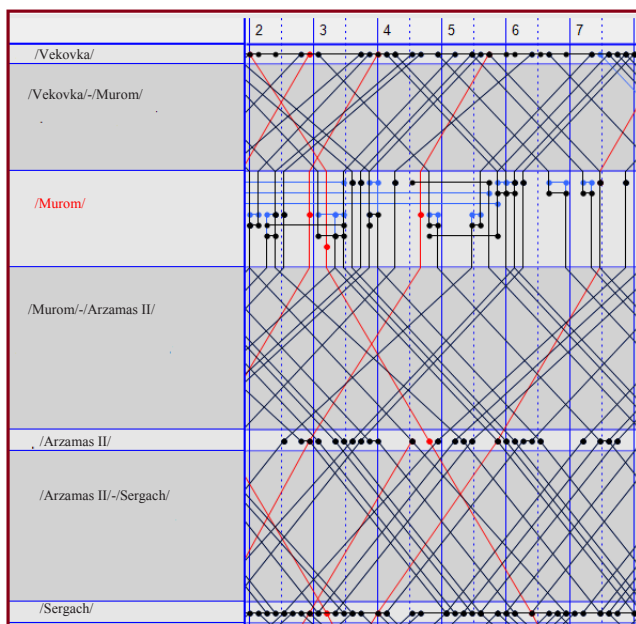
Locomotive	Useful operation	Settling	Lay -over with the set of wagons	Reserve run	Number of trains
2461	[5] 16:08	01:36	04:00	00:00	6
2453	[5] 16:08	03:04	02:48	00:00	6
2496	[5] 16:00	04:16	02:16	00:00	8
2507	[5] 15:36	02:56	03:20	00:00	6
2559	[5] 15:36	01:52	02:40	00:32	5
2454	[5] 15:20	01:12	06:24	00:00	6



**Pic. 7. Parameters of progress of a train flow.**



**Pic. 8. Train delays on individual jets of a flow.**



**Pic. 9. The graph of implemented motion of trains.**

Estimated period was nine days. The first and the last two were rejected as transient to take into account only the stable periods.

Parameters of use of locomotives are shown in Pic. 5.

We can see here, that reserve run is minimized. In case of manual control it is not possible to achieve such results, because the number of possible movements of locomotives for a few days are billions. Lay-over with the set of wagons includes technical operations and lay-over for overriding.

It is possible to see the parameters of work of each locomotive (Table 1).

The contents of the table can be sorted by any column. From each line, it is possible to come to the work schedule of any locomotive. A considerable number of the locomotives do not have empty runs (Pic. 6).

The useful operation does not include lay-over with the set of wagons for overriding. The same information is displayed for the movement of trains (Pic. 7).

As we can see, sets' of wagons waiting for locomotives is also minimized.

The model provides information on movement of trains in each direction (Pic. 8).

It is possible to see the implemented graph of motion and parameters of each train (Pic. 9).

The model allows to carry out various experiments, including – calculation of the optimal number of locomotives for the given cost parameters, determining the best location of the locomotive at the beginning of calculation period for a given structure of a train flow, assessment of impact of the number of locomotives on progress of trains, etc.

For use in operational planning the model should be connected to the appropriate information systems.

For additional study of achieved results (see, e.g., [4]) an imitation model of the same section was built with the same train flows and technological standards. Locomotive turnover control was simulated as close to real dispatch management as possible, simulating making a decision in an operational environment, seeking to find the most effective solutions. Of course,

Группы локомотивов				
Type	Number of loc-s	Full operation	Useful operation	Delays
B/80	150	23443:17(93%)	18854:47(74.8%)	3223:04

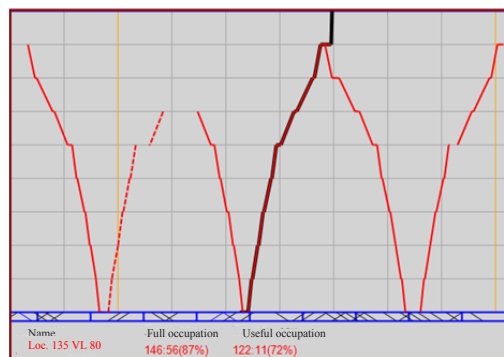
  

Группы локомотивов				
Type	Number of loc-s	Full operation	Useful operation	Delays
B/80	155	23151:04(88.9%)	18724:20(71.9%)	2656:14

Группы локомотивов				
Type	Number of loc-s	Full operation	Useful operation	Delays
B/80	159	22922:31(85.8%)	18662:39(69.9%)	2433:52

**Pic. 10. Parameters of locomotive operation (simulation model). The difference in such results can be understood by comparing the most typical turnover graphs in the optimization model (Pic. 6) and in the simulation model (Pic. 11).**

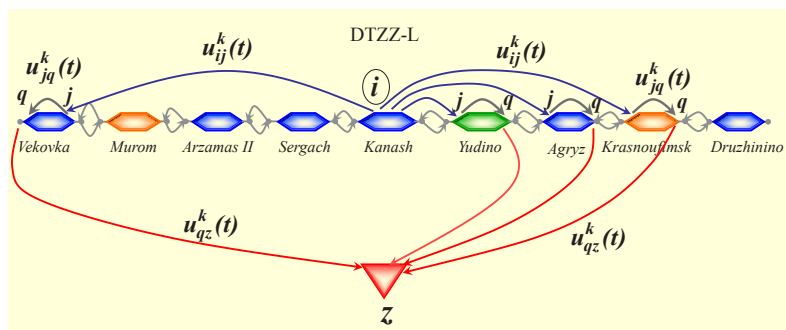


**Pic. 11. Typical graph of locomotive turnover in the simulation model (dotted line – backup runs, breaks – locomotive idle time in anticipation of set of wagons).**



**Pic. 12. The scheme for calculating the optimal turnover of locomotives with regard to maintenance (SP-1, SP-2, SP-3 – service programs).**

**Pic. 13. Construction of the calculation network for DTZZ-L.**



the dispatcher can not evaluate the influence of the decision taken for a more or less long-term perspective because of the huge multivariance.

Numerous experiments have shown that it is impossible to provide such a high level of useful use of locomotives during manual control (Pic. 10). Here, even with 150 locomotives, there are large train delays. The average useful operation is only about 75%. The delays decrease somewhat with the increase in the number of locomotives to 155 and 160.

On the basis of a dynamic transport task, it is not possible to optimize simultaneously the process of advancing the train flow and maintenance, since the flows in these processes are different. For them, it is necessary to set up production and consumption points with the rhythms of their work. In the Labyrinth model, flow elements are sets of wagons, locomotives are attached to them by appropriate constraints. And in the maintenance task, flows of locomotives are considered. Therefore, a three-step solution is proposed (Pic. 12).

The calculation period is the maximum operating time of the locomotive without maintenance (usually 4 days). The first stage is the calculation with the help of the Labyrinth program for the whole period. Then the SP-1 service program determines the stopping points for trains according to the limiting operating time of the locomotives and generates a number of points and departure moments for the dynamic transport task.

Then the dynamic transport task DTZZ-L is formed, where the elements of the flow are locomotives, the destination is the artificial drain, and the service depot is the transit point (Pic. 13).

Notation used in Pic. 13:

$u_{ij}^k(t)$  – flow of locomotives of  $k$ -type locomotives from the location to the maintenance point,

$u_{ij}^k(t)$  – locomotives awaiting maintenance,

$u_{qz}^k(t)$  – flow along an arc to a fictitious drain  $z$ ,

$c_{ij}^k$  – cost of movement of the locomotive to the point of maintenance,

$c_{ij}^k$  – cost of locomotive idle time in anticipation of maintenance,

$c_{jz}^k = 0$  – for locomotives of all types.

The routes of the locomotive to the source necessarily pass through the entire depot of possible maintenance. The criterion is the minimum of total costs for backup runs and downtime in anticipation of maintenance. The task will be multi-product, because different locomotives can be serviced in different depots.

After the calculation, SP-2 service program determines the service stations and moments of readiness of locomotives. These sets, as well as the set of stations and stopping times of trains, will be the initial information for the second calculation of the Labyrinth model.

The SP-3 service program generates common calculation results.

**Conclusion.** Each of the three stages within a Labyrinth method a process of strict optimization is implemented. There is no strict proof of the optimality of the combined process. But it, in all likelihood, will be either optimal or close to it.

The combined process can be transformed into an automated procedure and turned into an optimizing unit in a locomotive automated control system.

## REFERENCES

1. Kozlov, P. A., Milovidov, S. P. Optimization of transport flows structure in flow dynamics at the priority of consumers [Optimizacija struktury transportnykh potokov v dinamike pri prioritete potrebitel'ej]. *Ekonomika i matematicheskie metody*. Vol. XVIII, Iss. 3, Moscow, 1982, pp. 521–531.
2. Kozlov, P. A. Theoretical foundations, organizational forms, methods of optimization of flexible transport service of ferrous metallurgy plants [Teoreticheskie osnovy, organizacionnye formy, metody optimizacii gibkoj tehnologii transportnogo obsluzhivaniya zavodov chernoj metallurgii]. D.Sc. (Eng.) thesis. Lipetsk, LPI publ., 1986, 377 p.
3. Kozlov, P. A., Vakulenko, S. P. Calculation of optimal modes of locomotives operation when serving train flows [Raschet optimal'nykh rezhimov raboty lokomotivov pri obsluzhivanii poezdopotokov]. *Transport Urala*, 2016, Iss. 1, pp. 3–8.
4. Vakulenko, S. P. Model of calculation of optimal operation modes of train locomotives [Model rascheta optimalnykh rezhimov raboty poezdnykh lokomotivov]. *Novaya nauka: opyt, traditsii, innovatsii*, 2016, № 8–2, pp. 97–104. ●

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