NEW APPROACHES TO DEVELOPMENT OF TRANSPORT SCHEDULES

Klevansky, Nikolai N., *Saratov State Agrarian University n.a. N. I. Vavilov, Saratov, Russia.*
Antipov, Maksim A., *Saratov State Agrarian University n.a. N. I. Vavilov, Saratov, Russia.*

ABSTRACT

The authors justify the choice of methods of developing schedules of long-distance trains on the basis of heuristic approaches and taking into account noncyclicity of passenger and freight flows in the Russian context. In the course of algorithmization and development of software for solutions of problems presented the concept of ideology of greedy algorithms, congestion and uniformity, target programming in the framework of requirements of traffic safety management system, ranking of decision-making theory are used. The two-stage process of procedures includes forming the initial schedule and its subsequent optimization. A mathematical model and algorithms of implementing actions are offered according to the objectives.

Keywords: *transport, timetable, schedule development, methodological approaches, heuristics, mathematical model, algorithms.*

Background. *One of the representatives of transport systems with fairly rigid restrictions is a rail transport with its regular seasonal schedule of passenger trains. The task of development of train schedules in scientific publications is usually considered in two forms: cyclic and non-cyclic [1–3].*

Cyclic timetables are typical for the Western European countries with high population density and large passenger traffic. These schedules provide strict periodicity within a day or part of a day. In Russian conditions at large distances between major cities cyclical schedules of passenger transportation do not apply.

The schedules of long-distance passenger trains and freight trains are non-cyclic. Mostly their development is based on linear programming procedures. Both models lead to tasks of high dimensionality, for solutions of which different heuristics are used. For example, Lagrange relaxation [4], columns generation [5], unit problem [6].

Objective. *The article aims to introduce new heuristic methods of developing schedules of long-distance passenger rail transport.*

Methods. *The authors use general scientific and mathematical methods, comparative analysis, evaluation approach, modeling, greedy algorithms, ranking methods wthin decision-making theory.*

Results.

1.

In the process of algorithmization and development of software to generate schedules the authors applied the following concepts [7]: software solution to the problem within the framework of traffic safety control; two-step solution process; ideology of a greedy algorithm; concepts of congestion and uniformity; the use of ranking methods of decision-making theory.

A two-step problem solving process includes the development of an initial schedule and its subsequent optimization. The initial schedule is a schedule created by a program subject to mandatory restrictions. Methods of both phases are cyclical and are completed after inclusion of all routes in the initial schedule, or if it is impossible to further improve schedule.

The task of forming the initial schedule is solved by sequential selection of the next route and its subsequent inclusion in the schedule at the designated time of departure from the starting station. Route selection is based on the concept of congestion, i.e. at each step the busiest route for the required resources is determined. The timing of the inclusion of the route is based on the concept of uniformity of system resources use. That is, at each step of the formation of the initial schedule there are two selection operations.

The task of optimizing the initial timetable is solved with a consistent selection of the most irregular route and its subsequent rearrangement in the schedule at a designated time of departure from the starting station. Rearrangement of the route in the schedule is also based on the concept of uniformity. At each step of optimizing the schedule there are again two selection operations.

This approach is characteristic of greedy algorithms, and it is applicable to problems of formation of schedules and related resource allocation tasks. Using the ideology of greedy algorithms cyclicity of procedures for both phases of the solution of the problem of schedule development is assumed.

Operations of selection under the presented algorithms are multicriteria and for their implementation ranking methods apparatus is attracted [8].

2.

Initial task data:

$S=\{ s_i, \quad i=\overline{1,I} \}$ *– a set of stations of the railway network;*

$L=\{ l_j, \quad j=\overline{1,J} \}$ *– a set of hauls of the railway network;*

pl_i *– a number of platforms for boarding/unboarding of passengers of i-th station;*

$\Delta t_{i,j}$ *– minimum time interval between two successive trains at exit / entrance to j-th haul to the i-th station;*

$F=\{(s,l), \quad s\in S, \quad l\in L\}$ *– a plurality of incident pairs – station s is one of two boundary stations of the haul l. Obviously, the number of elements of the set F is* $n_f = 2\cdot J$*;*

n_r *– number of routes of the railway network;*

$R=\{ r_k, \quad k=\overline{1,n_r} \}$ *– set of vectors of route's applications;*

$n_{k,s}$ *– number of stations of the route* r_k*;*

$$r_k=\begin{pmatrix} f_{k,i}, \quad \forall f_{k,i}\in F, \quad i=\overline{1,n_{k,s}}, \\ k=\overline{1,n_r} \end{pmatrix}= =\left(\left(l_{k,i},s_{k,i}\right), \quad i=\overline{1,n_{k,s}}, \quad k=\overline{1,n_r} \right) - \qquad (1)$$

vector of applications of the route r_k *– sequence of pairs – haul/station from the starting to the ending station;*

n_d – number of days of the interval schedule;

$Day = (day_1, day_2, \ldots, day_{nd})$ – ordered by the time vector of the day;

$Interval = 24 \cdot n_d$ – interval of the schedule, hours;

n_k – number of days of departure of trains of the k-th route;

$Day_k = \{day_{k,i},\ \ i = \overline{1, n_k},\ \ day_{k,i} \in Day\},\ \ k = \overline{1, n_r}$ – a set of days of departure of trains of the routes;

$n_t = \sum_{k=1}^{n_r} n_k$ – number of trains of the schedule;

$U = \{(r, day),\ \ r \in R,\ \ day \in Day\}$ – a plurality of incident pairs, partially forming the applications of the trains of routes – the train of the route r should depart from the starting station on the day of day; using (1) we get a plurality of applications of routes of trains:

$$U = \left\{ \begin{matrix} day_{k,j}, \left(\left(l_{k,i}, s_{k,i}\right),\ \ i = \overline{1, n_{k,s}} \right), \\ k = \overline{1, n_r},\ \ j = \overline{1, n_k},\ \ day_{k,j} \in Day \end{matrix} \right\}; \qquad (2)$$

$d_{k,i} = \left(\left(l_{k,i}, s_{k,i}, \mathrm{int}_{k,i}\right),\ \ i = \overline{1, n_{k,s}},\ \ k = \overline{1, n_r} \right)$ – application to pass the route k along i-th haul to the i-th station, where $int_{k,i}$ – time interval of movement from (i-1)-th station to the i-th station; application of passage of train $u_{k,j}$ along i-th haul to the i-th station

$$d_{k,i,j} = \left(\begin{matrix} \left(l_{k,i}, s_{k,i}, \mathrm{int}_{k,i}, day_{k,j}\right),\ \ i = \overline{1, n_{k,s}}, \\ k = \overline{1, n_r},\ \ j = \overline{1, n_k},\ \ day_{k,j} \in Day \end{matrix} \right), \qquad (3)$$

where $day_{k,j}$ is taken equal to the day of departure of the train of the route.

The initial calculation data of the task:

$nt_i,\ \ i = \overline{1, I}$ – number of trains passing through the i-th station;

$nl_j,\ \ j = \overline{1, J}$ – number of trains passing through the j-th haul.

3.

Variable tasks:

ni – number of routes included in the initial or optimized (rearranged) routes in the schedule;

nr – number of routes not included in the initial schedule or non-optimized routes in the schedule;

nit – number of trains included in the initial schedule;

nit_i – number of trains included in the initial schedule, passing through the i-th station;

nil_j – number of trains included in the initial schedule, passing through the j-th haul;

ti, tf – time of arrival/departure of trains to the station;

til, tfl – time of train entrance to the haul and exit from the haul;

Tl_k – time of departure of trains of the route r_k from the starting station, $Tl_k = ti_{k,1}$.

The event of the schedule, generated by the application (3), on passage of the train $u_{k,j}$ of the route r_k along the i-th haul to the i-th station will be determined by the following expression

$$e_{k,i,j} = \left(\begin{matrix} \left(l_{k,i}, s_{k,i}, \mathrm{int}_{k,i}, day_{k,j}, ti_{k,i,j}, tf_{k,i,j}\right),\ \ i = \overline{1, n_{k,s}}, \\ k = \overline{1, n_r},\ \ j = \overline{1, n_k},\ \ day_{k,j} \in Day \end{matrix} \right); \qquad (4)$$

$day_{k,j}$, $ti_{k,i,j}$ and $tf_{k,i,j}$ in (4) are determined at each step of formation of the schedule;

assessment of occupancy of the i-th station on trains passing through it

$$c1_i = \frac{nt_i - nit_i}{nt - nit},\ \ i = \overline{1, nit_i}; \qquad (5)$$

assessment of occupancy of the j-th haul on trains passing through it

$$c2_j = \frac{nl_j - nil_j}{nt - nit},\ \ j = \overline{1, nil_j}; \qquad (6)$$

scalar assessment of occupancy of the k-th route for days of the schedule

$$c3_k = \frac{n_k}{n_d},\ \ k = \overline{1, n_r}; \qquad (7)$$

Assessment of uniformity of the event of the schedule of the i-th station k1

$$k1_{k,i,j} = \frac{nit_i \left(ti_{suc} + ti_{prec} - 2ti_{k,i,j}\right)}{2 * Interval}, \quad i = \overline{1, n_{k,s}},\ \ k = \overline{1, n_r},\ \ j = \overline{1, n_k}, \qquad (8)$$

where ti_{prec} and ti_{suc} – time of arrival to the i-th station of trains, preceding and following with respect to $ti_{k,i,j}$, in the interval of the schedule;

assessment of uniformity of the event $e_{k,i,j}$ in the periods of the schedule of the i-th station

$$k2_{k,i,j} = \frac{\sum_{j=1}^{n_d} q\left(s_{k,i}, t_{k,i,j}, day_{k,j}\right)}{n_d}, \qquad (9)$$

where

$q = q(s_{k,i}, t_{k,i,j}, day_{k,j}) =$ 1, if at least one train during time $t_{k,i,j}$ of the day $day_{k,j}$ is at the station $s_{k,i}$; 0 – otherwise;

$q \in Q_s$ – a set of Boolean notations of the events of stations;

assessment of uniformity of distribution of events $K1_i$ of the schedule i-th station in the interval of the schedule

$$K1_i = \sqrt{\frac{1}{nt_i} \sum_{j=1}^{nt_i} \left(Int_i - Intr_{i,j}\right)^2},\ \ i = \overline{1, I}, \qquad (10)$$

where $Int_i = \dfrac{Interval}{nt_i},\ \ i = \overline{1, I}$ – average value of the interval between arrivals of trains to the i-th station;

$Intr_{i,j} = \left(ti_{j+1} - ti_j\right),\ \ j = \overline{1, nt_i},\ \ i = \overline{1, I}$ – value of the interval between arrivals at the station of two trains closest in time. For $j = nt_i$ $ti_{j+1} = ti_1$;

assessment of uniformity of the event of the schedule of the i-th haul $k3_{k,i,j}$

$$k3_{k,i,j} = \frac{nil_i \left(ti_{suc} + ti_{prec} - 2ti_{k,i,j}\right)}{2 * Interval}, \quad i = \overline{1, n_{k,s}},\ \ k = \overline{1, n_r},\ \ j = \overline{1, n_k}, \qquad (11)$$

where ti_{prec} and ti_{suc} – time of start of motion of trains on the i-th haul of preceding and following in relation to time $ti_{k,i,j}$;

assessment of uniformity of distribution of the event $e_{k,i,j}$ in the periods of the schedule of the i-th haul at entrance/exit from the station

$$k4_{k,i,j} = \frac{\sum_{j=1}^{n_d} q\left(l_{k,i}, s_{k,i}, t_{k,i,j}, day_{k,j}\right)}{n_d}, \qquad (12)$$

where

$$q = q(s_{k,i}, t_{k,i,j}, day_{k,j}) = \begin{cases} \text{1, if at least one train on } t_{k,i,j} \text{ of the day } day_{k,j} \text{ enters or leaves the station } s_{k,i} \text{ to the haul } l_{k,i}; \\ \text{0 – otherwise;} \end{cases}$$

$q \in Q_l$ – a plurality of Boolean notations of the events of the haul;

Assessment of distribution of events $K2_i$ of the schedule of the i-th haul in the interval of the schedule

$$K2_i = \sqrt{\frac{1}{nl_i}\sum_{j=1}^{nl_i}\left(Int_i - Intr_{i,j}\right)^2}, \quad i = \overline{1,J}, \qquad (13)$$

where $Int_i = \dfrac{Interval}{nl_i}, \quad i = \overline{1,J}$ – average value of the interval between the start of motion of trains on the i-th haul;

$Intr_{i,j} = \left(ti_{j+1} - ti_j\right), \quad j = \overline{1,nl_i}, \quad i = \overline{1,J}$ – value of interval between start of motion of trains on the i-th haul of two trains closest in time.

4.

The task of formation of the initial schedule *is a step-wise choice of another route r_{ni+1} and formation of the schedule $S = \left\{ TI_i, \quad i = \overline{1, ni+1} \right\}$, which minimized multicriterially four vectors of assessment of uniformity of events of stations and hauls of the selected route*

$$\min \begin{cases} \left(k1_{ni+1,i,j}, \quad i = \overline{1, n_{ni+1,s}}, \quad j = \overline{1, n_{ni+1}}\right) \\ \left(k2_{ni+1,i,j}, \quad i = \overline{1, n_{ni+1,s}}, \quad j = \overline{1, n_{ni+1}}\right) \\ \left(k3_{ni+1,i,j}, \quad i = \overline{1, n_{ni+1,s}}, \quad j = \overline{1, n_{ni+1}}\right) \\ \left(k4_{ni+1,i,j}, \quad i = \overline{1, n_{ni+1,s}}, \quad j = \overline{1, n_{ni+1}}\right) \end{cases} \qquad (14)$$

at mandatory restrictions

$$\forall s, \forall day, \forall t \quad \sum_{t \in Interval} q \le pl_i; \qquad (15)$$

$$\forall l, \forall day, \forall t \quad \left(til_i - til_{i-1}\right) \ge \Delta t_{i,j}$$
$$\forall l, \forall day, \forall t \quad \left(tfl_i - tfl_{i-1}\right) \ge \Delta t_{i,j}. \qquad (16)$$

The objective function (14) provides a multi-criterion minimization of assessment of uniformity of events of the schedule of stations and hauls at inclusion of the next route in the initial schedule. Restrictions (15) relate to the simultaneous location of trains at the station in the range of not more than the number of tracks with passenger platforms. Restrictions (16) reflect required intervals between successive entrances and exits of trains from the station and to the station.

Assessment of occupancy of stations (5) and hauls (6) form a plurality of the first and second vector components of routes' congestion:

$$\left\{ \left(c1_{1,k}, c1_{2,k}, \ldots, c1_{i,k}, \quad i = \overline{1, n_{k,s}}\right), \quad k = \overline{1, nr} \right\}; \qquad (17)$$

$$\left\{ \left(c2_{1,k}, c2_{2,k}, \ldots, c2_{i,k}, \quad i = \overline{1, n_{k,s} - 1}\right), \quad k = \overline{1, nr} \right\}. \qquad (18)$$

Multicriteria ranking of vectors (17) and (18) generates a plurality of ranks of routes according to occupancy of stations and hauls:

$$\left\{ rank1_k, \quad k = \overline{1, nr} \right\}; \qquad (19)$$

$$\left\{ rank2_k, \quad k = \overline{1, nr} \right\}. \qquad (20)$$

Reverse sorting of expressions (7) generates a plurality of ranks of routes according to occupancy of days of the interval of the schedule

$$\left\{ rank3\ , \quad k \quad \overline{1, nr} \right\}. \qquad (21)$$

Ranks (19, 20, 21) form a set of vectors of routes

$$\left\{ \left(rank1_k, rank2_k, rank3_k\right), \quad k = \overline{1, nr} \right\}. \qquad (22)$$

Route senior in rank, obtained due to multi-criteria ranking of vectors (22), is the busiest and becomes the next candidate r_{ni+1} for inclusion in the initial schedule.

To determine time of departure from the starting station TI_{ni+1} of trains of the route r_{ni+1} the following actions are performed cyclically:

for $ti_{ni+1,1} = 0$ in increments of = 0,1 hours to 24 hours determination of values of estimates (8, 9, 11, 12) for all hauls and stations of the route with account of restrictions (15, 16) and accumulation of a set of vectors

$$\left\{ ti_{ni+1,1}, \left(k1_{ni+1,i,j}, \quad i = \overline{1, n_{ni+1,s}}, \quad j = \overline{1, n_{ni+1}}\right) \right\}; \qquad (23)$$

$$\left\{ ti_{ni+1,1}, \left(k2_{ni+1,i,j}, \quad i = \overline{1, n_{ni+1,s}}, \quad j = \overline{1, n_{ni+1}}\right) \right\}; \qquad (24)$$

$$\left\{ ti_{ni+1,1}, \left(k3_{ni+1,i,j}, \quad i = \overline{1, n_{ni+1,s}}, \quad j = \overline{1, n_{ni+1}}\right) \right\}; \qquad (25)$$

$$\left\{ ti_{ni+1,1}, \left(k4_{ni+1,i,j}, \quad i = \overline{1, n_{ni+1,s}}, \quad j = \overline{1, n_{ni+1}}\right) \right\}. \qquad (26)$$

Multicriteria ranking of vectors (23, 24, 25, 26) form sets of vectors of ranks for the time of starting motion of trains of the route r_{ni+1}

$$\left\{ \begin{array}{l} ti_{ni+1,1}, \left(rank1, rank2, rank3, rank4\right), \\ ti_{ni+1,1} = \overline{0, 240} \end{array} \right\}. \qquad (27)$$

Multicriteria ranking of vectors (27) determines the starting time TI_{ni+1} of motion of trains of the route r_{ni+1} in the initial schedule.

5.

The task of optimizing the initial schedule *is a stepwise choice of a next route r_{ni+1} and formation* of a schedule $S=\left\{ TI_i, \quad i=\overline{1, ni+1} \right\}$ that minimizes multicriterially two vectors of assessment of uniformity of stations and hauls of the selected route

$$\min \begin{cases} \left(K1_i, \quad i=\overline{1,I}\right) \\ \left(K2_i, \quad i=\overline{1,J}\right) \end{cases} \qquad (28)$$

at mandatory restrictions (15) and (16).

The objective function (28) minimizes the mean-square deviations of intervals between events at stations and hauls. The objective function is related to the necessity of multi-criteria ranking of derived vectors (28). The completion of the initial transport schedule optimization is due to action strategy taken.

Estimates of uniformity of distribution of events of station schedules (10) and hauls (13) form a plurality of first and second vector components of uniformity of routes on the next step in optimizing the initial schedule

$$\left\{ \begin{matrix} \begin{pmatrix} \alpha_{s,1}K1_{1,k}, & \alpha_{s,2}K1_{2,k},\dots, \\ \alpha_{s,k}K1_{i,k}, & i=\overline{1,n_{k,s}} \end{pmatrix}, \\ k=\overline{1,nr} \end{matrix} \right\}, \qquad (29)$$

where $\alpha_{s,i}$ – coefficient of importance of assessment,

$$\alpha_{s,i}=\frac{nt_i}{\sum_{j=1}^{I} nt_j}.$$

$$\left\{ \begin{matrix} \begin{pmatrix} \alpha_{l,1}K2_{1,k}, & \alpha_{l,2}K2_{2,k},\dots, \\ \alpha_{l,k}K2_{i,k}, & i=\overline{2,n_{k,s}} \end{pmatrix}, \\ k=\overline{1,nr} \end{matrix} \right\}, \qquad (30)$$

where $\alpha_{l,i}$ – coefficient of importance of assessment,

$$\alpha_{l,i}=\frac{nl_i}{\sum_{j=1}^{J} nl_j}.$$

Multicriteria ranking of vectors (29) generates a plurality of ranks of routes in uniformity of stations

$$\left\{ rank1_k, \quad k=\overline{1,nr} \right\}. \qquad (31)$$

Multicriteria ranking of vectors (30) generates a plurality of ranks of routes in uniformity of hauls

$$\left\{ rank2_k, \quad k=\overline{1,nr} \right\}. \qquad (32)$$

Ranks (31, 32) form a set of vectors (uniformity criteria) of routes

$$\left\{ \left(rank1_k, rank2_k\right), \quad k=\overline{1,nr} \right\}. \qquad (33)$$

The route senior in rank, obtained by multicriteria ranking of vectors (33), is the most unequal at taken estimates and criteria of uniformity. It becomes a next candidate r_{ni+1} for rearrangement in the initial schedule.

To determine the time of departure from the starting station TI_{ni+1} of trains of the route r_{ni+1} the following actions are performed cyclically:

for $ti_{ni+1,1}$ = 0 in increments of = 0,1 hours to 24 hours determination of values of estimates (10, 13) for all hauls and stations of the route with account of restrictions (15, 16) and accumulation of a set of vectors

$$\left\{ ti_{ni+1,1}, \begin{pmatrix} \alpha_{s,i}K1_{ni+1,i,j}, \\ i=\overline{1,n_{ni+1,s}}, \quad j=\overline{1,n_{ni+1}} \end{pmatrix} \right\}; \qquad (34)$$

$$\left\{ ti_{ni+1,1}, \begin{pmatrix} \alpha_{l,i}K2_{ni+1,i,j}, \\ i=\overline{1,n_{ni+1,s}}, \quad j=\overline{1,n_{ni+1}} \end{pmatrix} \right\}. \qquad (35)$$

Multicriteria ranking of vectors for sets of vectors of ranks for the initial time of starting motion of trains of the route r_{ni+1}

$$\left\{ ti_{ni+1,1}, \left(rank1, rank2\right), \quad ti_{ni+1,1}=\overline{0,240} \right\}. \qquad (36)$$

Multicriteria ranking of vectors (36) determines the desired starting time TI_{ni+1} of motion of trains of the route r_{ni+1} in the optimized schedule of the railway network.

Conclusion. The proposed mathematical model and algorithms of formation and optimization of the initial schedule are to be implemented in software as part of the traffic safety control system.

REFERENCES

1. Lazarev, A. A., Musatova, E. G., Gafarov, E. R., Kvaratshelia, A. G. Scheduling Theory. Tasks of transport system control [*Teorija raspisanij. Zadachi upravlenija transportnymi sistemami*]. Moscow, Faculty of Physics, Moscow State University, 2012, 159 p.

2. Peeters, L. Cyclic Railway Timetable Optimization, ERIM Ph.D. series Research in Management, Erasmus Research inst. of Management (ERIM), 2002.

3. Caprara, A., Fischetti, M., Toth, P. Modeling and solving the train timetabling problem, Operations Research, 2002, 50 (5), pp. 851–861.

4. Fischer, F., Helmberg, C., Jansen, J., Krostitz, B. Towards solving very large scale train timetabling problems by Lagrangian relaxation, paper presented at the ATMOS2008 – 8th Workshop on Algorithmic Approaches for Transportation Modeling, Optimization, and Systems, Dagstuhl, Germany.

5. Cacchiani, V., Caprara, A., Toth, P. A column generation approach to train timetabling on a corridor, 2008, 4OR, 6 (2), pp. 125–142.

6. Burdett, R., Kozan, E. A sequencing approach for creating new train timetables, OR Spectrum, 2010, 32 (1), pp. 163–193.

7. Klevansky, N. N. Main concepts of implementation of tasks of schedule formation [*Osnovnye koncepcii realizacii zadach formirovanija raspisanij*]. *Obrazovatel'nye resursy i tehnologii*, 2014, Iss. 2, pp. 9–21.

8. Klevansky, N. N. Methods of ranking in the tasks of schedule formation [*Metody ranzhirovanija v zadachah formirovanija raspisanij*]. *Proceedings of the XII All-Russia meeting on management problems governance: VSPU – 2014.* Moscow, 2014, pp. 8040–8050.

Information about the authors:

Klevansky, Nikolai N. – Ph.D. (Eng.), professor of the department of Economic Cybernetics Saratov State Agrarian University n.a. N. I. Vavilov, Saratov, Russia, nklevansky@yandex.ru.

Antipov, Maksim A. – Ph.D. student of Saratov State Agrarian University n.a. N. I. Vavilov, Saratov, Russia, antipovmaxim@ya.ru.

Article received 27.06.2016, accepted 30.08.2016.