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Development of the Theory of Macrosystems as a Necessary Condition for Improving Quality of Transport Modelling

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ABSTRACT

The issues of development of the theory of transport macrosystems are considered regarding results obtained in the works of A. G. Wilson, Yu. S. Popkov, A. V. Gasnikov, E. V. Gasnikova and others. The transport macrosystem is considered a complex multicomponent system to which series of thermodynamic analogy can be applied (equilibrium state, information entropy as a function of state parameters, the presence of basic phenomenological schemes for filling of states with elements, etc.). To further develop the theory of development of transport modelling, which is the objective of the article, it is proposed to consider circumstances that reflect modern trends in development of transport systems: diversity of transport systems, dynamic nature of its functioning, many different elements that can obey different models for filling of states. To implement this task, various methods are used: equation of the transport process introduced by the author, which makes it quite easy to go to quasi-dynamic formulations of transport modelling problems, as well as a general formal representation of the system as a set of elements, which is determined based on the analysis of many works of domestic authors. The conclusion is dedicated to discussion of issues of further development of the theory of transport macrosystems in a dynamic setting.

<u>Keywords:</u> transport system, theory of macrosystems, transport modelling, mathematical model, classification of transport systems.

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Background. The development of the theory of macroscopic systems as systems with a large number of stochastic elements is primarily related to statistical physics. Classical and quantum systems, consisting of a large number of molecules, atoms, ions, elementary particles, can be considered within the framework of the molecular-kinetic or thermodynamic approaches. A feature of physical systems is the possibility of existence of deterministic macroscopic states of the system as a whole with stochastic behaviour of particles.

The use of the molecular kinetic analogy for description of transport systems was first proposed, apparently, in the works of Alan Geoffrey Wilson [1–5]. They were based on the idea that many regularities of physical macrosystems can be found in complex systems of various nature. In the preface to the Russian edition of the book [1] Yu. S. Popkov pointed out the systems of exchange and distribution of economic resources. In fact, any transport system at the level of a city or a region exhibits certain properties that allow it to consider it as «*macroscopic*». It is worth noting that in the English-language scientific literature, the theory of macroscopic systems refers primarily to physical systems [6]. The Soviet and modern Russian interpretation of this term has two meanings. First interprets them in the same manner, as physical systems [7], and the second considers them as complex systems involved in resource transportation and allocation. It is to the second class of systems that the most famous works of Academician of RAS Yu. S. Popkov belong [9–13]. In foreign literature, the analogue of this term is the concept of «*urban system*» [8], but it does not cover the entire range of transport systems considered within the framework of the macroscopic approach.

One of the most significant contributions of A. G. Wilson to the theory of transport systems is the *entropy approach*, the meaning of which is to maximize the function responsible for the most probable (equilibrium) state of elements in a macroscopic system [1]. This function can be built on the basis of, for example, information entropy, and its specific form depends on what type of state and elements are present in the system, and also, mainly, on the way the filling of possible state with elements is accomplished. As a result, a formulation of the problem can be obtained, in which the equilibrium of a system consisting of chaotically acting elements is determined.

If we talk about transport systems, then they have both deterministic and chaotic (more precisely, undefined) behaviour of participants in transport processes. As it was said in above mentioned work [1, p. 8], «*no matter how high the degree of centralization is, the economic system of exchange is so complex that there are always random (uncontrollable) factors*». However, a common property of such macrosystems is their ability, by analogy with physical ones, to transform the chaotic actions of elements into a certain deterministic process.

If we indicate the main results of the works of A. G. Wilson, then it is necessary to mention:

1) development of a large number of different transport models, considering splitting by types of travel, types of transportation and routes;

2) development of various models of interregional exchange (essentially those that we now call «transport and logistics systems»);

3) extension of the entropy method to nonequilibrium states of transport systems;

4) use of the concept of «systems with maximum utility» for macroscopic systems, etc. [1–5].

Yu. S. Popkov's contribution to further development of the theory of macrosystems is determined by a systematic presentation of the apparatus of phenomenological schemes (Bose–Einstein, Fermi–Dirac, Boltzmann statistics) in the framework of transportation and distribution, economic, demographic systems, a complete study of properties of their stationary states, numerical methods of solving equilibrium problems, generalization of the results to the case of dynamic behaviour of transport macrosystems [9–13].

It is worth mentioning that at present there is a huge layer of transport problems that explicitly or implicitly use the main results of the theory of transport macrosystems. Among them, we should note the works [14–20], which we single out with the purpose to only point out a wide range of problems solved by the theory of macrosystems, but in no way we pretend to present a complete bibliography on this topic. At the same time, there is an urgency in the new conditions of digital transport to rethink the legacy of the cited works and to outline ways for development of transport modelling technologies. This is *the objective* of this article.

Table 1

Elements and states of a transport macrosystem (examples) (compiled by the author)

No.	Elements	States
1	Vehicle	a. Element *i* is in the transport area *j*; b. Element *i* is in the transport area *j* and the zone of the transport area *k*; c. Element *i* is on the haul *j* of the street-road network; d. Element *i* is in the vehicle *j*; e. Element *i* is on the route *j*; f. Element *i* is at the stop point *j* etc.
2	Route vehicle	
3	Non-route vehicle	
4	Vehicle driver	
5	Passenger	
6	Pedestrian	
7	Route	

Note: Some states are impossible for certain elements.

Brief definition of the transport macrosystem

Let us consider a generalized transport macrosystem with continuous time *t* containing *Y* elements of the same type with some kind of behaviour from the set $B(t) = \{\beta_d(t), \beta_s(t)\}$. Let each element have a state from among ρ classes $K_1, ..., K_\rho$. The classification of states is such that these classes do not overlap.

Let $\sigma^1, ..., \sigma^\rho$ denote the sets of states, where $\sigma^i \in K_i$. We will further assume that the sets of states are discrete for $\beta_s(t)$ (or continuous for $\beta_d(t)$) and contain a finite number of elements (or infinite number, respectively).

In this section, we restrict ourselves to the case of a *homogeneous* macrosystem, whose elements can take on the states of a single class only. For example, the class of states can mean «location in the transport area-zone» (source-sink of the system elements). This class of states can be used for elements with $\beta_s(t)$ behaviour type. Another example of the concept «*class of state*» can be «average speed of the transport flow wihin a haul», used for elements of the type «vehicle» or «transportation flow» with a deterministic type of behaviour $\beta_d(t)$. This class corresponds to a continuous (infinite) *set of states, deterministically related to the sets of states of other classes*. Thus, systems with a deterministic type of behaviour within the framework of the adopted definition cannot be homogeneous since they are simultaneously associated with several classes of states.

So, the homogeneous system has a single set of states of elements σ, the subsets of which $\sigma_1, ..., \sigma_m$ are such that their union coincides with σ, and the intersection of any pairs is empty.

Elements of the macrosystem can randomly and independently of each other fall into any state provided by the subsets $\sigma_1, ..., \sigma_m$. For each fixed subset σ_n for an element, there are two possibilities: to get into any state from σ_n with a priori probability a_n and not to get there with probability $(1 – a_n)$.

Let us consider as an example a set of elements of the transport macrosystem of automobile (urban) transport, which can be in appropriate states (Table 1), and that contains the main elements and states that can be supplemented depending on the content of the task. The formulation of a problem in which all the elements (and the corresponding sets of states) are present is hardly expedient, and even hardly possible in principle from the point of view of its solvability. A more rational way is formulation of special cases, which contain no more than 2–3 elements. For example, it is possible to consider solving the problem of loading routes with vehicles taking into account occupancy. Then in the formulation of the problem we will use elements 2, 5, 7 and the corresponding possible states *{d, e, f}*.

The theory of macrosystems has the main stages of construction, which we indicate in accordance with the work of Yu. S. Popkov [8]:

1) terms and concepts, a phenomenological scheme are introduced;

2) the basic concept of «macrostate probability» is introduced as a value $P(N) = \Pi_{n=1}^{m} \cdot P_n \cdot (N_n)$, that is expressed through the probabilities of states of individual elements $n = 1, ..., m$;

3) to determine the function *P(N)*, the mechanism of filling of states in σ_n is considered, depending on the type of states

(Fermi, Einstein, or Boltzmann states), and on this basis, the function *P(N)* is determined, and then the physical and information entropy are determined;

4) the obtained probabilistic characteristics allow one to take into account the peculiarities of the phenomenological scheme of a homogeneous isolated macrosystem, which consist in the unequally probable choice of the corresponding states by the elements of the system in subsets $\sigma_n (n \in \overline{1,m})$;

5) it is concluded that the generalized entropy for a macrosystem has a single maximum, the «sharpness» of which increases with the number of elements in the system;

6) distribution of elements of a homogeneous system by subsets of close states is established, which can be associated, for example, with consumption of various resources, presence and operation of «price functions» of the transport network, etc., that is, specific problems associated most often with determination of the equilibrium of the macrosystem are solved. An example of such a problem is a classical formulation of the problem of traffic load on the street-road network [20], which uses the entropy approach when taking into account distribution of the travel distance [1].

Among the main shortcomings of the traditional theory, one should point out practically insufficient attention to the nonequilibrium states of systems, while consideration of transport processes is accomplished in dynamics.

Quasi-dynamic model of a transport macrosystem

It is worth noting that an increase in accuracy of transport modelling can be achieved through the complex use of various measures aimed at reducing influence of the factor of «hardly measured» parameters of the state of the transport system:

1) use of methods for constructing high-quality correspondence matrices;

2) use of data stored in the databases of intelligent transport systems;

3) development of a theoretical apparatus for solving problems of the theory of transport macrosystems in a nonequilibrium and/or dynamic formulation;

4) the use of new splitting problems, which would develop more broadly the concept of splitting by type of travel, modes of transportation, etc., proposed by A. G. Wilson.

To implement this direction of development, a new description of the «transport system» model is required. In [21], based on a detailed analysis of the structure of the term «transport system», an approach is proposed that makes it possible to describe transport and transport-logistics systems based on the transportation process equation, taking into account intensity of an arbitrary number of operations of any number of vehicles included in the structure of the transportation process. Such opportunities for obtaining information are created by modern technology (IoT, Big Data).

Let us present a description of a transport system model that generalizes the results obtained in [21] and provides a basis for its use in a wide range of transport modelling problems.

The transport system is described as a set of elements:

a) logical and spatial connection between sources and flows of transport (in a broader description as a resource of some kind);

b) temporal characteristics of the links (start time and duration of each link);

c) appropriate carrying (transit) capacity;

d) transportation process plan;

e) criteria for effectiveness of functioning of systems (or presence of an objective).

In accordance with this definition, it is possible to draw up a generalized structure of a model of an arbitrary transport system, which should include equations that determine:

1) road network graph;

2) a matrix of connections as logical (Boolean) functions of time;

3) carrying capacity of the system (as an alternative description, equations for the traffic flow transit capacity of road and street systems or equations for the flow rates can be specified here);

4) degree of fulfillment of the transport task (equations of transportztion processes);

5) performance criteria or equilibrium conditions.

The number of vehicles and the number of routes are included in the model as parameters. Then the model of the generalized transport system can be written as:

$$\begin{cases} \tilde{A} = \tilde{A}(t); \\ \tilde{n} = \tilde{n}(t), \quad \tau_0 \le t \le \tau_0 + \Delta\tau; \\ \Delta\tau = \sum_{k=1}^{n} \Delta\tau_k; \\ \Delta\tau_k \in \{\Delta\tau_1; \Delta\tau_2; ...; \Delta\tau_n\}; \\ q_{ij} = q_{ij}(t); \\ \pi_v = \pi_v(t); \\ k = 1, ..., l. \end{cases} \quad (1)$$

Here:

t is time that is continuous in the model with discrete (highlighted) moments selected for calculating or determining the state of the system;

τ_0 is start time for the system monitoring;

$\Delta\tau$ is duration of the transport system work (duration of existence, modelling, etc.);

$\Delta\tau_k$ are intervals between points of calculation (consideration) of the system;

k is time interval index;

$\rho = \rho(t)$ is matrix of dimension $i \cdot j$ of transport links;

q_{ij} is transportation flow (flow intensity; transit capacity of a network element; carrying capacity of a flow of vehicles);

i, j are indices of nodes of the transport network, between which the value of q_{ij} is measured;

$\pi_\upsilon = \pi_\upsilon(t)$ is equation of the transportation process, expressing the share (degree) of completion of the trip (transportation);

υ is vehicle index.

Let us note that system (1) does not explicitly have a correspondence matrix, either a set of routes, etc. Instead, the matrix $\rho = \rho(t)$ can be used in conjunction with the flows q_{ij}. Efficiency criteria, equilibrium and optimization conditions, or other extreme conditions are not yet included in system (1), we will do it later.

Model (1) should, for each specific case, acquire a complete form that allows one to make calculations or solve an optimization problem. Therefore, for the selected system, these equations are supplemented with auxiliary conditions (distribution functions; balance equations; variational equalities, etc.). One of the options is to formulate system (1) in a quasi-dynamic setting and considering information about consumption of resources during the execution of transportation processes.

For linear consumption of resources:

$$G_{pq}(x) = \sum_{i,j}^{m,n} t_{q_{ij}} x_{p_{ij}} = g_{pq}, q \in \overline{1,r}, \quad (2)$$

where r is the number of types of resources;

g_{pq} is stock of resource of q-th type;

t_{qij} is parameter of consumption function;

ρ is route index;

q is index of resource type.

For non-linear consumption of resources the corresponding expression looks as follows:

$$G_{pq}(x) = \phi_{pq}\left(x_{p_{11}}, ..., x_{p_{ij}}, ..., x_{p_{mn}}\right) = g_{pq}. \quad (3)$$

Then the equations (1) can be written in a more general form depending on time (in the case of a single type of resource):

$$\begin{cases} \tilde{A} = \tilde{A}(t); \\ \rho = \rho(t), \quad \tau_0 \le t \le \tau_0 + \Delta\tau; \\ \Delta\tau = \sum_{k=1}^{l} \Delta\tau_k; \\ \Delta\tau_k \in \{\Delta\tau_1; \Delta\tau_2; ...; \Delta\tau_n\}; \\ q_{ij} = q_{ij}(t), \quad q(t) = \sum_{i,j}^{m,n} q_{ij}(t); \\ G_p(x) = \phi_p\left(x_{p_{11}}(t), ..., x_{p_{ij}}(t), ..., x_{p_{mn}}(t)\right) \le \mathbf{g}_p; \\ H\left(x^*(\Delta\tau_k)\right) \to \max; \\ k = 1, ..., l. \end{cases} \quad (1a)$$

Let us show the connection between $G_p(x, t)$ and $\pi_\upsilon = \pi_\upsilon(t)$. Let us introduce a vector $V(t) = (V_1, ..., V_a, ..., V_p)$, which groups all vehicles by routes: $V_a = \sum v(t) | v \in p_a$. For each vehicle there is an individual resource consumption function that depends on the value of the transportation process function. For simplicity, we will assume that all vehicles are homogeneous in type, and then we can assume that there is the one and the same function for all vehicles that depends on π_υ. Obviously, each component of the vector V can be associated with the value of the current resource consumption $G_p(x,t) \to V \otimes \overline{\pi_p}(t)$. Here $\overline{\pi_p}(t)$ is a vector of a share of a fulfilled transportation process, averaged for all vehicles on each route. The measure of compliance is the specific value of the individual resource consumption: $G_p(x,t) = g_{vp} \otimes V \otimes \overline{\pi_p}(t)$. Let us note that the components of the vector $g_{\upsilon p}$ depend on the route. The constraints g_p determine limit values for resource consumption that can be reached on routes during a cycle of the transport system operation. Of course, other ways of determining $G_p(x, t)$ through the functions of the transportation process can be chosen. This may be a particular topic for consideration.

Then (1a) will be written in a final form:

$$\begin{cases} \tilde{A} = \tilde{A}(t); \\ \rho = \rho(t), \quad \tau_0 \le t \le \tau_0 + \Delta\tau; \\ \Delta\tau = \sum_{k=1}^{l} \Delta\tau_k; \\ \Delta\tau_k \in \{\Delta\tau_1; \Delta\tau_2; \ldots; \Delta\tau_n\}; \\ q_{ij} = q_{ij}(t), \quad q(t) = \sum_{i,j}^{m,n} q_{ij}(t); \\ \pi_v = \pi_v(t), \quad v = 1, \ldots, n_v(t); \\ V(t) = \{V_1, \ldots, V_P : V_\alpha = \sum v(t) | v \in p_\alpha\}; \\ G_p(x,t) = g_{vp} \otimes V \otimes \eth_p(t) \le g_p; \\ H\left(V^*(\Delta\tau_k)\right) \to \max; \\ k = 1, \ldots, l. \end{cases} \quad (1b)$$

Obviously, formula (1b) is more general than (1a), since detailed information about transportation processes always allows us to determine the function of resource consumption. The converse is not always possible to perform unambiguously. The procedure for maximizing entropy is written here as an extreme condition, which depends on the specific type of states (phenomenological scheme of the macrosystem) adopted in the system. The Boltzmann distribution is usually used as an assumption, which, of course, requires a separate discussion in each case (see, for example [8, p. 77]). Of course, the extreme condition can be written in a different form, which does not limit the generality (1b).

Thus, the present formulation asserts the existence of a sequence of equilibrium states of a transport system (with Fermi; Einstein or Boltzmann states), defined on a generalized graph $\tilde{\Gamma} = \tilde{\Gamma}(t)$ and having transit or carrying capacity $q_{ij} = q_{ij}(t)$, resources that are consumed in accordance with $\pi_\upsilon = \pi_\upsilon(t)$, which determine the functions $G_p(x, t)$, and which are generally nonlinear. This formulation corresponds to the quasi-dynamic description of the system.

Conclusion. Generalization of models of type (1) can be developed towards dynamic theory of transport macrosystems, when the nonequilibrium dynamics of transport systems will be taken into account. It is worth noting that very often the hypothesis of equilibrium of transportation processes can be questioned. For example, during peak hours or during traffic incidents, non-equilibrium structures appear in the flow, which lead to significant differences between the results of modelling using predictive or optimization models and the real picture of distribution of flows.

The dynamic theory can be built based on the following basic provisions.

1) For the transport system model, the phase space «generalized coordinates – generalized impulses» is introduced. The dimension of the space depends on the conditions of specific problems. It is assumed that in the phase space there is a function of distribution of elements by «coordinates» and «impulses», which depends on time.

2) The existence of a generalized kinetic equation (equations of Boltzmann type) is established if validity of the thermodynamic approach is postulated for the macrosystem under consideration (the possibility of describing the macroscopic state of a system by several parameters consisting of many elements).

3) It is assumed that the macrosystem defined in this way passes through time evolution, which can be characterized by a change in the *degree of chaos* with the help of entropy of a nonequilibrium process. Entropy is expressed through the distribution function introduced above.

4) Changing the degree of chaos in the system is one of the basic concepts of the dynamic theory of transport systems, which is explicitly absent in the traditional theory of macrosystems. For externally closed systems, an analogue of the Boltzmann *H*-theorem is established, which states that during time evolution to an equilibrium state, entropy of the system increases and remains unchanged when it is reached.

5) For open transport macrosystems under conditions of continuous interaction with systems of a higher level, it is postulated that there is an analogue of the concept of «energy», possibility of changing the degree of chaos in the process of evolution and of the use of the apparatus of physics of open systems, in particular, of the *S*-theorem of Klimontovich [22].

6) To consider the dynamics in macrosystems, the concept of «active particle» is

introduced, and then there are possibilities of applying the results of the corresponding theory [23]. In particular, for transport systems, this makes it possible to more fully consider probabilistic features of behaviour of such particles, whose role can be assumed by any element in Table 1.

In conclusion, we will note that the above model (1) was tested for a section of the street-road network, considering daily dynamics of adjoining territories, as main centers for generating vehicles [24].

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