# WAVELET ANALYSIS IN VIBRO-ACOUSTIC CONTROL METHOD

**Kvashnin, Mikhail Ya.** – Ph.D. (Tech.), associate professor at the department of Transport construction, Kazakh Academy of Transport and Communications n. a. M. Tynyshpayev, Almaty, Kazakhstan.
**Kvashnin, Nikolai M.** – Ph.D. (Tech.), head of the department of LLC «EXPERT», Moscow, Russia.
**Bondar, Ivan S.** – external Ph.D. student at the department of Bridges and tunnels of Moscow State University of Railway Engineering (MIIT), Moscow, Russia.

## ABSTRACT

*The article shows advantages of continuous wavelet analysis in comparison with fast Fourier transform in the vibro-acoustic control of multi-layer constructions. Evidence of above mentioned advantages was obtained, in particular in laboratory conditions using a model in the form of a marble slab and supporting marble cubes. Wavelet spectrum makes it possible to investigate in detail all high-frequency components, which are located at the beginning of the timeline and quickly die out, while the amplitude of the oscillations of the slab significantly reduces on the dominant mode.*

## ENGLISH SUMMARY

**Background.** *Vibro-acoustic method of non-destructive control has found wide application in the detection of defects in binding of coverage and slabs both in mining and transport construction. It is based on the point shock excitation of a free surface of the object of research and the spectral analysis of vibro-acoustic response of the object to impact.*

*Traditionally, in data processing of vibro-acoustic method Fourier transform was used. However, in some cases, to assess the state of the object of control Fourier spectrum is not sufficient.This is due to the fact that the transform gives an idea of the proportion of the spectral components of the entire length of the signal without revealing their behavior in time. Consequently, it is not adapted in its traditional type for the analysis of nonstationary signals including signals, localized at a certain time interval. Therefore, spectral analysis of actual signals should be carried out in relation to both frequency and time. Advantages of such an analysis are obvious.*

*There is a tool that makes it possible to get a time-frequency representation of the signal. The problem is solved in the framework of wavelet analysis.*

**Objective.** *The objective of the authors is to show advantages of continuous wavelet analysis as a tool of nondestructive vibro-acoustic check in solving problems of transport construction over Fourier transform.*

**Methods.** *The authors use analytical method, comparison, descriptive method, mathematical analysis and special methods of vibro-acoustic control and wavelet analysis.*

**Results.** *The Fourier transform represents an analyzed signal as a linear combination of trigonometric functions which are restricted in frequency, but are not limited in time. The wavelet transform uses functions (wavelets), which are localized in frequency and time domains, and thus provides a two-dimensional representation of the test signal in the plane of frequency- position. It allows us to split large and small parts of signals, at the same time locating them on a timeline.*

*In the wavelet analysis two types can be distinguished: continuous and orthogonal. Continuous wavelet analysis as compared with the orthogonal is redundant in terms of signal reconstruction and requires a lot of computing capacity, but at the same time it gives more detailed information about the signal.*

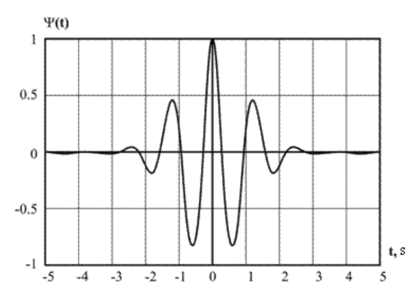
*Modern computers have sufficient capacity for fast continuous wavelet transform (hereinafter-CWT), so it is advisable to use it in the analysis of data obtained in the course of non-destructive control.*

*In CWT a signal is multiplied with the function (wavelet) and then the time integral of their product is taken. These operations are repeated for different values of the wavelet scale and its shift. The scale refers to a coefficient that determines how many times this wavelet is spaced out in time in relation to the original (mother) wavelet; the shift sets its position relative to the origin of time. Mathematically, CWT is determined as follows [1]:*

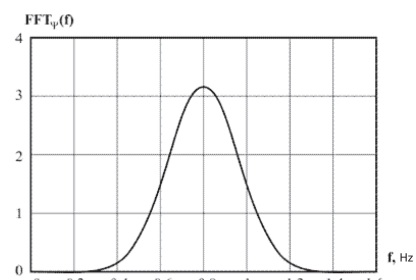
$$CWT(\tau,s)=\frac{1}{\sqrt{s}}\cdot\int x(t)\psi\left(\frac{t-\tau}{s}\right)dt, \qquad (1)$$

*where s – scale; t – time; τ – shift of a wavelet relative to the origin of time; function Y(t) – mother wavelet; component (1/s) is introduced for normalization in order that a signal, represented at different scales, has a constant energy.*

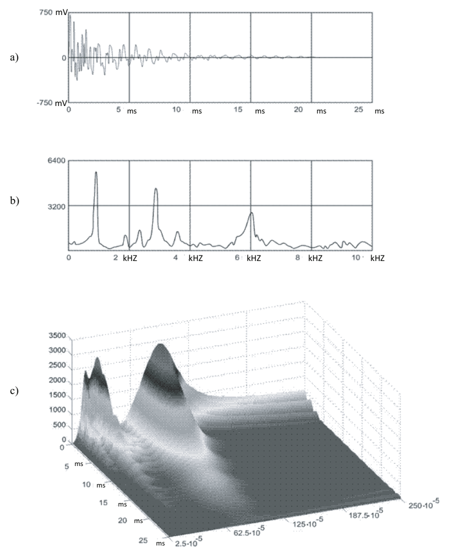
*Scale can also be called an analogue of frequency in Fourier transform, but with the difference that the smaller is the scale, the higher is the frequency. Mother wavelet is a function that is a prototype for functions used in the transform. All wavelets are formed by scaling and shifting of the mother wavelet.*



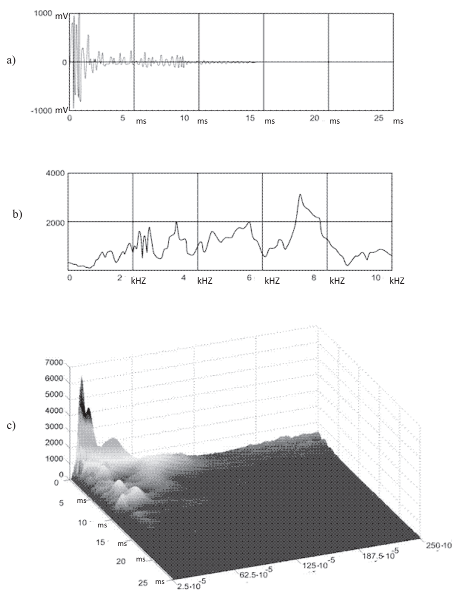

***Pic. 1. Graph of the function «Morlet wavelet».***




***Pic. 2. Spectrum of the function «Morlet wavelet».***




**Pic. 3. Results of processing of vibration impulse data received above the defect.**
**a – oscillogram; b – Fourier spectrum; c – wavelet spectrum.**




**Pic. 4. Results of processing of vibration impulse data received outside the defect.**
**a – oscillogram; b – Fourier spectrum; c – wavelet spectrum.**

*There are some functions that are used as mother wavelets. For data analysis of vibro-acoustic control method Morlet wavelet [2] was chosen with a frequency of 5. Analytical expression of this function is:*

$$\Psi(t) = e^{-\frac{t^2}{2}} \cdot \cos(5 \cdot t). \qquad (2)$$

*A graph of this function is shown in Pic. 1, and Fourier spectrum is in Pic. 2.*

*As it can be seen from the graph, the function is a damping cosine wave; a greater part of its energy is concentrated in the time interval from –4 to 4 seconds. Function spectrum is also localized, it is a singular peak, the maximum of which occurs at a frequency of 0,8 Hz. Here we can find a link between the scale of the wavelet and the frequency of the maximum of its spectrum:*

$$f = \frac{0{,}8}{s}. \qquad (3)$$

*From (3) it follows that scales and frequencies are related nonlinearly. This fact shows that carrying out a wavelet transform on a large scale ensures a good frequency resolution, but according to the Heisenberg principle, the temporal resolution will decrease.*

*In order to determine the effectiveness of the wavelet transform in the analysis of vibro-acoustic control data, a series of measurements was made in the laboratory conditions.*

*Measurements were carried out on the model used to optimize the vibro-acoustic control method [3]. The model is a marble slab 680 × 330 × 50 mm, rigidly connected at the edges with two continuous marble cubes 253 × 253 × 253 mm, so that in the center of the slab a defect of connection occurred in the form of cavity with horizontal dimensions 253 × 315 mm. Registration of vibration impulse was conducted with the use of piezoelectric accelerometer KD-32 (natural frequency is 10* kHz, the thickness oscillations of the piezoelectric ceramic).

One of the points of the sensor installation was located directly above the defect, and the second was located above one of the cubes. Surface of the plate was excited by impact force and signal was got from the output of the converter that was further digitized by ADC plate installed on the user's computer. The data were converted into a convenient format for the calculation and stored on the hard drive of the computer. At each point of the sensor installation ten identical impulses were performed.

Calculations were carried out in MATLAB. To conduct a continuous wavelet analysis, the built-in function «CWT» was used. Scales were taken from $2{,}5 \cdot 10^{-5}$ to $2{,}5 \cdot 10^{-3}$ in steps $2{,}5 \cdot 10^{-5}$, thus covering the frequency range from 320 Hz to 32 kHz. Absolute values of all elements of the resulting array were calculated. Then using those values the surface chart was constructed. In addition to the continuous wavelet transform fast Fourier transform (hereinafter-FFT) of the original signal was performed, the result of which was later also visualized.

Form of vibration impulses, recorded above the defect and outside the defect, and the results of their wavelet transform and the Fourier transform are shown in Pic. 3 and 4.

In the graph of the Fourier spectrum obtained above the defect, three local maxima (950 Hz, 2700 Hz, 6000 Hz) can be clearly revealed, corresponding to three bending modes of the slab. In the graph of the wavelet spectrum there are three formations, maxima of which are located within scales $8{,}4 \cdot 10^{-4}$, $3 \cdot 10^{-4}$, $1{,}3 \cdot 10^{-4}$, which correspond to frequencies 952 Hz, 2666 Hz, 6153 Hz. And their initial amplitudes are about the same, but with an increase in the oscillation frequency the attenuation coefficient increases, this is manifested in the Fourier spectrum in the form of different heights of the spectral spikes.

In case of measurements carried out outside the defect, as expected, on the Fourier spectrum such clear maxima, as in the measurements above the defect, are not detected and the shift of the spectrum to the high-frequency region is observed. However, origin of the remaining spectral spikes remains unclear. In the wavelet spectrum it can be seen that all high-frequency components are located at the beginning of the timeline, die out quickly and vibration amplitude on the dominant mode of the slab decreased significantly (approximately by 10 times).

**Conclusion.** On the basis of the described test research we can conclude that the wavelet transform, in the first place, more clearly describes the physical processes within vibro-acoustic method of control and, secondly, in case of the wavelet transform spectral spikes in the low-frequency region become more apparent, which is most typical for vibro-acoustic method of control.

Keywords: transport construction, multilayered structures, vibro-acoustic control, Fourier spectrum, Fourier transform, wavelet transform, wavelet analysis.

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Координаты авторов (contact information): Квашнин М. Я. (Kvashnin, M. Ya.) – kvashnin_mj55@mail.ru, Квашнин Н. М. (Kvashnin, N. M.) – kvashnin_nm@mail.ru, Бондарь И. С. (Bondar, I.S.) – ivan_sergeevich_08@mail.ru.

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