



# Interaction of Post Foundations and Frozen Soils



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## ABSTRACT

The frozen soils' stress-strain state varies with time due to the internal soil rheological processes. Those processes become active within the thawing period of the active soil layer causing increase in settlement of the engineering structures' foundations. Hence, creep processes and thawing of frozen soils should be considered when designing the transportation facilities for regions of the Far North and Siberia.

The objective of the research is to develop a procedure for evaluating the variation in time of the stress-strain state of the frozen soil under the post footing of a bridge pier's foundation considering the frozen soil creep and thawing.

The interaction of the bridge pier post foundations and frozen silt-loam soil is modelled and studied. The research is based on the example of an existing overpass over the M-56 Lena motor road situated at Amga-Samyrdah stage of Tommot-Yakutsk section of the Berkakit-Tommot-Yakutsk railway line. This overpass has piers with post

foundations. The above railway line is in the area of hard frozen soils.

The study focuses on changes in principal normal compressive stresses with the course of time, as well as on the frozen soil movements under the post footing. The time allotted for the above system behavior study is limited to five months. There are two design cases: a) considering the frozen soil thawing up to a depth range of 1,5 to 4 m; b) without considering the frozen soil thawing.

The research has shown that the thawing of the frozen soil up to a comparatively low depth as compared to natural level results in a significant increase (by 2÷2,5 times) in the values of post foundation settlement as compared with the design case without thawing. At the same time, it was found that small values of thawing have a subtle effect on the frozen soil's stress state under the post footing. Besides, all reviewed design cases (with / without thawing revealed that stress of the frozen soil under the post footing decreases with time (stress relaxation).

**Keywords:** transport construction, frozen soil's stress-strain state, rheological processes, creep, thawing, bridge pier, post foundation settlement, earth cover model, modulus of deformation.

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**Background.** It is well known that the frozen soils' stress-strain state varies with time due to the internal soil rheological processes [1; 2]. Those processes start during the period of thawing of the active soil layer causing increase in settlement of the engineering structures' foundations. Hence, creep processes and thawing of frozen soils should be considered when designing the transportation facilities for regions of the Far North and Siberia.

It is worth noticing that the results of investigation of the frozen soil creep are presented in [3–5].

Thus, John M. Ting [3] examined linear correlation observed between the logarithm of the minimum creep velocity and the logarithm of the time to reach this minimum creep velocity rate in soil, ice, and frozen soil.

The authors of [4] studied the damage during creep of the frozen soil and deduced a damage evolution equation, taking the variation of ice content into consideration.

The authors of [5] propose a rheological model for frozen soils by combining Maxwell, Kelvin, and Bingham models. The papers [6–8] review the frozen soil behavior during thawing period. The article [6] experimentally identifies the major factors in determining the thawing soil settlement behavior in an area. The article [7] is dedicated to the experimental studies of the freeze-thaw cycles' effect on the main mechanics properties of different soils. The article [8] suggests a method for calculating the thaw

depth of permafrost at the foundation of multi-layer pavements based on the analytical solution for transient heat conduction in the multi-layer medium. The articles [9; 10] cover various aspects of the use of finite element method to analyze frozen soils.

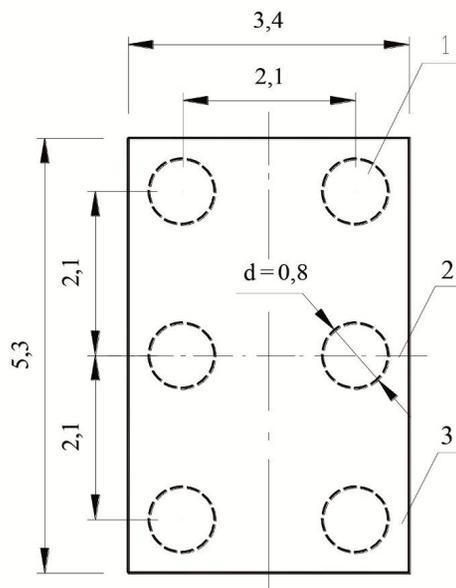
The present research has the *objective* to evaluate the changes in stresses and deformations occurring during the determined period within the frozen soil under the foundation of a bridge pier, considering its viscoelastic properties and ambient temperature fluctuations.

## Results

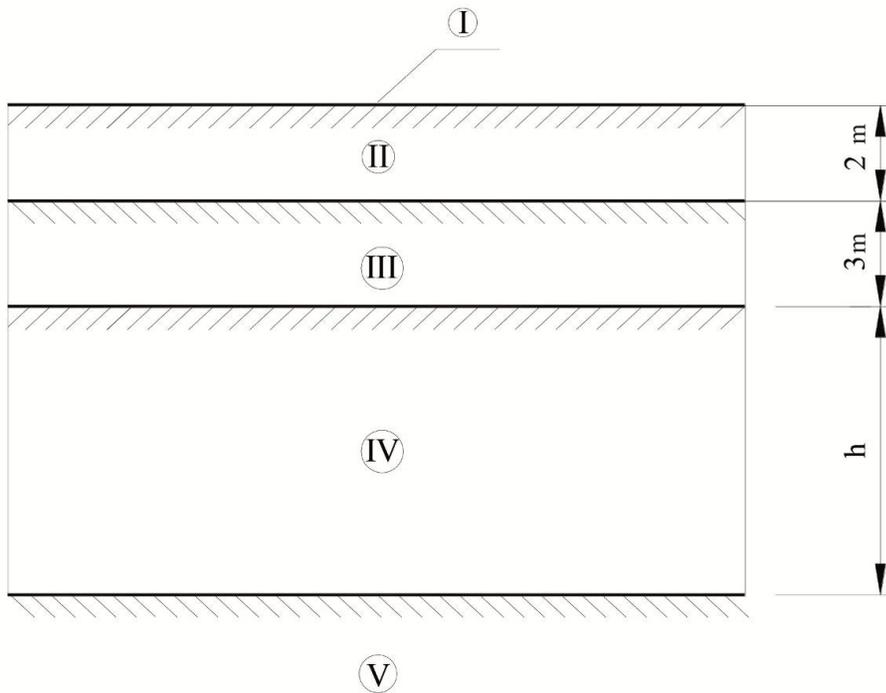
### Model Description and Justification of its Parameters

For studying the interaction of the bridge piers' post foundations and the soil, the research based on the data on the pier No. 2 of the overpass over the M-56 Lena motor road at the stake (station distance) PK4819+25 at Amga–Samyrdah stage of Tommot–Yakutsk section of the Berkakit–Tommot–Yakutsk railway line. However, the purpose of scrupulously following the initial data, linked to specific site, was not intended, because those data (especially regarding geologic properties) can change significantly when studying various sites.

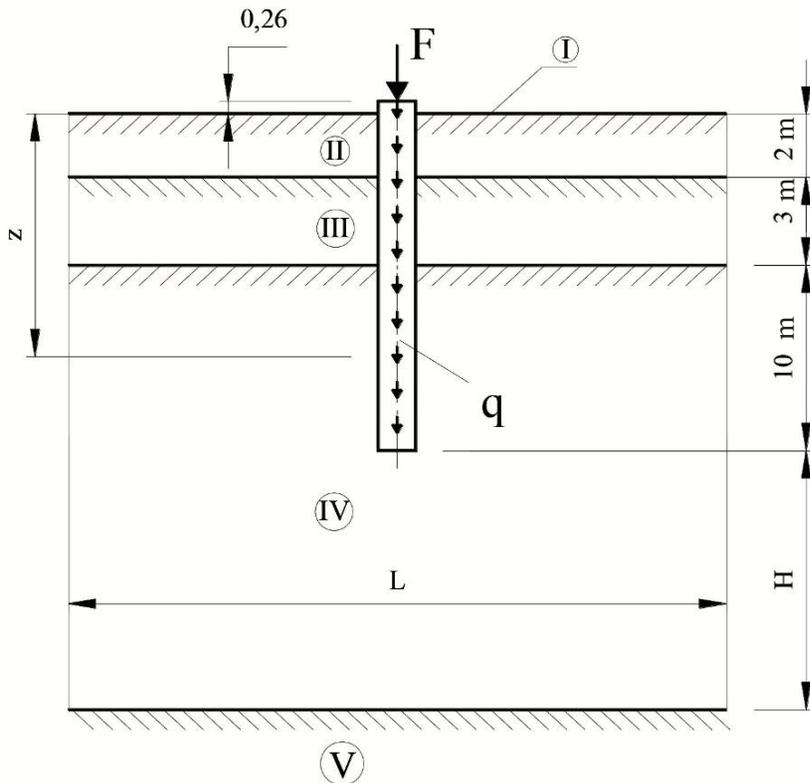
The steel reinforced concrete pier under the study has the post foundation. Six precast steel reinforced concrete posts ( $\varnothing 0,8$  m,  $l \approx 15$  m) are integrated by high cast-in-place grillage



**Pic. 1.** Location of the grillage and posts of pier No. 2: 1 – post, 2 – bridge center line, 3 – grillage.

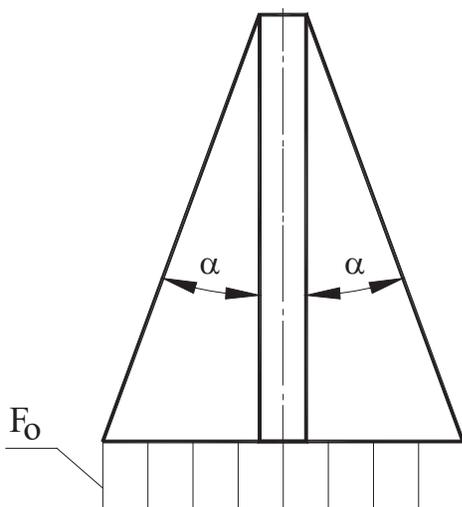


**Pic. 2.** The earth cover model (I – terrain, II – filled-up crushed stone hard-frozen soil containing up to 30 % of sandy loam, when thawing, the sandy loam is flowing, III – average-density and middle-sized hard-frozen sand, when thawing it is saturated with water, IV – hard-frozen sandy loam and clay loam strata, when thawing, they are flowing, V – incompressible stratum).



**Pic. 3.** Design model of the interaction of a post with the earth cover:  $H$  – depth of stratum IV under the post footing ( $H = h - 10$  m);  $L$  – soil mass width considered in the model;  $t$  – linear, standard, dead weight of the post;  $F$  – force due to the dead weight of the overlying overpass structures, transmitted to the pier.





**Pic. 4. Diagram of transmission of load to the plane of lower end of a post.**

(reinforced-concrete slab thickness is 1,65 m). Distance from the grillage footing up to the filled-up crushed stone soil is 26 cm. The posts' location in cross-section view is shown in Pic. 1 (dimensions are shown in meters).

Based on the available data, let us adopt the earth cover model as illustrated in Pic. 2.

Let us assume that hard-frozen silt-loam strata (sandy loam and clay loam) are spread to a sufficiently large depth: from a few dozen to several hundred meters. The above assumption is substantiated by the fact that the site under review is in the area between Tommot and Yakutsk, where the permafrost stratum depth is from 120 m to 250 m [1]. Let us also assume that there is a incompressible soil stratum below silt-loam strata.

As the hard-frozen soils are near-incompressible with E deformation modulus varying within the range of 300 MPa to 30000 MPa [1], and so deformation modulus is ten and hundred times higher than the deformation moduli of the non-frozen soils, then, when analyzing the stress-strain state of the post foundation interacting with the soil, it is possible to analyze behavior of a single post only. Then the following design model can be used (Pic. 3). Here, the Roman numerals mark the soil strata shown in Pic. 2.

Let us proceed from the assumption used in the hanging pile calculation [11], which considers that when the post is subjected to loadings the stressed soil body emerges around

the post; it is limited at the sides by the conic frustum or frustum of pyramid and from below by the convex curved surface (Pic. 4).

In calculations, the pressure in the plane of a post footing is taken as evenly distributed ( $F_0$ ), and the area of the reaction pressure profile is determined proceeding from the assumption that friction forces are transmitted to the post footing plane at an angle:

$$\alpha = \varphi_0/4, \tag{1}$$

where  $\varphi_0$  is the averaged value of the angle of internal friction.

Hence, it can be considered that the soil body thickness in the section between the terrain surface and the post footing shown in the design diagram in Pic. 3 (lateral dimension vertical to the model plane), varies following the law:

$$\delta(z) = d + 2 \cdot z \cdot \operatorname{tg} \alpha, \tag{2},$$

where  $z$  is the distance from the ground surface to the soil layer which thickness is studied;

$d$  is the post diameter.

The soil body thickness below the post footing is taken equal to:

$$\delta = d + 2 \cdot l \cdot \operatorname{tg} \alpha. \tag{3}$$

Here  $l \approx 15$  m is the post penetration depth.

The proper assessment of the frozen soil temperature is important when calculating the substructures and foundations for the frozen soils. The overpass certificate states the specified value of the average annual permafrost soils temperature as follows:  $t^{\circ}\text{C} = -1,7^{\circ}\text{C}$  (average annual temperature at the depth of 10 m). However, according to the research data there is a gradual temperature increase of the frozen soil with increasing the depth of measurement up to the values closed to  $0^{\circ}\text{C}$  [1]. In the suggested design model, the temperature of the soil stratum below the post footing is the most important. Accordingly, the frozen soil temperature is taken as  $\theta^{\circ}\text{C} = -0,2^{\circ}\text{C}$  (let us notice that as per Table 2 in Appendix 1 [12] the soil starting freezing temperature is equal to  $T_{\text{br}} = -0,1^{\circ}\text{C}$  for non-saline sandy loam and  $T_{\text{br}} = -0,2^{\circ}\text{C}$  for non-saline clay loam).

Another important issue is associated with a correct determination of the value of  $\varphi_0$ . The work [1] states that at temperature close to  $0^{\circ}\text{C}$  the angle of internal friction of the frozen soil is practically equal to the angle of internal friction of the non-frozen soil. Roughly, the value  $\varphi_0$  may be taken as equal to the angle of internal friction  $\varphi$  for silt-loam soils with yield value  $J_L = 0$  (let us notice that  $J_L < 0$  for hard-frozen silt-loam soils).

Table 1

Overpass properties

Description	Weight by volume, kN/m <sup>3</sup>	Weight per unit length, kN/m	Weight of one piece, kN	Volume, m <sup>3</sup>	Length, m	Total number	Weight, kN
Rails	–	1,275	–	–	22,09	–	28,2
Timber sleeper	–	–	0,69	–	–	67	46,2
Ballast	17,66	–	–	30,41	–	–	537,0
Metal bridge superstructures	–	24,45	–	–	13,8	–	337,4
Steel reinforced concrete bridge superstructures	24,525	–	–	18,1	–	–	443,9
Walkway slabs, floor blocks and bridge refuge blocks	24,525	–	–	1,47	–	–	36,0
Footway cantilever brackets, hand railings and gutters (metal)	–	–	–	–	–	–	35,5
Cast in-situ grillage	24,525			30			735,8
Precast pier body blocks	24,525			17,415			427,1
Cast-in-situ filling concrete	24,525			10,1			247,7
Cast-in-situ bent cap	24,525			12,2			299,2

As per the overpass certificate data, porosity ratio varies within the range of 0,47÷2,78 for hard-frozen sandy loam and clay loam. Let us consider this ratio as equal to 1,0. Then, according to the Table 2 data in Appendix 1 [13] the value of  $\varphi_0$  is taken as  $\varphi_0 \approx 20^\circ$ . It should be noted that the angle of internal friction  $\varphi_0$  decreases when the porosity factor value increases, that causes decrease of the post load carrying force.

Let us also assume that modulus of deformation  $E$  and Poisson ratio  $\mu$  values are the same for all soil strata.  $E$  is determined by the following formula [1] used for the frozen silt-loam soils:

$$E = \gamma + \beta \cdot |\theta|, \tag{4}$$

where  $\theta = -0,2^\circ\text{C}$ ;  $\gamma = 392,4 \text{ MPa}$ ;  $\beta = 1373,4 \text{ MPa}$  (when compression  $\sigma \approx 0,2 \text{ MPa}$ ). Then,  $E = 667,08 \text{ MPa}$ .

According to the test data [1], the Poisson ratio  $\mu$  varies within the range of 0,13÷0,35 for the frozen silt-loam soils. The value of  $\mu$  is so taken as  $\mu = 0,3$ .

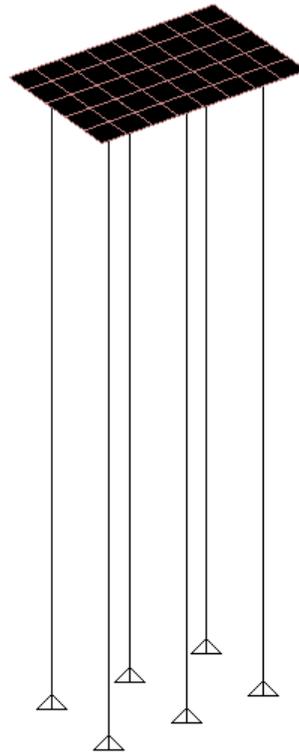
To take into consideration the creep processes in the frozen silt-loam soils, let us use a relation associating the relative axial deformation  $\varepsilon$  and normal stress  $\sigma$  stated in [14]:

$$\varepsilon(t) = [(\sigma \cdot t^\lambda) / \xi]^m, \tag{5}$$

where

$$\xi = \omega \cdot (|\theta| + 1)^k. \tag{6}$$

Here  $t$  means time. According to [14] for the frozen sandy loam it may be taken as follows:



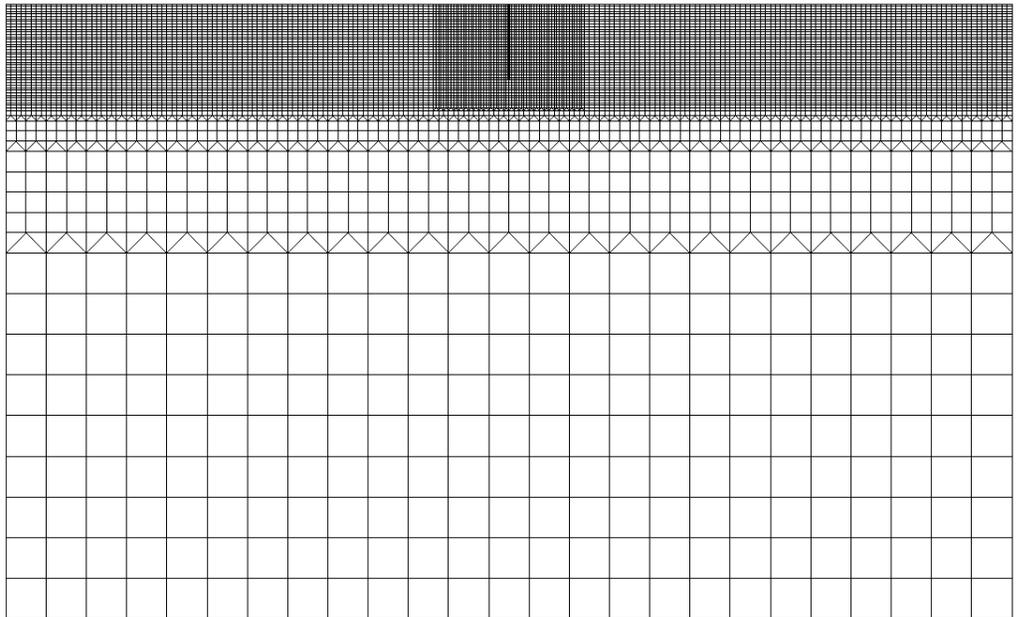
Pic. 5. A spatial model of the post foundation.

$$\lambda = 0,1; m = 3,704; \omega = 9; k = 0,89.$$

Then, at  $\theta = -0,2^\circ\text{C}$  we shall obtain  $\xi = 10,59 \text{ (kg/cm}^2\text{)} \cdot \text{hour}^{0,1} = 0,7557 \text{ MPa} \cdot \text{day}^{0,1}$ .

Summary of the specified load from the overpass dead weight is stated in Table 1.





Pic. 6. Finite element model illustrating the interaction of the post and the earth cover.

Table 2

Dependency of the stress-strain behavior factors of the soil under the post footing on H and L parameters (see Pics. 3, 6).

Parameter L, m	Parameter H, m	Post footing vertical movement Δ, mm	Principal compressive stress σ <sub>2</sub> of the soil under the post footing, MPa
200	106	0,6142	0,03510
200	170	0,7320	0,03505
200	226	0,8250	0,03505
312	106	0,6061	0,03508
408	106	0,6050	0,03508

When summing up the last column values stated in Table 1, we get  $Q = 3174$  kN (specified weight transferred to the grillage of the pier No. 2 considering the grillage weight).

Let us find force  $F$  (Pic. 3) transmitted to a post. To do this, it is necessary to solve a spatial problem. Pic. 5 shows a spatial finite element model of the post foundation (the MSC NASTRAN software was used). The post footings are fixed-ended. On the top the posts are rigidly joined by a grillage slab. The grillage slab is uniformly loaded:  $q_0 = Q/A = 3174 \text{ kN}/18,02 \text{ m}^2 = 176,1 \text{ kN/m}^2$ . Here  $A = 5,3 \text{ m} \cdot 3,4 \text{ m} = 18,02 \text{ m}^2$  is the horizontal surface area of the grillage slab. As a result of the calculation, the most loaded posts are in the middle row. The force of 546,4 kN is transmitted to each of them, while the force applied to each of the outer posts is of 520,1 kN. Thus,  $F = 546,4$  kN.

Load  $q$  (dead weight per meter of a post of  $\varnothing 0,8$  m) is determined as follows:

$$q = \rho \cdot V/L = 24,525 \text{ kN/m}^3 \cdot 7,5417 \text{ m}^3/15 \text{ m} = 12,33 \text{ kN/m}$$

where  $\rho$  is the weight by volume;  $V$  is the volume;  $L$  is the post length.

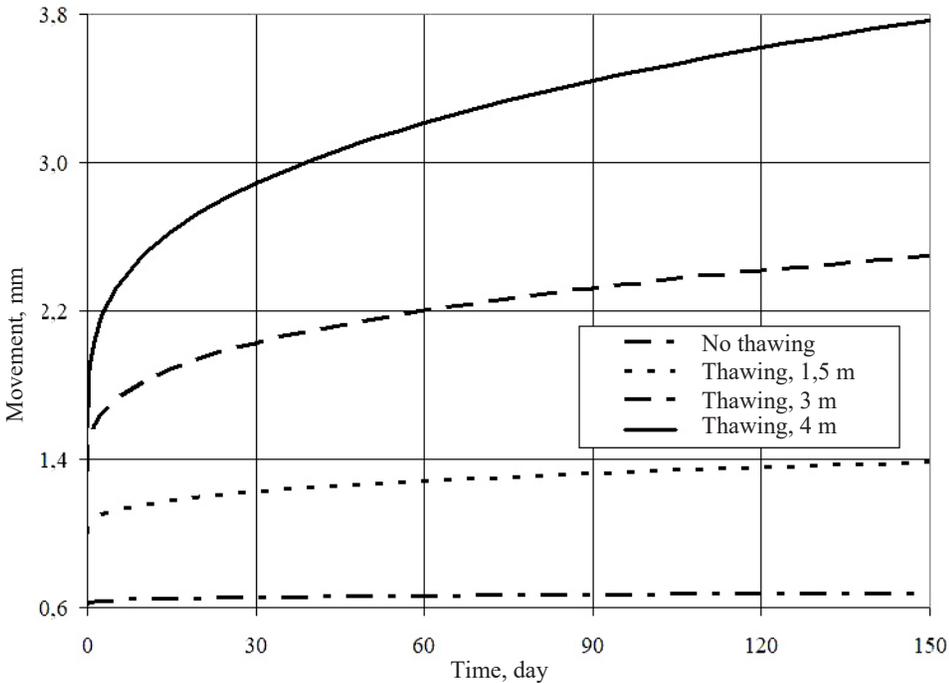
Let us analyze the effect of the design parameters on the assessment of the stress-strain behavior under the post footing (Pic. 3):  $L$  (earth cover width) and  $H$  (depth of the earth cover under the post footing). Let us analyze it using the finite element model (Pic. 6).

The model consists of 11071 finite elements and 10923 nodes. Near the post there is the finite element mesh clustering (15 m left and right from the post axis and 20 m deep from the terrain surface). Beyond this area the finite element mesh becomes rarer. The post is simulated with rod-type elements, the soil is simulated using the plane triangular and quadrilateral elements,

Table 3

Variation in time of parameters  $\Delta$  and  $\sigma_2$

Time, day	The post footing vertical movement $\Delta$ , mm (the principal compressive stress $\sigma_2$ of the soil under the post footing, MPa) at various values of thaw penetration depth of S			
	a) S = 0	b) S = 1,5 m	c) S = 3 m	d) S = 3 m
0	0,6142 (0,0351)	0,9942 (0,0630)	1,3341 (0,0900)	1,5749 (0,1109)
30	0,6487 (0,0302)	1,2196 (0,0497)	2,0239 (0,0674)	2,8895 (0,0795)
60	0,6572 (0,0297)	1,2774 (0,0491)	2,1969 (0,0668)	3,2136 (0,0784)
90	0,6631 (0,0294)	1,3179 (0,0488)	2,3187 (0,0659)	3,4401 (0,0776)
120	0,6679 (0,0292)	1,3502 (0,0486)	2,4151 (0,0655)	3,6187 (0,0770)
150	0,6718 (0,0291)	1,3774 (0,0484)	2,4961 (0,0654)	3,7683 (0,0765)



Pic. 7. Post settlement vs time curves.

thickness of which is determined using the ratio (2) from the terrain surface to the post footing and using the ratio (3) below the post footing. The model lower end is provided with rigid fixing.

Concentrated force  $F$  and distributed load  $q$  (Pic. 3) are conventionally not shown. This model was used both for calculating at the elastic stage and for calculating at the soil creep and thawing stages. Let us notice that when calculating thawing stage, the number of the finite elements and nodes decreased because of removing the thawed layers from the model.

Table 2 shows the values of the post footing vertical movement  $\Delta$  and the principal normal compressive stress  $\sigma_2$  of the soil under the post footing. Those values are obtained at  $L = 200$  m

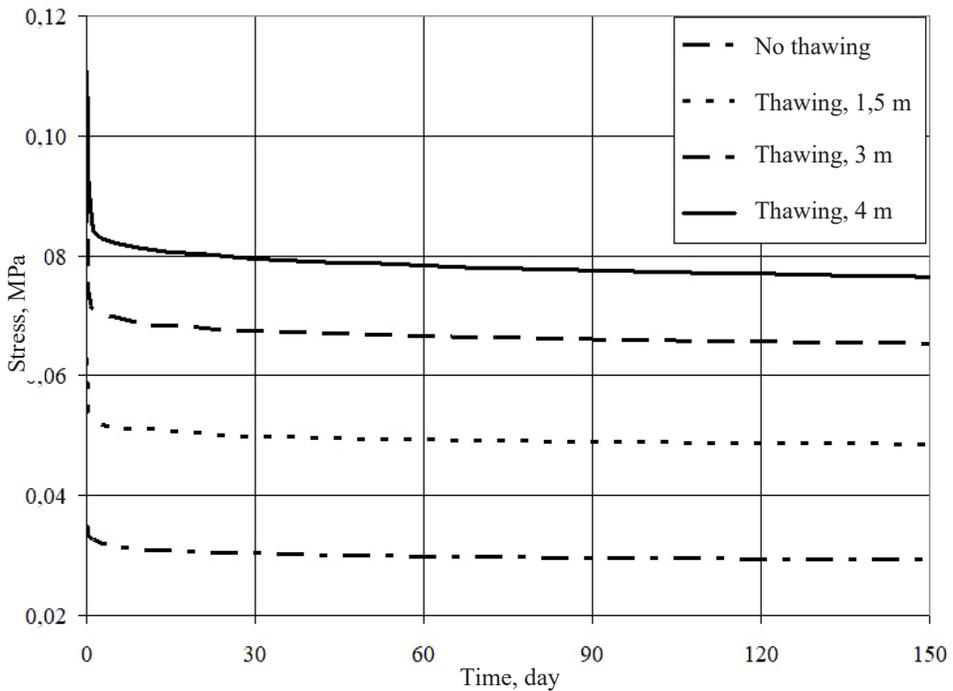
and for various values of  $H$  (here and elsewhere  $\sigma_2$  is indicated in absolute values). It is seen that parameter  $\Delta$  increases with the increase of the  $H$  value. At the same time, the soil stress state under the post footing changes slowly.

Besides, Table 2 shows also values of  $\Delta$  and  $\sigma_2$  obtained at  $H = 106$  m and for various values of  $L$ . From the above data, it follows that increase of  $L$  has little effect on the stress-strain behavior under the post footing. Let us take  $L = 200$  m;  $H = 106$  m. The so assumed value of the parameter  $H$  can be justified by the above mentioned depth of the permafrost soil in the region concerned ( $120 \div 250$  m).

**Results of Creep and Thaw Calculation**

Study of the variations in time of the stress-strain behavior of the model shown in Pics. 3





Pic. 8. Principal soil compressive stress under post footing curves.

and 6 was conducted within the time interval [0; 150 days] for the following loadings:

- a) The effect of the specified dead load of the part of the structure transferred to the post.
- b) The effect of the specified dead load of the part of the structure transferred to the post supplemented with the effect of the negative frictional load of the thawing soil applied to the side surface of the post.

For calculations of the above item b), the value of the linear negative frictional load of the thawing soil's *i*-layer applied to the side surface of the post is determined according to paragraph 4,38 [12] as follows:

$$f_{neg,i} = u_p \cdot f_{n,i} \tag{7}$$

where  $u_p$  is a cross-sectional perimeter of the post, m;

$f_{n,i}$  is a negative friction of the thawing soil's *i*-layer applied to the side surface of the post, kPa (it is allowed to look up for this value in Table 2 [15]).

It was assumed that  $u_p = \pi \cdot d = 3,14$  m, where  $d = 1$  m is a diameter of the well filled with sand-cement mortar and in which the post is immersed.

The thaw calculations do not consider the negative force applied to the side surface of

the post from the side of the filled-up crushed stone soil (layer II in Pic. 3), and the thaw depth *S* is measured from the natural terrain surface. In this case, the design model is refined as follows: the thawed soil layer is replaced with the corresponding negative frictional force  $f_{neg,i}$ . Removing the thawed layer entailed changes in the stressed soil body formed around the post under loading, that is, the indicated soil body was limited by a horizontal plane separating the thawed soil from the frozen soil as well as by a horizontal plane getting through the post footing. When determining the soil strata thicknesses (refer to (2)), the distance *z* (Pic. 3) was also measured from the thawed and unthawed soil strata boundaries and parameter *l* in relation (3) was taken as follows:  $l = 15 - 2 - S$ .

The following three thawing cases have been considered:

- 1) *S* = 1,5 m;
- 2) *S* = 3 m;
- 3) *S* = 4 m;

The definition of the negative frictional forces for the above cases is shown below:

- 1) Thawing of the layer III (Pic. 3) to *S* = 1,5 m. For medium coarse and medium density

sand at average thaw penetration of 0,75 m according to Table 2 [15] we get  $f_{n,2} \approx 35$  kPa;  $f_{neg,2} = 35$  kPa  $\cdot$  3,14 m = 109,9 kN/m.

2) Thawing of the layer III to  $S = 3$  m. The average depth of thawed layer is equal to 1,5 m;  $f_{n,2} \approx 38,5$  kPa;  $f_{neg,2} = 38,5$  kPa  $\cdot$  3,14 m = 120,89 kN/m.

3) Thawing of the layer III to 3 m complemented by thawing of the layer IV to 1 m ( $S = 4$  m). The average depth of thawed layer III is equal to 2 m;  $f_{n,2} \approx 42$  kPa;  $f_{neg,2} = 42$  kPa  $\cdot$  3,14 m = 131,88 kN/m. The average depth of thawed layer IV (silt-loam stratum) is equal to 3,5 m;  $f_{n,3} \approx 5$  kPa;  $f_{neg,3} = 5$  kPa  $\cdot$  3,14 m = 15,70 kN/m.

Variation in time of the stress-strain behavior under the post footing caused by soil creep was studied for four design cases:

a) Thawing is absent ( $S = 0$ );

b) Thawing case (1) (refer to point 1 above) at thaw penetration depth of  $S = 1,5$  m;

c) Thawing case (2) at thaw penetration depth of  $S = 3$  m;

d) Thawing case (3) at average thaw penetration  $S = 4$  m.

Table 3 shows the values of the vertical post footing movement  $\Delta$  (post settlement) and the principal normal compressive stress  $\sigma_2$  of the soil under the post footing respectively obtained for all the above design cases at varying times elapsed after loadings. Term curves of the post settlement  $\Delta$  and of parameter  $\sigma_2$  are shown in Pics. 7, 8.

## Conclusion

It follows from the above described results that while there is no thaw or it exists at relatively shallow depths, the post foundation settlement caused by the overpass dead load is insignificant ( $\leq 1$  mm). Besides, the increase in foundation settlement caused by the soil creep is also insignificant within the specified time interval. However, an increase in the thaw penetration up to 3÷4 meters from the terrain level results in a more significant settlement (1,3÷1,6 mm). Considering the evolving creep of the frozen soil, this settlement can reach 2,5÷3,8 mm in 120÷150 days after thawing, that is, it can grow by about 2÷2.5 times. At the same time, change of the frozen soil stress state when the post is subjected to loadings with  $F$  and  $q$  loads is insignificant even if the thaw penetration depth  $S$  is substantial ( $\sigma_2 \leq 0,111$  MPa). Over time, the principal compressive stress  $\sigma_2$

decreases for all the above design cases, that is, a stress relaxation occurs.

So, the conducted research may result in the conclusion that the thawing process developing during comparatively short periods (several months) can considerably affect the strained state of the frozen soil under the foundation footing but have low effect on its stress state.

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