DATA PROCESSING CENTER IN TRANSIENT MODE

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ABSTRACT

The article provides an investigation of transient (non-stationary) mode of data processing center (hereinafter- DPC). The center, subject to certain assumptions and conditions, is considered as a queuing system of M/M/1/∞ type. Numerical integration of Kolmogorov equations shows that the stationary mode is reached in 1,5 seconds after the start of operation.

ENGLISH SUMMARY

Background. *Data Processing Center (DPC), working as part of a complex system – a corporate network, with an assumption that random streams of requests are exponentially distributed between the receipt of the request, as well as the exponential distribution of processing time, can be represented as independently functioning queuing system (hereinafter- QS) of M/M/1/∞ type.*

Objective. *The objective of the author is to investigate transient mode of data processing center, which is considered as a queuing system of M/M/1/∞ type.*

Methods. *The author uses mathematical method and analysis.*

Results. *Conditional M/M/1/∞ is a Poisson input stream, exponential service time distribution, one servicer; the system itself is open (disconnected). Then change of states of QS should be represented as a graph shown in Pic. 1 (S_i, i=0,1,2,3, ... is QS state, when there are i requests in the system).*

In Pic. 1 symbol λ denotes the intensity of the flow of events – receipt of requests for processing, in which the durations of intervals between times of the events have an exponential distribution with probability density function:

$$f(x) = \lambda\, e^{-\lambda x}, \qquad (1)$$

where λ is a parameter of stream distribution. Average interval of the input stream is 1/λ.

Symbol μ denotes the intensity of the service time distribution. The intensity has the probability density function:

$$f(x) = \mu\, e^{-\mu x}, \qquad (2)$$

where μ is distribution parameter (service rate). Average service time is $1/\mu_i$.

In transient (non-stationary) mode queuing system is described by a system of differential equations of Kolmogorov [1, 2]:

$$\frac{dp_0(t)}{dt} = -\lambda\, p_0(t) + \mu\, p_1(t)\,;$$

$$\frac{dp_i(t)}{dt} = -\lambda\, p_i(t) - \mu\, p_i(t) + \mu\, p_{i+1}(t) + \lambda\, p_{i+1}(t),\quad i = 1,2,3... \qquad (3)$$

Here pi (t) is the probability that QS is in state Si at time t.

When $t \to \infty$ left sides of Kolmogorov equation tend to 0, in other words, in the steady (stationary) mode, when λ/μ<1, the system of differential equations is transformed into a system of algebraic equations:

$$0 = -\lambda p_0 + \mu p_1;$$

$$0 = -\lambda p_i - \mu p_i + \mu p_{i+1} + \lambda p_{i+1},\ i=1,2,3... \qquad (4)$$

where p_i is stationary probability that QS is in state S_i.

There is [1] solution of the system of equations (4):

$$p_0 = (1 - \lambda/\mu);$$

$$p_i = p_0 (\lambda/\mu)^i,\ i>0 \qquad (5)$$

When solving a system of differential equations (3) the probabilities $p_i(t)$ when $t \to \infty$ should tend to p_i, i = 0,1,2,3,..., which may serve as a test of correctness of the result.

Time during which the derivatives of the probabilities of the states are set to 0, is a time during which DPC is in a transient state. Therefore, solving the system of equations (1) and building graphs of state probabilities change as a function of time, we answer the question, how long after DPC turns on, a stationary regime is set.

The system of equations (3) can be solved numerically, using one of the known methods for the numerical solution of systems of differential equations. The most proven method is Runge-Kutta method of the 4th order [3, 4]. We perform the calculation for average values of time, i. e. time intervals in the input stream and service time taken from real measurements in a corporate network of JSC «Russian Railways». We take 1 / λ = 0,504 s, 1 / μ = 0,03705 s. The results of solving a system of differential Kolmogorov equations for these values of λ and μ are shown in Pic. 2 (only functions $p_0(t)$ and $p_1(t)$ are indicated – in order to save space). The probability of lack of requests in the system p_0 for steady mode is calculated using the formula, taken from [1]:

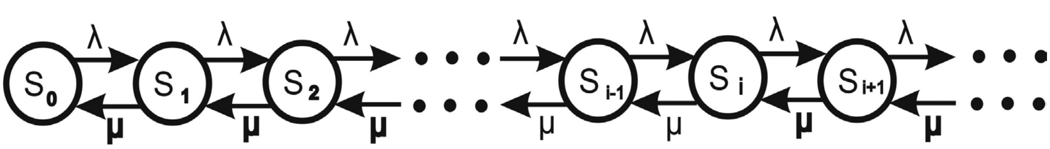
$$p_0 = 1 - \lambda/\mu. \qquad (6)$$

For the assumed values of λ and μ probability is 0, 9264881. As can be seen from Pic. 2, curve $p_0(t)$ with increasing time values tends to this value.

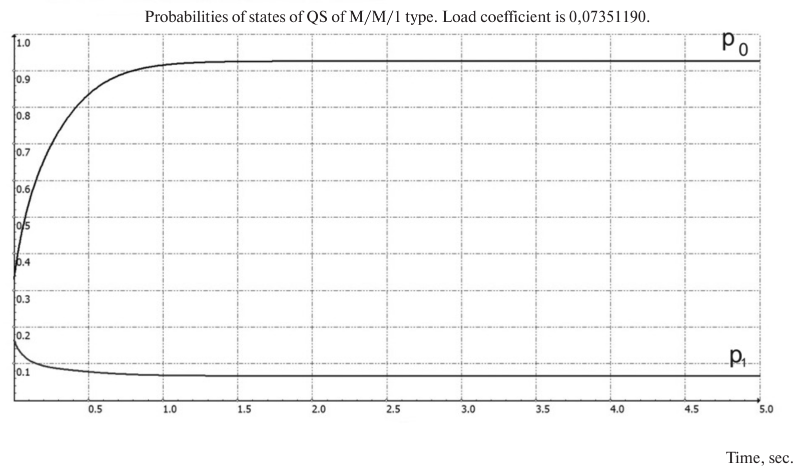
Pic. 3 shows the values of the average number of requests in the system. Average number of requests in DPC $\overline{n}(t)$ was calculated using the formula for the mean of a discrete random variable:

$$\overline{n}(t) = \sum_{i=1}^{\infty} i p_i(t)\,. \qquad (7)$$

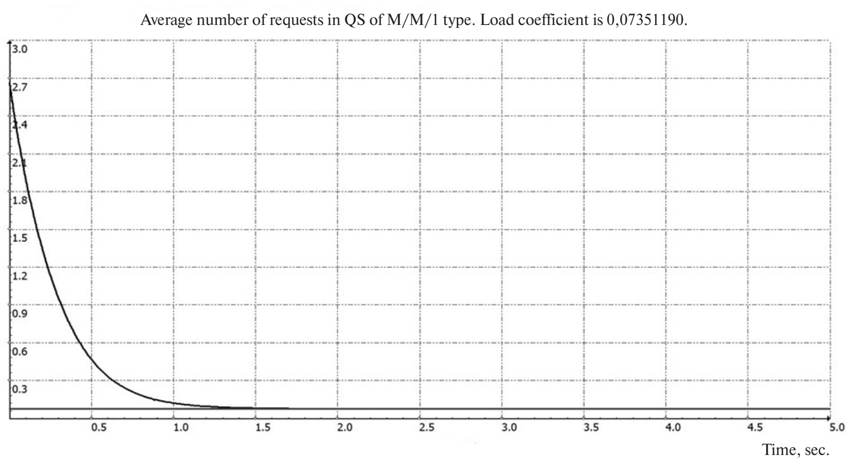
In the steady (stationary) mode of operation, the average number of requests in the system is calculated by the formula [1]:



Pic. 1. Graph of states of QS of M/M/1/∞ type.



Pic. 2. Probabilities p0 (t) and p1 (t) as a time function.



Pic. 3. Average number of requests as a time function.

$$\overline{n} = \frac{\lambda/\mu}{1-\lambda/\mu} = \frac{0{,}03705}{0{,}96295} = 0{,}0385 \,. \qquad (8)$$

In Pic. 3 line parallel to the axis t, is the steady value of the average number of requests in the system, equal to 0,0385, which a time function $\overline{n}(t)$ *asymptotically tends to.*

The results given in Pic. 2 and 3 show that the transition process in DPC is almost completed in 1,5 seconds after the start of integration (DPC turning on).

Conclusions. *By proceeding with numerical integration of equations (3) under different start conditions, input flow parameters, and service time it is possible to obtain assessment of the time of achievement by DPC of stable oepration mode and of mean values of requests in the system. This allows calculating necessary parameters of devices of storage of requests which are queuing or processed.*

Keywords: *queuing theory, data processing center, corporate network, request, input stream, integration, o system of Kolmogorov equations.*

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