TRANSVERSE OSCILLATION OF A BASE SLAB SECTION OF BALLASTLESS TRACK

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ABSTRACT

The study is devoted to construction of a mathematical model of a slab of a ballastless base of a railway track, assembled from separate segments fastened together. Each rectangular segment is modeled by a transversely isotropic pre-stressed plate. An equation has been obtained that allows researchers to study the natural oscillation frequencies of a rectangular plate, which is especially important from the point of view of choosing a final design of a ballastless track for high-speed main lines. An important issue solved is damping of supporting structures and separation of frequencies of natural and forced oscillations over different intervals of the frequency spectrum. To solve the set tasks, the decomposition method was used, its efectiveness and relative simplicity were proved.

<u>Keywords</u>: railway, high-speed main line, ballastless track, frequency spectrum, mathematical model, Lame constants, decomposition method, integral and differential operators, viscoelasticity.

Background. *One of the most promising areas for development of railway transport is associated with high-speed traffic. In the world practice of building high-speed rail for speeds of up to 350 km/h, a block-shaped ballastless base prevails. It can provide for many types of structures: a rail can rest on separate supports or lie on a solid foundation, plates can be prefabricated or monolithic, a rail is held by fasteners or fixed in a plate chute. Despite the relatively high initial cost – twice as expensive as a rail-sleeper track, ballastless structures imply low maintenance costs. In addition, international experience in laying, research and operation of a ballastless track has shown its reliability and expected service life of up to 60 years.*

Objective. *The objective of the authors is to consider transverse oscillations of a slab section at a ballastless track base, to determine the normal displacements of the specified points of the structure in order to provide a model of reinforced concrete foundation for various engineering and geological factors.*

Methods. *The authors use general scientific and engineering methods, comparative analysis, evaluation approach, mathematical methods, specific methods like Maxwell viscoelastic model, differential equations, decomposition method.*

Results.

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The construction of Moscow–Kazan HSR with an extension to Yekaterinburg was declared to be a pilot project of HSR in Russia. For conditions of high speeds up to 400 km/h, work in a cold climate zone with temperature differences up to 100˚C, mixed traffic with different loads from the wheel set on rails, and taking into account safety, reliability, strength, maintainability and adaptability, the design of a ballastless track of CRTS III RUS type (the newest Chinese development of Er Yuan) was adopted: a slab of self-compacting concrete, the composition of which was adapted to the Russian conditions of operation. The element-wise circuit design is shown in Pic. 1.

Further development of high-speed traffic involves the use of design of a ballastless track, the development of which implies a thorough calculation of properties of the material of a subrail base. The problem is a live question for both Russia and Kazakhstan.

In this study a slab of a ballastless base of a railway track is considered as a set of interconnected individual segments of reinforced concrete. A typical rectangular plate segment modeled by a transversely isotropic plate prestressed in one of the directions is studied in more detail, as it corresponds to the most demanded typical structures of a ballastless track [1, p. 62; 2, p. 74].

An important aspect is the fact that the plate material has rheological properties described by the Maxwell viscoelastic model. This option allows to choose mechanical properties so that, on the one hand, to avoid internal resonance, and on the other hand, to obtain the characteristic values of normal movements, which are within the standard interval depending on the speed of rolling stock and the load on an individual wheel set [3, p. 5; 4, p. 792].

The object of our attention is a transversely isotropic pre-stressed plate lying on a deformable base, with dimensions in the plan $-l_1 \le x \le l_1$; $-l_2 \le y \le l_2$ made from a viscoelastic material. Let's suppose that the external forces are zero, the initial conditions are non-zero. In this case, there are free vibrations of a plate lying on an elastic base, and they are described by the equation [4, p. 793] for $W^{(1)}$, the right side of which must be set equal to zero.

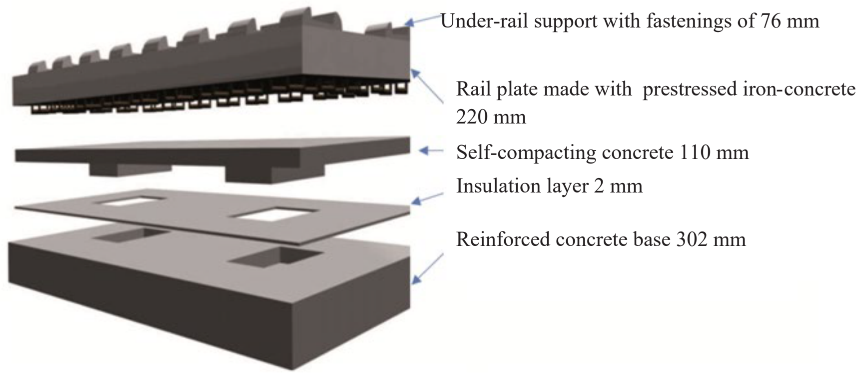
The plate material, as already noted, is viscoelastic and satisfies the Maxwell model:

$$\tilde{A}_{ij} = \tilde{a}_{ij}\left[\zeta(t) - \frac{1}{\tau}\int_0^t e^{\frac{-t-\zeta}{\tau}} \zeta(\xi)\, d\xi\right],$$

where τ – relaxation time.

Pre-dividing both sides of the equation [5, p. 15] by $(1+\alpha_0)$, we get

$$\rho_1\left(\frac{\partial^2 W^{(1)}}{\partial t^2} + \frac{1}{\tau}\frac{\partial W^{(1)}}{\partial t}\right) + \frac{h^2}{6}\left[A_1'\left(\frac{\partial^4}{\partial t^4} + \frac{2}{\tau}\frac{\partial^3}{dt^3} + \frac{1}{\tau}\frac{\partial^2}{\partial t^2}\right)W^{(1)} + A_2'\left(\frac{\partial^2}{\partial t^2} + \frac{1}{\tau}\cdot\frac{\partial}{\partial t}\right)\cdot \Delta W^{(1)} + A_3'\Delta^2 W^{(1)}\right] + \frac{s}{2h}\left\{\left(\frac{\partial W^{(1)}}{\partial t} + \frac{1}{\tau}W^{(1)}\right) + \frac{h^2}{2}\left[(1+c_2)^{-1}A_{44}^{-1} + 3(1+a_0)^{-1}A_{33}^{-1}\cdot\left(\frac{\partial^3 W^{(1)}}{\partial t^3} + \frac{2}{\tau}\frac{\partial^2 W^{(1)}}{\partial t^2} + \frac{1}{\tau}\frac{\partial W^{(1)}}{\partial t}\right) - 4(1+c_2)^{-1}A_{11}A_{33}^{-1}\left(\frac{\partial}{\partial t} + \frac{1}{\tau}\right)\Delta W^{(1)}\right]\right\} = 0, \quad (1)$$



Pic. 1. Adopted design of a ballastless track superstructure at HSR Moscow–Kazan, CRTS III RUS.

where $A_1^{-1}=\rho_1^2\left[(1+c_2)^{-1}A_{11}A_{33}^{-1}+3(1+a_0)^{-1}A_{44}^{-1}\right]$;

$$A_2^{\cdot}=\rho_1\begin{bmatrix}3\left(A_{13}^2-A_{11}A_{33}\right)A_{33}^{-1}A_{44}^{-1}+\\+A_{13}A_{33}^{-1}-2(1+c_2)(1+a_0)^{-1}\end{bmatrix}; \qquad (2)$$

$$A_3'=2\left[(1+c_2)\left(A_{11}A_{33}-A_{13}^2\right)A_{13}^{-1}\right].$$

A rectangular plate is pivotally supported along the contour and in this case the boundary conditions [6, pp. 84, 164] for the displacement $W^{(1)}$ *get the form:*

$$W^{(1)}=0;\quad \frac{\partial^2W^{(1)}}{\partial x^2}=0 \text{ at } x=-l_1;\ x=l_1;$$

$$W^{(1)}=0;\quad \frac{\partial^2W^{(1)}}{\partial y^2}=0 \text{ at } y=-l_2;\ y=l_2, \qquad (3)$$

and initial conditions:

$$W^{(1)}=\varphi_1(x,y);\ \frac{\partial W^{(1)}}{\partial t}=\varphi_2(x,y);\ \frac{\partial^3W^{(1)}}{\partial t^3}=\frac{\partial^2W^{(1)}}{\partial t^2}=0$$

at t = 0. (4)

Moreover, the functions ϕ_1, ϕ_2 *must also satisfy conditions (3).*

The solution of equation (1), satisfying the boundary conditions (3), is sought in the form of an infinite double series [7, p. 155; 8, p. 317]:

$$W^{(1)}=\sum_{n=0}^{\infty}\sum_{n=0}^{\infty}W_{n.m}(t)\cos\frac{(2n+1)\pi x}{2l_1}\cos\frac{(2m+1)\pi y}{2l_2}. \qquad (5)$$

Substituting (5) into (1), for $W_{n,m}$ *we obtain an ordinary differential equation:*

$$W_{n.m}^{V}+B_3W_{n.m}^{'''}+B_2W_{n.m}^{''}+B_1W'+B_0W_{n.m}=0, \qquad (6)$$

where Bj (j = 0...3):

$$B_3=\frac{\tau}{2}+\frac{3s}{h_1^3}\begin{bmatrix}(1+c_2)^{-1}A_{44}^{-1}+\\+3(1+a_0)^{-1}A_{33}^{-1}\end{bmatrix}(1+c_2)A_{33}+ \qquad (7a)$$

$$+(a_0+1)A_{44};$$

$$B_2=1+A_2A_1^{-1}+\frac{6}{h_1^2}A^{-1}\rho+\frac{\tau}{8}\bullet\frac{3s}{h_1^3}; \qquad (7b)$$

$$B_1=\left(A_2A_1^{-1}\gamma_{n.m}\frac{1}{\tau}+\frac{1}{\tau}\bullet\frac{6}{h^2}A_1^{-1}\rho\right)+ \qquad (8a)$$

$$+\frac{s}{h_1^3}\left[(1+c_2)^{-1}A_{44}^{-1}+3(1+a_0)^{-1}A_{33}^{-1}\right];$$

$$B_0=A_3A_1^{-1}\gamma_{n.m}^2-\frac{2s}{h_1}\left[(1+c_2)^{-1}A_{11}A_{33}^{-1}\frac{1}{\tau}\gamma_{m.n}^2\right]; \qquad (8b)$$

where $h_1=(1+c_2)h;\ \gamma_{n.m}=\left[\frac{(2n+1)^2}{l_1^2}+\frac{(2n+1)^2}{l_2^2}\right]\frac{\pi^2}{4}$.

To find the dimensionless natural frequency of the plate [8, p. 216; 9, p. 121], lying on a deformable base, the solution of equation (6) is sought in the form:

$$W_{n.m}(t)=\exp\left(\frac{b_1}{h}\xi t\right); \qquad (9)$$

$$b_1=\sqrt{\frac{A_{44}}{\rho}},$$

where ξ *– dimensionless complex quantity that contains both the real part and the imaginary part, while the real one characterizes damping of oscillations [10, p. 112; 11, p. 961; 12, p. 130], and the imaginary part describes natural oscillation frequencies.*

Substituting (9) into (6), we obtain for ξ *the frequency equation:*

$$\xi^4+C_4\xi^3+C_2\xi^2+C_1\xi^2+C_0=0, \qquad (10)$$

where C_j, *(j = 0...4) are equal:*

$$C_0=\frac{A_3\gamma_0^2}{A_1b_4}+\frac{2s}{h}\left[(1+c_2)^{-1}A_{11}A_{33}^{-1}\gamma_1\right];$$

$$C_1=\frac{1}{\tau_0}\left[\frac{6\rho_1}{(1+c_2)^2A_1}+\frac{A_2\gamma_0}{A_1b^2}\right]+$$

$$+\frac{2s}{h}\left(A_2A_1^{-1}\gamma_0\frac{1}{\tau_0}+\frac{1}{\tau_0}\frac{1}{h^2}A_1^{-1}\right);$$

$$C_2=\frac{6\rho_1}{(1+c_2)^2b_1^2}+\frac{A_0\gamma_0}{A_1b_1^2}+\frac{1}{\tau_0^2}-\frac{3s}{h^2}; \qquad (11)$$

$$C_3=\frac{2}{\tau_0}+\frac{2hs}{3};$$

$$\gamma_0=\left[\frac{(2n+1)^2h^2}{l_1^2}+\frac{(2m+1)^2h^2}{l_2^2}\right]\frac{\pi}{4}.$$

As it can be seen from (11), the coefficients c_j *in the equation (10) are positive, and, according to the Hurwitz's theorem, the real parts of the roots of this equation are negative, and that characterizes damping of oscillations [13, p. 20; 14, p. 488; 15, p. 418].*

We write the general solution (6) in the form:

$$W_{n.m}(t)=e^{-\alpha_1^t}\left(a_{m,n}\sin\beta_1t+b_{m,n}\cos\beta_1t\right)+$$

$$+e^{-\alpha_1^t}\left(c_{n,m}\sin\beta_2t\ +\ d_{m,n}\cos\beta_2t\right), \qquad (12)$$

where $\alpha_1=Re\xi_1$; $\alpha_2=Re\xi_2$; $\beta_1=Im\xi_1$; $\beta_2=Im\xi_2$.

Integration constants $a_{m,n}$, $b_{m,n}$, $c_{m,n}$, $d_{m,n}$ [7, p. 98; 12, p. 131; 16, p. 5] are found from the initial conditions (4).

Conclusions. *Conducted research on the natural frequencies of vibrations allows not only to correctly select the geometric dimensions of the segments of the slab of a ballastless base, but also to calculate the mechanical characteristics of the materials used, taking into account their viscoelastic properties. The proposed method for calculating the behaviour parameters of the plate can also be adapted in order to account for the moving load, which will help to more accurately describe the nature of deformation of the track structures and predict their condition after multiple cycle and high-frequency effects of the vehicle wheel sets taking into account the anisotropy and prestress of the plate.*

The decomposition method has proven itself efficient in solving such problems and can be recommended when calculating the mechanical characteristics in different directions of anisotropy for a ballastless base plate, its reinforcement and expected loads depending on the planned vehicle speeds and axle load, which is especially important when designing and building high-speed rail lines.

The equation obtained in this paper allows one to obtain the natural oscillation frequencies of the slab section of the ballastless base of the railway track, as well as to determine the normal displacements of the specified points of the structure. Changing the initial conditions and conditions for fixing the slab section along the contour will provide a model of reinforced concrete foundation for various engineering and geological factors depending on the area of railway construction.

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