ABSTRACT

The study is devoted to construction of a mathematical model of a slab of a ballastless base of a railway track, assembled from separate segments fastened together. Each rectangular segment is modeled by a transversely isotropic pre-stressed plate. An equation has been obtained that allows researchers to study the natural oscillation frequencies of a rectangular plate, which is especially important from the point of view of choosing a final design of a ballastless track for high-speed main lines. An important issue solved is damping of supporting structures and separation of frequencies of natural and forced oscillations over different intervals of the frequency spectrum. To solve the set tasks, the decomposition method was used, its effectiveness and relative simplicity were proved.

Keywords: railway, high-speed main line, ballastless track, frequency spectrum, mathematical model, Lame constants, decomposition method, integral and differential operators, viscoelasticity.

Background. One of the most promising areas for development of railway transport is associated with high-speed traffic. In the world practice of building high-speed rail for speeds of up to 350 km/h, a block-shaped ballastless base prevails. It can provide for many types of structures: a rail can rest on separate supports or lie on a solid foundation, plates can be prefabricated or monolithic, a rail is held by fasteners or fixed in a plate chute. Despite the relatively high initial cost – twice as expensive as a rail-sleeper track, ballastless structures imply low maintenance costs. In addition, international experience in laying, research and operation of a ballastless track has shown its reliability and expected service life of up to 60 years.

Objective. The objective of the authors is to consider transverse oscillations of a slab section at a ballastless track base, to determine the normal displacements of the specified points of the structure in order to provide a model of reinforced concrete foundation for various engineering and geological factors.

Methods. The authors use general scientific and engineering methods, comparative analysis, evaluation approach, mathematical methods, specific methods like Maxwell viscoelastic model, differential equations, decomposition method.

Results. ...
where \( A_1 = \rho_1 \left[ (1+c_1)^{-1} A_{11} + 3(1+a_1)^{-1} A_{13} \right] \); \( A_2 = \rho_1 \left[ (1+c_1)^{-1} A_{11} + (1+a_1)^{-1} A_{13} \right] \); \( A_3 = 3(1+c_1)^{-1} A_{11} + 2(1+c_2)(1+a_1)^{-1} A_{13} \); \( A_4 = 2 \left( (1+c_1)(1+a_3 - A_{11}^{-1}) A_{13} \right) \);

A rectangular plate is pivotally supported along the contour and in this case the boundary conditions (3), is sought in the form of an ordinary differential equation:

\[
W(t) = 0; \quad \frac{\partial^2 W}{\partial x^2} = 0 \text{ at } x = -l; \quad x = l;
\]

\[
W(t) = 0; \quad \frac{\partial^2 W}{\partial y^2} = 0 \text{ at } y = -l; \quad y = l;
\]

and initial conditions:

\[
W(0) = \phi_0(x, y); \quad \frac{\partial W}{\partial t}(0) = \phi_1(x, y); \quad \frac{\partial^2 W}{\partial t^2} = 0 \text{ at } t = 0.
\]

Moreover, the functions \( \phi_0, \phi_1 \) must also satisfy condition (3).

The solution of equation (1), satisfying the boundary conditions (3), is sought in the form of an infinite double series [7, p. 155; 8, p. 317]:

\[
W(t) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} W_{n,m}(t) \cos \frac{2\pi n x}{2l} \cos \frac{2\pi m y}{2l}.
\]

Substituting (5) into (1), we obtain an ordinary differential equation:

\[
W_{n,m} + B_{n,m} W_{n,m} + B_{n,m} W_{n,m} = 0,
\]

where B\((j = 0...3)\):

\[
B_1 = \frac{\tau^2 + 3 s}{2 H^2} \left[ (1+c_1)^{-1} A_{11} + \frac{3(1+a_1)^{-1} A_{13}}{2} \right];
\]

\[
B_2 = 1 + \frac{6}{H^2} A_{11} \rho + \frac{\tau}{8 H^2} + \frac{3 s}{8 H^2} \left[ (1+c_1)^{-1} A_{11} + 3(1+a_1)^{-1} A_{13} \right];
\]

\[
B_3 = \left( A_{11}^{-1} \gamma_{1,m} + 1 \frac{6}{H^2} A_{11} \rho \right) + \frac{s}{H^2} \left[ (1+c_1)^{-1} A_{11} + (1+a_1)^{-1} A_{13} \right];
\]

\[
B_4 = A_{11}^{-1} \gamma_{1,m} - \frac{2 s}{H^2} \left[ (1+c_1)^{-1} A_{11} + 1 \frac{1}{\gamma_{1,m}} \right];
\]

and \( \gamma_{1,m} \), (j = 0...4) are equal:

\[
\gamma_{1,m} = \frac{C_j}{\rho} + \frac{2 s}{H^2} \left( (1+c_1)^{-1} A_{11} + 1 \frac{1}{\gamma_{1,m}} \right);
\]

where

\[
C_0 = \frac{1}{\rho} + \frac{6}{H^2} A_{11} \rho + \frac{1}{H^2} A_{11} \rho + \frac{3 s}{H^2} \left( (1+c_1)^{-1} A_{11} + 3(1+a_1)^{-1} A_{13} \right);
\]

As it can be seen from (11), the coefficients \( c \) in the equation (10) are positive, and, according to the Hurwitz’s theorem, the real parts of the roots of this equation are negative, and that characterizes damping of oscillations [12, p. 20; 14, p. 488; 15, p. 418].

We write the general solution (6) in the form:

\[
W_{n,m}(t) = e^{-\xi_{1,m} t} \left( a_n \sin \beta t + b_n \cos \beta t \right) + e^{-\xi_{2,m} t} \left( c_n \sin \beta t + d_n \cos \beta t \right),
\]

where \( a_1 = Re \xi_{1,m}; a_2 = Re \xi_{2,m}; b_1 = Im \xi_{1,m}; b_2 = Im \xi_{2,m} \).
Integration constants $a_{m,n}, b_{m,n}, c_{m,n}, d_{m,n}$ [7, p. 98; 12, p. 131; 16, p. 5] are found from the initial conditions (4).

Conclusions. Conducted research on the natural frequencies of vibrations allows not only to correctly select the geometric dimensions of the segments of the slab of a ballastless base, but also to calculate the mechanical characteristics of the materials used, taking into account their viscoelastic properties. The proposed method for calculating the behaviour parameters of the plate can also be adapted in order to account for the moving load, which will help to more accurately describe the nature of deformation of the track structures and predict their condition after multiple cycle and high-frequency effects of the vehicle wheel sets taking into account the anisotropy and prestress of the plate.

The decomposition method has proven itself efficient in solving such problems and can be recommended when calculating the mechanical characteristics in different directions of anisotropy for a ballastless base plate, its reinforcement and expected loads depending on the planned vehicle speeds and axle load, which is especially important when designing and building high-speed rail lines.

The equation obtained in this paper allows one to obtain the normal oscillation frequencies of the slab section of the ballastless base of the railway track, as well as to determine the normal displacements of the specified points of the structure. Changing the initial conditions and conditions for fixing the slab section along the contour will provide a model of reinforced concrete foundation for various engineering and geological factors depending on the area of railway construction.

REFERENCES


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