ON MODELING OF CAR'S WHEEL SET MOVEMENT

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ABSTRACT

The article considers a refined mathematical model of motion of a wheel set with a variable structure of a system of nonlinear differential equations, as well as design diagrams describing interaction of elements of a railway vehicle and a track. The model also takes into account factors that are relevant for speed movement, such as gyroscopic moment and imbalance of a wheel set. The «vehicle–track» system is presented as complex and multi-mass one, with a different structure of interconnections. The track is studied in the form of a free curved surface on the Winkler base as a system of separate inertial elastic supports with dry friction dampers. The model will allow to obtain the correct quantitative results, to evaluate the dynamic properties of rolling stock.

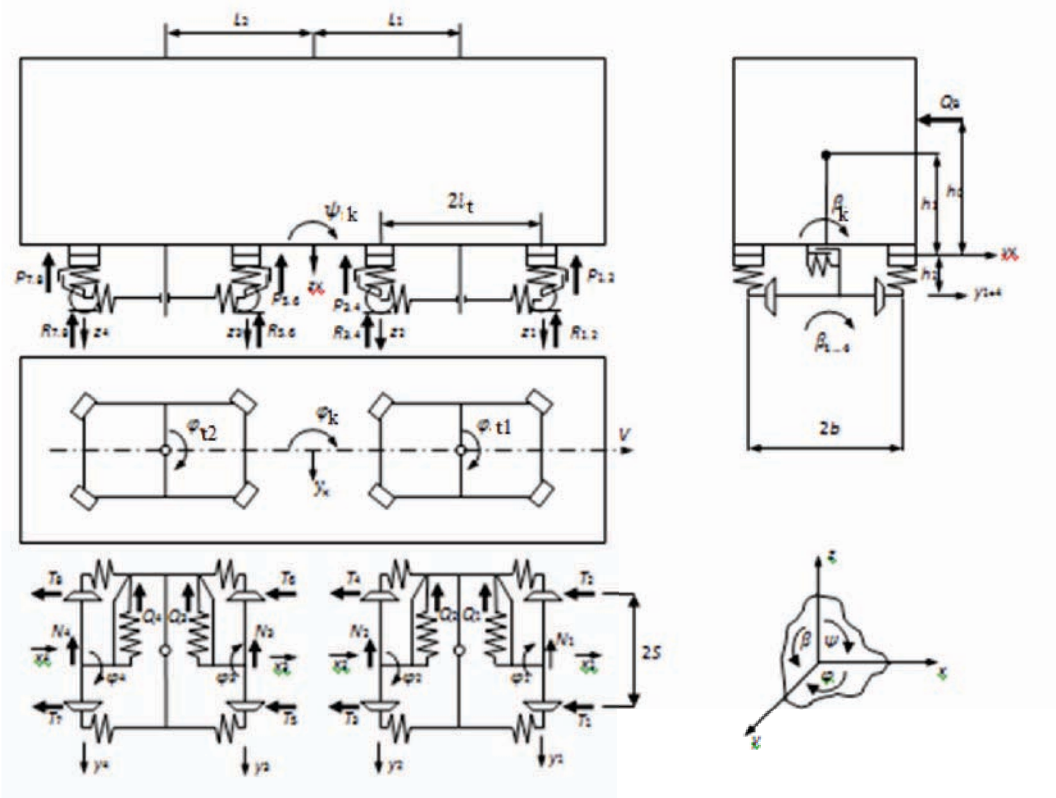
Keywords: *railway, high-speed traffic, railway track, mathematical model, dynamic properties of a car, wheel set.*

Background. *A modern railway track is characterized by the use of heavy-type rails, reinforced concrete sleepers, changes in the standards for track construction, bearing capacity of the roadbed, increased tolerances for track gauge maintenance, deviations in the plan and profile [1, p. 29; 2, pp. 27–30]. As a result, the reduced mass of track, dynamic loads have increased, affecting the operating conditions of rolling stock, first of all its running gear [3, p. 1584], and led to an actual reduction in the design speed. These factors require consideration when creating an innovative rolling stock (RS) and predicting expected dynamic indicators.*

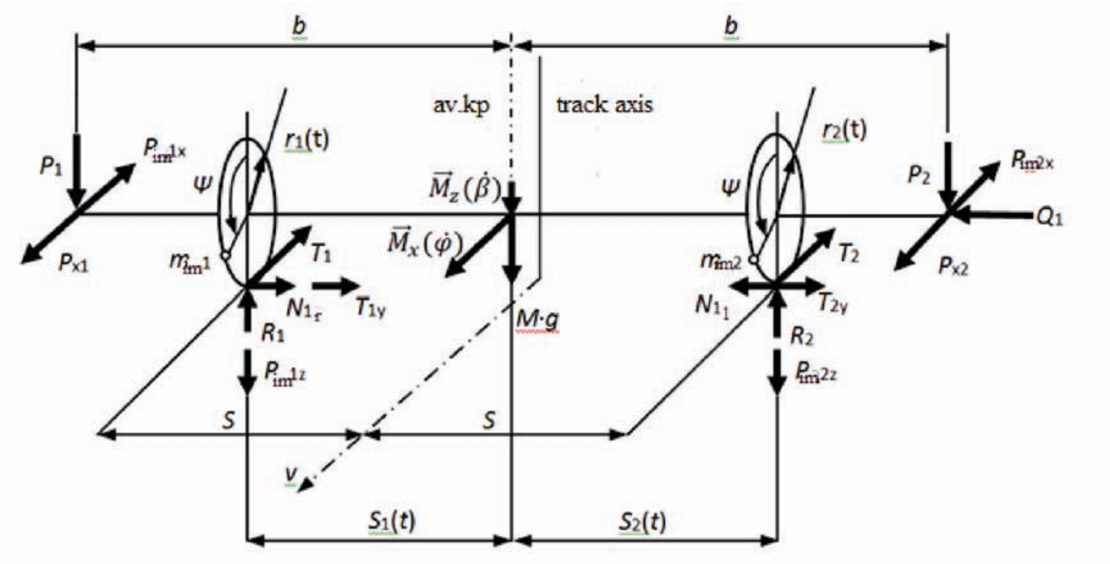
In this regard, it is necessary to have a reliable methodology and a mathematical model of wagon movement, which adequately describes the processes in the complex dynamic system «vehicle–track».

To simulate the dynamics of RS, specialized software systems are used, among which are CONTACT, FASTSIM software applications and MEDYNA, VAMPIRE, NUCARS, GENSYS, SIMPACK, ADAMS/Rail software systems, and the domestic «Wagon» (Research Institute of Railway Transport), «DiONiS» («Dynamics, optimization, loading and statics», Russian University of Transport), «UM» «Universal Mechanism» (Bryansk State Technical University) software systems.

In a number of cases, we will immediately say, linearized models are used that do not take into account important aspects of dynamic loading of elements of cars under typical operating load conditions. It is known, in particular, that the results obtained on linearized models apply only to vibrations with small amplitudes and provide a qualitative, rather



Pic. 1. Calculation model of a four-axle car (an example with a body leaning on axle boxes).



Pic. 2. Scheme of forces applied to the wheel set.

than a qualitative picture of phenomena of tortuous movement and forced vibrations [4, p. 38]. Therefore, when developing an adequate design model, one should keep in mind the existing nonlinearities in differential equations and constraints.

A key element determining reliability of results obtained in modeling movement of a railway vehicle is correct description of dynamics of wheel sets. Therefore, we consider implementation of the model at the example of a nonlinear model of the motion of a wheel set (WS) along rails and its interaction with a track.

Objective. *The objective of the authors is to consider different aspects of modeling of car's wheel set movement.*

Methods. *The authors use general scientific and engineering methods, comparative analysis, mathematical method.*

Results.

1.

The tendency to increase speed of movement of RS requires that simulation of WS movement takes into account such factors as the gyroscopic moment and the run-out of wheels due to imbalance of the wheel set.

When developing a design model as an integral part of a car model (Pic. 1 shows a variant of a car having bogies with direct support of a body on axle boxes of wheel sets) and a system of differential equations, two conditions are priori:

- *WS are absolutely rigid;*
- *it is assumed that the forces shown in Pic. 2 affect each WS.*

The spatial oscillations of WS (unsprung masses of a bogie falling on a wheel set) can be described by a system of nonlinear differential equations, using the example of the first WS.

$$\left(M + M_r\right)\cdot\ddot{z}_1 + R_1 + R_2 + P_{\text{im1z}} + P_{\text{im2z}} - P_1 - P_2 - M\cdot g = 0; \qquad (1)$$

$$M\cdot\ddot{y}_1 + N_1 - Q_1 = 0\,; \qquad (2)$$

$$M\cdot\ddot{x}_1 + T_1 + T_2 - P_{\text{x1}} - P_{\text{x2}} + P_{\text{im1x}} + P_{\text{im2x}} = 0\,; \qquad (3)$$

$$J_z\cdot\ddot{\varphi}_1 - T_1\cdot s_1(t) + T_2\cdot s_2(t) + \left(P_{\text{x1}} - P_{\text{x2}}\right)\cdot b + M_{z1}^{gyr} = 0; \qquad (4)$$

$$\left(J_{\text{x}} + J_{\text{1r}}\right)\cdot\ddot{\beta}_1 + R_1\cdot s_1(t) - R_2\cdot s_2(t) + \left(P_2 - P_1\right)\cdot b - \left(N_{\text{1r}} + T_{\text{1y}}\right)\cdot r_1(\text{t}) + \left(N_{\text{1l}} - T_{\text{2y}}\right)\cdot r_2(\text{t}) + M_{x1}^{gyr}\cdot\text{sign}\left(\dot{\beta}_1\right) = 0; \qquad (5)$$

$$\dot{\psi}_1 = \frac{2\dot{x}_{c1}}{r_1(t) + r_2(t)}; \quad \psi_{1(\text{i+1})} = \psi_{1\text{i}} + \dot{\psi}_{1(\text{i+1})}\cdot h; \quad \dot{x}_{c1} = V - \dot{x}_1, \qquad (6)$$

where M – mass of a wheel set;

g – gravitational acceleration;

M_r – reduced mass of a track;

x, y, z, $\dot{x}, \dot{y}, \dot{z}, \ddot{x}, \ddot{y}, \ddot{z}$ – coordinates of the position of the centers of mass of wheel sets and their derivatives with respect to time;

R_1, …, R_8 – vertical reactions of rails under wheels;

P_{im1z}, …, P_{im8z} – vertical components of forces due to imbalance of wheel sets;

P_1, …, P_8 – vertical reactions of spring sets;

N_1, …, N_4 – horizontal reactions acting on wheel sets;

Q_1, …, Q_4 – horizontal transverse reactions of spring sets;

T_1, …, T_8 – longitudinal components of friction forces arising from wheel slip on rails;

P_{x1}, …, P_{x8} – horizontal longitudinal reactions of the axle box connections to the bogie frame (determined taking into account dry friction forces arising between the bogie frame and axle box body and gaps in the guides);

P_{im1x}, …, P_{im8x} – horizontal longitudinal components of forces due to imbalance of wheel sets;

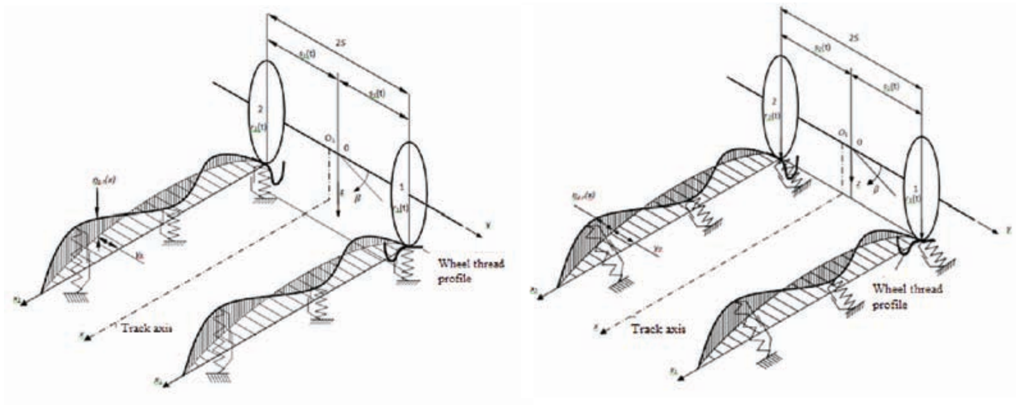
J_x, J_y, J_z – moments of inertia of the wheel set relative to the corresponding axes of coordinates;

$s_1(t)$, …, $s_8(t)$ – distance from the middle of wheel sets to the contact points of the wheel with the rail;

b – distance from the longitudinal axis of symmetry of the car to the fulcrum of the body on the axle box;

$M_{x1}^{gyr}, \ldots, M_{x4}^{gyr}, M_{z1}^{gyr}, \ldots, M_{z4}^{gyr}$ – gyroscopic moments of rotating wheel sets with respect to the respective axes;

J_{rx} – reduced moment of inertia of the track;



a) b)

Pic. 3. Simplified design scheme of the system «vehicle–track»: a) vertical track irregularities; b) horizontal track irregularities.

N_{1l}, ..., N_{4l}, N_{1r}, ..., N_{4r} – horizontal components of the reaction, acting on wheel sets; the reaction is due to conicity of the rolling profile of the left and right wheels;

T_{1y}, ..., T_{8y} – transverse components of friction forces arising from wheel slip along rails;

ψ_1, ..., ψ_4; $\dot{\beta}_1, \ldots, \dot{\beta}_4$; $\dot{\psi}_1, \ldots, \dot{\psi}_4$; $\ddot{\beta}_1, \ldots, \ddot{\beta}_4$ – angles of rotation of wheel sets and their derivatives with respect to time;

$r_1(t)$, ..., $r_8(t)$ – radii of wheels rolling circles;

$\dot{x}_{c1}, \ldots, \dot{x}_{c4}$ — speed of movement of the center of mass of the wheel set;

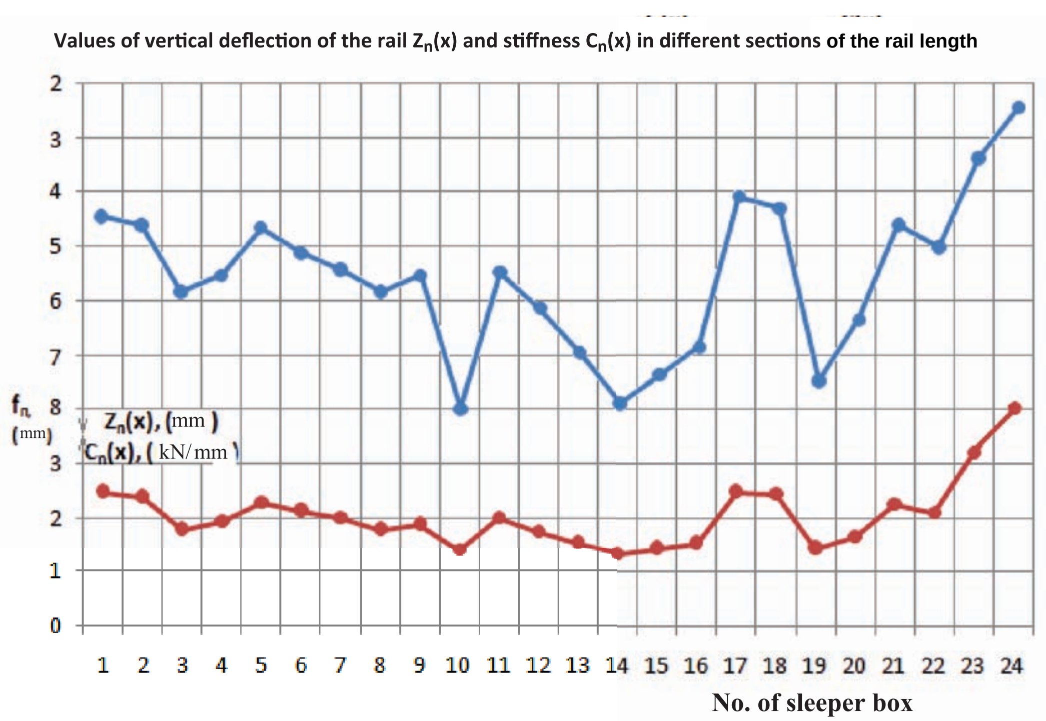
V – speed of a car along a track;

h – geometric size;

i, i+1 – index denoting the iteration sequence number.

When determining vertical reactions of spring sets and vertical reactions from the side of the track, a one-sided connection is taken into account by introducing the conditions of deloading if the sum of static deflection and dynamic deformation is equal to or less than zero.

Horizontal reactions N_1, ..., N_4, acting on wheel sets, are determined from the expression (7) taking into account the horizontal component due to conicity of the rolling profile N_{1r}, ..., N_{4r} of the right and N_{1l}, ..., N_{4l} of the left wheels, and transverse components of friction forces T_{1y}, ..., T_{8y}, arising from wheel slip along rails.



Pic. 4. Characteristics of the track obtained during the field experiment.

Vertical dynamic deformations $\Delta_{p1}, \ldots, \Delta_{p8}$, horizontal transverse dynamic deformations $\Delta_1, \ldots, \Delta_8$, $\Delta_{x1}, \ldots, \Delta_{x8}$ and horizontal longitudinal dynamic deformations are determined by dependencies (8), (9), (10). They also record the rates of dynamic deformations, but the coordinates in this case are replaced by the values of their first derivatives with respect to time. The vertical dynamic reactions of the track under wheels follow from (11).

$$N_1 = N_{1r} - N_{1l} + T_{1y} + T_{2y}; \qquad (7)$$

$$\left.\begin{aligned} \Delta_{p1} &= z_k + \psi_k \cdot (L_1 + L_T) + \beta_k \cdot b_1(t) - z_1 - \beta_1 \cdot b \\ \Delta_{p2} &= z_k + \psi_k \cdot (L_1 + L_T) + \beta_k \cdot b_2(t) - z_1 - \beta_1 \cdot b; \end{aligned}\right\} \qquad (8)$$

$$\Delta_1 = y_k + \phi_k \cdot (L_1 + l_T) + \phi_{t1} \cdot l_t - y_1; \qquad (9)$$

$$\left.\begin{aligned} \Delta_{x1} &= (\phi_1 - \phi_{t1}) \cdot b + x_1 \\ \Delta_{x2} &= (\phi_{t1} - \phi_1) \cdot b + x_1; \end{aligned}\right\} \qquad (10)$$

$$\left.\begin{aligned} \Delta_{R1} &= z_1 + \beta_1 \cdot s_1(t) - z_{pr1z}(x) + \eta_{p1z}(x) + r_1(t) \\ \Delta_{R2} &= z_1 + \beta_1 \cdot s_2(t) - z_{pr2z}(x) + \eta_{p2z}(x) + r_2(t); \end{aligned}\right\} \qquad (11)$$

$$\left.\begin{aligned} \Delta_{S1} &= y_1 + \phi_1 \cdot s_1(t) - z_{pr1y}(x) + \eta_{p1y}(x) + s_1(t) \\ \Delta_{S2} &= y_1 + \phi_1 \cdot s_2(t) - z_{pr2y}(x) + \eta_{p2y}(x) + s_2(t). \end{aligned}\right\} \qquad (12)$$

where z_{prz}, z_{pz}, z_{pry}, z_{py} – track parameters in the vertical and transverse directions.

The vertical reactions of the track regarding a wheel set depend on the parameters of the track and the coordinates that determine the position of the unsprung parts of the bogies. This connection is carried out through the values of dynamic deformations.

To find the trajectory of the wheel set, one should look for solutions of a system of six differential equations. However, in order to accurately determine the forces of resistance to rolling of a wheel set along an elastically deformed rail, it is necessary to significantly complicate the computational models of the track, representing them as complex multi-mass systems with different structures of interconnections. This complication allows to more fully reflect the physical side of the task, the state of the real track, to get the correct quantitative results.

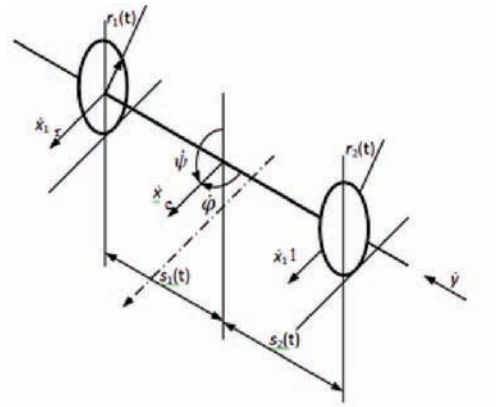
To account for the elastic and inertial properties of the track between the wheel and the base of the railroad track, an additional sprung mass of the track superstructure, reduced to the wheel, is introduced. The rail is considered as a free curved surface on a Winkler base.

In the design model of the track (Pic. 3), the rail has geometrical irregularities, the base stiffness variable in length (Pic. 4) is taken into account, which allows simulating the real track when the necessary data are available [5, p. 21]. The magnitude of the reduced mass of the track M_r involved in the force interaction with the wheel is assumed to be dependent on deflection of the rail, this mass may change when the car moves. Vertical and horizontal irregularities are set separately on the right and left rails.

2.

When solving many problems associated with assessment of dynamic qualities and determination of the maximum values of coefficients of vertical dynamics, horizontal frame forces, stability reserves from derailment, it is advisable to use deterministic irregularities corresponding to the length of the rail.

The track can be represented as a system of individual inertial elastic supports with dry friction dampers. The free surface of such supports in the adopted coordinate system is determined analytically or is given in the function $\eta(x)$. The stiffness and dissipative characteristics of the individual elastic supports can be different and can be set depending on x coordinate. Consequently, in a fixed section of the track x, the corresponding elastic support will have a certain stiffness $c_n(x)$ and a static deflection $z_n(x)$, which also depends on the static load on the vehicle's wheel (Pic. 4).



Pic. 5. The design scheme for determining the wheel slip speeds.

Rail track stiffness can be represented as a dependency [6, p. 306]:

$$c_n(x) = c_n^{'}(x) + c_n^{'}(x) \cdot \delta_n \cdot (\delta_{sec} - 0{,}5); \qquad (13)$$

$$c_n^{'}(x) = c_0 + \left| B_n \cdot \sin\left| \frac{\pi}{L} \cdot x \right| + \left| B_n \cdot \sin\left| 3 \cdot \frac{\pi}{L} \cdot x \right\| \right.\right.,$$

where c_0 – rail stiffness at the junction;

A_n and B_n – amplitude of change of stiffness in the first and second harmonics;

L – length of the rail;

x – section of the track, changing with a step equal to the length of the sleeper box;

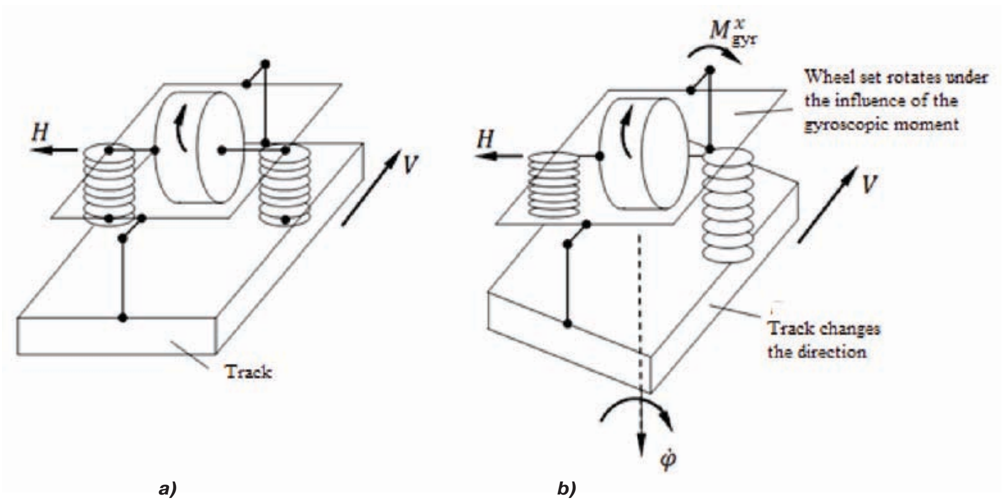
Δ_n – coefficient showing the maximum possible change in stiffness from one sleeper box to another (in general, this change in stiffness is random);

Δ_{sec} – pseudo-random number varying from 0 to 1 (random number generator is used to obtain them).

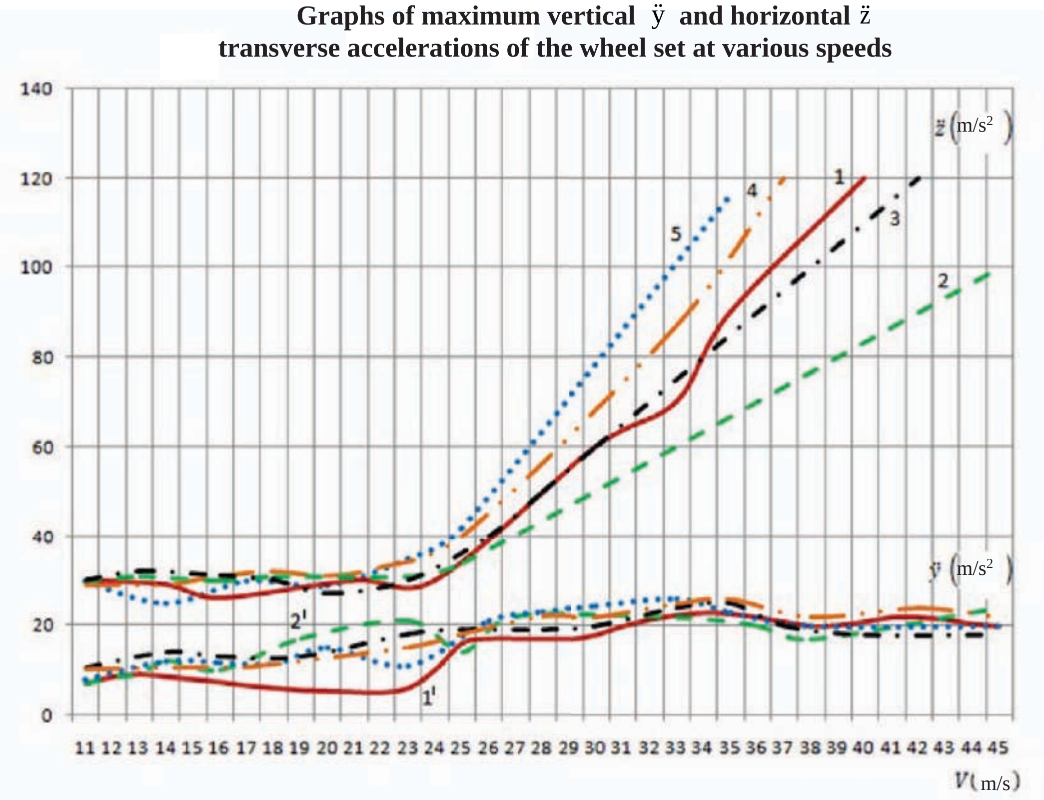
It is convenient to use such a representation of track stiffness, for example, for conducting a comparative assessment of influence of individual parameters of a car on its dynamic qualities.

It turns out that vertical and horizontal perturbations in the model vehicle–track are not only defined by the initial geometric irregularities of the unloaded track, but also by the variation of rigid parameters of the track along the rail length (parametric oscillations). Consequently, the trajectory of the wheel (the so-called dynamic unevenness) will be determined by the parameters of the entire system vehicle–track. In this case, the design model does not take into account the distribution properties of the real track.

The position of the wheel set, determined by z, y and β coordinates, depends on the vertical and horizontal dynamic irregularities of the right and left rail threads, as well as on the radii of rolling circles r(t), the cross-sectional deviation of the wheel set, determined by the coordinate y, the track width 2S in each section and geometric irregularities of rail threads in terms of the plan y_p. The radii of rolling circles r(t), in turn, depend on the given wheel profile and all other coordinates of the wheel set itself and the roughness of the track. This connection is made through size s(t), which helps to find the position of the point of contact of the wheel on the rail. From the design model it can be seen that the nature of the disturbances in the study of the spatial oscillations of the car will be determined not only by the independent



Pic. 6. The manifestation of the gyroscopic properties of the wheel set.



Pic. 7. Results of calculation.

parameters of the track, but also by the parameters of the vehicle under study and its position in the adopted coordinate system.

When finding friction forces at the wheel–rail contact points of the design model, to simplify the problem, we used a scheme for determining slip rates taking into account rolling of the wheel set (Pic. 5).

The direction of the friction force is perpendicular to the normal pressure force R_N at the point of contact wheel–rail and opposite to the direction of sliding speeds $\dot{x}$ and $\dot{y}$. Due to the smallness of the slope of rolling relative to XOY plane in the presence of gaps between the ridge and rail, the vertical component of the wheel–rail friction force is not taken into account.

Taking the assumption that the speed of the car along the axis of the track V is constant, it is possible from the expression (6) to obtain the speeds of the center of mass of each wheel set along the axis of the track $\dot{x}_{c1}, \ldots, \dot{x}_{c4}$, taking into account bouncing of wheel sets within the elastic gaps in the longitudinal direction. Then from the expressions (14)–(16) it is possible to determine sliding speed of each of the wheels. The relative sliding speeds are found by dividing the values from expressions (16) by the corresponding value $\dot{x}_{c1}, \ldots, \dot{x}_{c4}$.

$$\left.\begin{aligned} \dot{x}_{1r} &= \dot{\psi}_1 \cdot r_1(t) - \dot{\varphi}_1 \cdot s_1(t); \\ \dot{x}_{1l} &= \dot{\psi}_1 \cdot r_2(t) - \dot{\varphi}_1 \cdot s_2(t); \end{aligned}\right\} \quad (14)$$

$$\dot{y}_{1r} = \dot{y}_1 + \dot{x}_{c1} \cdot \varphi_1; \; \dot{y}_{1l} = \dot{y}_1 + \dot{x}_{c1} \cdot \varphi_1; \quad (15)$$

$$\left.\begin{aligned} V_{01r} &= \sqrt{(\dot{x}_{1r} - \dot{x}_{1c})^2 + \dot{y}_{1r}^2}; \\ V_{01l} &= \sqrt{(\dot{x}_{1l} - \dot{x}_{1c})^2 + \dot{y}_{1l}^2}. \end{aligned}\right\} \quad (16)$$

Knowing the value of the relative sliding speed, one can determine the coefficient of friction between the wheel and the rail.

The appearance of the gyroscopic moment and necessity of taking it into account in the model is due to the fact that at high speeds, the rapidly rotating wheel set begins to show gyroscopic properties. WS actually becomes a rotor of a two-step gyroscope [7, p. 288], the axis of which lies in the plane perpendicular to the direction of the track. For the sake of explanation, let us imagine that the direction of movement of the car changes, WS rolls in from the straight section into the curve (Pic. 6). At the initial stage, the gyroscope axis (WS) resists fitting into the curve. Through the wheel–rail contact, the track acts on the resisting gyroscope, creating a moment of forces to the vertical axis z, which causes the gyroscopic momentum vector H to turn down (Pic. 6b). A gyroscopic moment M^{x}_{gyr} appears, which should be taken into account in the differential equations of WS rotational motion.

The values of gyroscopic moments of rotating wheel sets entering into differential equations are determined from the dependences

$$\left.\begin{aligned} M^{gyr}_{z1,\ldots,4} &= J_y \cdot \frac{\dot{x}_{c1,\ldots,4}}{r_{av}} \dot{\beta}_{1,\ldots,4}; \\ M^{gyr}_{x1,\ldots,4} &= J_y \cdot \frac{\dot{x}_{c1,\ldots,4}}{r_{av}} \dot{\varphi}_{1,\ldots,4}. \end{aligned}\right\} \quad (17)$$

In addition, at speeds above 140 km/h, WS imbalance must be taken into account. The impact of imbalance occurs periodically and has the condition of a wheel turn. In one half of the turn, the impact goes on the track, on the other – the track, on the contrary, is unloaded (Pic. 7). The horizontal and vertical components of the forces caused by WS imbalance are determined depending on the mass imbalance m_{im} by the formulas

$$\left.\begin{aligned} P_{im\,jx} &= m_{im} (\dot{\psi}_j)^2 \cdot r_{av} \cdot \cos\psi_j; \\ P_{im\,jz} &= m_{im} (\dot{\psi}_j)^2 \cdot r_{av} \cdot \sin\psi_j. \end{aligned}\right\} \quad (18)$$

Direct support of the body by the axle boxes can significantly reduce the mass of unsprung parts. For this case, the problem of determining the influence of imbalance and gyroscopic moment on the level of vertical $\ddot{z}$ and horizontal $\ddot{y}$ accelerations acting on the wheel set is of interest. Such values can be used, for example, in determining the stability factor of the wheel set versus derailment. The calculation results in the form of dependences $\ddot{y}$ and $\ddot{z}$ on the speed of movement are presented in Pic. 7. From the analysis of the data it is clear that the mass imbalance and the gyroscopic effect have a greater effect on the vertical than on the horizontal accelerations. From the comparison of curves 1, 2 and 5 it follows that to a greater extent the magnitude of imbalance affects the maximum values $\ddot{z}$.

Conclusion. Thus, the refined non-linear model of movement of the wheel set of a car makes it possible to realistically evaluate the dynamic properties of rolling stock at high speeds, taking into account the influence of the gyroscopic moment and the imbalance of the wheel set.

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