CALCULATED DEPENDENCIES AND ESTIMATES OF FREQUENCIES AND VIBRATION MODES OF SHELLS OF RAIL TANK WAGONS

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ABSTRACT

The article gives the procedure for calculating the frequencies of natural oscillations of a shell of a railway tank wagons, taking into account its incomplete filling with liquid cargo. This task is relevant, since it is related to the conditions of safe operation of rolling stock. In this formulation, the solution of the problem is, of course, not done by the finished tracing paper. Including using the hypotheses of the absence of annular deformations of the shear of the shell of the tanks. The reliability of the calculated results is also confirmed by the experimental data obtained in time when testing eight-axis tanks.

Keywords: *rolling stock, tank car, rail tank wagon, tank oscillations, semimuscular theory of shells, natural frequencies, underfilling of liquid cargo.*

Background. *If we single out the essence, we consider the formation of calculated dependencies that allow us to estimate the frequency and shape of the natural oscillations of the cylindrical parts of the railway tank wagons taking into account the level of their filling with liquid cargo, and the results of numerical data that are compared with the experimental ones. The proposed calculations provide possible ways of estimating the stress state and strength of the shell under the action of dynamic loads.*

Objective. *The objective of the authors is to consider dependencies and estimates of frequencies and vibration modes of shells of rail tank wagons.*

Methods. *The authors use general scientific methods, comparative analysis, evaluation approach, mathematical methods.*

Results.

1.

To derive the calculated dependencies, an energy approach is used to solve the problems of construction mechanics, which, when selecting functions that approximate displacements with allowance for the kinematic boundary conditions, leads to a system of equilibrium equations relative to the approximation parameters. It has the following form for the dynamics problems [1]:

$$\left\{[R]-\omega^2[M]\right\}\vec{u}-\vec{P}=0\,. \qquad (1)$$

In this equation, in the case of the four-dimensional problem (for the Cartesian coordinate system of the three-dimensional components x, y, z and time t), the matrix [R] is obtained using the dependencies

$$[R]=\int_0^T\int_x\int_y\int_z\left|\Phi(x,y,z,t)\right|^T[D]\left|\Phi(x,y,z,t)\right|dtdxdydz. \qquad (2)$$

It is assumed that the elements of the matrix characterizing the deformation field of the elastic node vary in time according to the law sinωt. The energy is counted within the period T = 2ϖ/ω.

[D] – matrix of stiffness parameters reflecting the material and geometric characteristics of the node under consideration.

The matrix of inertial characteristics is defined by the formula:

$$[M]=\int_0^T\int_x\int_y\int_z\left|f(x,y,z,t)\right|^T[\gamma]\left|f(x,y,z,t)\right|\;dtdxdydz\,, \qquad (3)$$

where |f(x, y, z, t)| – matrix of functions approximating the velocity field obtained by differentiating with respect to time t the displacement field |ψ(x, y, z, t)|.

The matrix of the functions |ψ(x, y, z, t)| of the displacement field is chosen so as to correspond to the kinematic boundary conditions, and it is the initial one for formation of the matrix of the functions |ϕ(x, y, z, t)|. The latter reflects the operations of differentiation in obtaining expressions for calculating the deformations of the node under consideration.

In the formula (3), the matrix [γ] represents the inertial parameters of the node (mass, moments and inertia).

In the expression sinωt, which reflects the nature of the change in the work of the node in time, ω is the frequency of natural oscillations. The eigenvibrations will be determined if the vector of external influences in the equation (1) is zero. In the case of steady-state forced oscillations, ω^2 is given and equal to the known frequency of external influences.

As is known [2], the coefficients of the matrices [R] and [M] are obtained as a result of minimization of the potential and kinetic energy in terms of the approximation parameters u_{mn} of the considered elastic node under the action of the load $\vec{P}$.

$$\vec{P}=\int_0^T\int_x\int_y\int_z\left|\vec{q}\right|^T\left|\psi(x,y,z,t)\right|\;dtdxdydz\,, \qquad (4)$$

where $\left|\vec{q}\right|^T$ – vector of loads, the elements of which are external loads, oriented along the corresponding components of the displacement field.

In accordance with the problem posed earlier, it becomes necessary to choose a version of the theory of shells that provides sufficiently acceptable results for engineering tasks without the expense of large calculated resources.

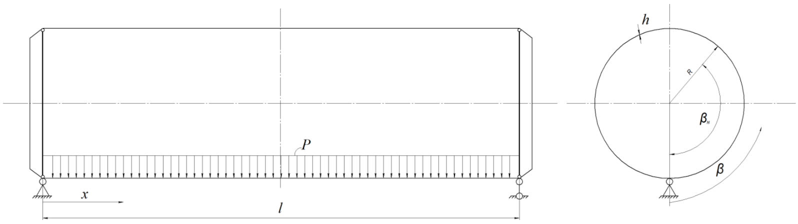
It follows from the scientific literature [3] that for these purposes the half-shell theory of shells is well suited [4]. Under certain conditions connected with the features of approximation of the displacement field for a cylindrical shell, in the design scheme of which we use the support for ideal diaphragms (Pic. 1), one can choose as one of three displacements w, v, u – radial, tangential and longitudinal.

Using the hypotheses of the absence of ring deformations of the shear of the middle surface of the shell, two others can be expressed through one displacement. In our case (as it seems), as the resolver, it is expedient to accept the displacement u.

$$u=\sum_{m=1}^{\infty}\sum_{n=2}^{\infty}u_{mn}\cos\lambda x\cos n\beta\sin\omega t\,, \qquad (5)$$

where u_{mn} – required coefficients of the series (i.e., the above-mentioned approximation parameters);

$$\lambda=\frac{m\pi x}{l},$$



Pic. 1. Calculation scheme of the cylindrical part of the shell of the railway tank wagon.

Table 1

The calculated values of the vibration frequencies for different forms of m and n depending on the filling angle βf.

m	n	β_f = 180	β_f = 157	β_f = 90	β_f = 0
1	2	2,85	3,643	5,902	11,438
	3	2,665	3,198	6,098	10,694
	4	4,302	4,916	9,435	17,264
	5	6,565	7,413	14,967	26,709
	6	9,569	10,632	21,174	38,402
m	n	β_f = 180	β_f = 157	β_f = 90	β_f = 0
2	2	10,561	13,498	21,868	42,38
	3	5,277	6,333	12,074	21,175
	4	5,008	5,723	10,983	20,095
	5	6,854	7,634	15,414	27,506
	6	9,636	10,706	21,322	38,671
m	n	β_f = 180	β_f = 157	β_f = 90	β_f = 0
3	2	23,393	29,899	48,44	93,874
	3	10,829	12,996	24,779	43,455
	4	7,317	8,362	16,048	29,362
	5	7,658	8,529	17,221	30,731
	6	9,923	11,025	21,957	39,822
m	n	β_f = 180	β_f = 157	β_f = 90	β_f = 0
4	2	40,905	52,281	84,703	164,151
	3	18,868	22,645	43,175	75,717
	4	11,401	13,029	25,006	45,753
	5	9,489	10,568	21,339	38,079
	6	10,659	11,842	23,585	42,774

where m – harmonic number of the displacement expansion in a row along the length of the cylindrical part of the tank's shell; l – length of the shell of the cylindrical part of the tank; n – number of the term in the expansion series along the arc of the cross section.

Such a choice takes into account the kinematic boundary conditions – the absence of longitudinal displacement in the middle section of the cylindrical part of the tank and the periodicity of movement along the circumference.

2.

From the hypothesis of the absence of a shear, we get:

$$v=\sum_{m=1}^{\infty}\sum_{n=2}^{\infty} v_{mn}\sin\lambda x\sin n\beta\sin\omega t\,, \qquad (6)$$

where $v_{mn}=\dfrac{n}{R\lambda}u_{mn}$.

From the hypothesis that there is no extension of the contour of the sections, we have:

$$w=\sum_{m=1}^{\infty}\sum_{n=2}^{\infty} w_{mn}\sin\lambda x\sin n\beta\sin\omega t\,, \qquad (7)$$

where $w_{mn}=-\dfrac{n^2}{R\lambda}u_{mn}$.

The application of the proposed displacement approximation leads to the matrix [R]. When deriving formulas, the orthogonality condition of trigonometric functions is used.

$$[R]=\left[\frac{D}{R^5}\frac{\pi l}{2}\frac{n^2}{\lambda^2}+\frac{Bl\pi R}{2}\lambda^2\right], \qquad (8)$$

where $D=\dfrac{Eh^3}{12\left(1-\mu^2\right)}$ *– cylindrical rigidity;*

E – modulus of elasticity;
h – thickness of the shell;
μ – the Poisson's ratio;

$B=\dfrac{Eh}{1-\mu^2}$ *– longitudinal stiffness of a strip of unit width.*

It should be noted that when calculating deformations in the considered version of shell theory, the potential energy depends on the deformation of the change in the curvature of the cross section of the shell and on the deformation of the longitudinal elongation of the shell element. Both these deformations, in accordance with the hypotheses used, are expressed in terms of the displacement u and therefore the matrix [R] has one element.

The matrix [M] reflects the effect of two displacements w and u. It is assumed that the motion of the liquid cargo is excited by the movement of the wettable surface of the tank. To simplify the solution, we introduced a hypothesis according to which the velocity of the liquid particles corresponds to the velocity of points on the contact line of the layer with the walls of the tank.

$$\left[M\right]=\left[\frac{\pi lR}{2}\frac{\gamma_{sh}h}{g}\left(\frac{n^4}{R^2\lambda^2}+1\right)+\frac{\gamma_l l}{2g}F\right], \qquad (9)$$

where γ_{sh} – specific weight of the shell material;

γ_l –specific weight of liquid cargo;

$F=\int_0^{\beta_n} F_{\sec}\cos^2 n\beta d\beta$ *– a parameter that takes into account the level of liquid cargo filling.*

We have carried out the integration in a general form, but because of the cumbersome nature of this expression, it is not given in the article. In the calculations, we calculated this integral numerically.

The cross-sectional area of the liquid cargo filling the cylindrical part is determined by the formula

$$F=R^2\left(\beta_f{}^2-\frac{1}{2}\sin 2\beta_f\right),$$

where β_f – the angle of filling the circular section of the tank with liquid cargo. It is counted as a coordinate of β.

The coefficients of the matrix [M] are obtained as a result of differentiation of the kinetic energy of the tank shell and the liquid cargo filling it. In the case under consideration, as in the matrix of reactions [R], the matrix [M] consists of one element, represented as two terms: the first term reflects the kinetic energy of the shell, and the second – of the liquid cargo.

We provide a table of calculated values (Table 1) for different forms of oscillations along the circle n and along the generator of the cylinder m for different β_f. In calculation, the shell of a tank of an eight-axis tank wagon is taken, which was tested on the experimental ring of the All-Russian Scientific Research Institute of Railway Transport [5].

Analysis of the results shows that the values of the calculated and experimental frequencies for a tank completely filled with a liquid are satisfactorily agreed. When fully filled, the minimum frequency is 2,66 Hz with one wave of approximation along the length and three waves along the circumference; for the empty tank, the minimum frequency is 10,694 Hz with the same oscillation modes.

In the table four forms of approximation along the length are considered, the lower frequency corresponds to four waves of n oscillations along the circumference.

The report on the results of tests of the eight-axis tank wagon indicates the value of the oscillation frequency at full loading of 3 Hz.

Conclusion. *Summarizing the work done, it can be concluded that the dependencies obtained allow us to estimate with sufficient accuracy the frequencies and shapes of natural oscillations of the tanks of railway tank wagons, taking into account the level of filling.*

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