MATHEMATICAL MODELING OF TRANSVERSAL CREEP OF RAILWAY WHEELS

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ABSTRACT

Analyzing the analytical expression of transverse creep used to model the forces of interaction between a wheel and a rail, it is possible to note that the formula for its calculation does not physically correspond to the definition of creep as the difference between kinematic and actual displacements. In fact, the formula contains the difference between real transverse and longitudinal rotation, multiplied by an angle, displacements of the center of mass of the wheel set. If the direction of movement of the wheel set does not coincide with the track axis, the deformation of the contacting fibers of the wheel and the rail occurs in the direction of movement of the center of each wheel. However, the formula for calculating the transverse creep, in contrast to the longitudinal creep, is in no way connected with the contact between the wheel and the rail. Two physically justified variants of calculating the transverse creep are proposed, which can lead to nonlinear differential equations describing the plane motion of a wheel set or a bogie.

Keywords: railway, rail, wheel, contact spot, actual displacement, creep, track axis, modeling, projection.

Background. *The study of safety and stability of movement of railway vehicles, their constructive changes presupposes creation of adequate mathematical models that most accurately describe the interaction of both the track superstructure and the vehicle and the elements of the rolling stock structure itself.*

In this case, the mathematical model of wheel-rail interaction is being refined, which serves as the foundation for more complex models describing movement of either a wheel set, or a vehicle, or a car.

Objective. *The objective of the author is to consider mathematical modeling of transversal creep of railway wheels.*

Methods. *The author uses general scientific methods, comparative analysis, evaluation approach, engineering qnd mathematical methods, particularly non-linear differential equations.*

Results.

I.

The wheel, interacting with the rail, transfers the vertical load to the track superstructure through a small part of the rail surface, called the contact spot. The shape of the contact spot, as shown in [1], is an ellipse, usually drawn out on the rolling surface of the rail along the track axis. Because of the small area of the surface of the spot, a large specific pressure occurs, under the influence of which elastic deformations occur.

If no torque or horizontal force is applied to the wheel, the spot (contact zone) is symmetrical with respect to the vertical axis.

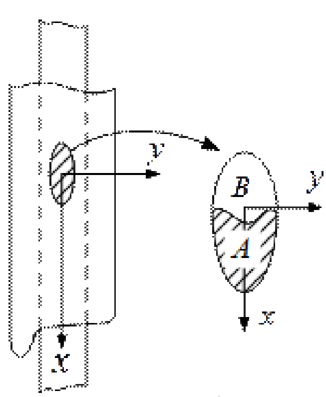
When the wheel moves under the action of the torque, the symmetry of the strain distribution is violated. The contact zone is divided into two areas (Pic. 1). In the area «A», adhesion is maintained and elastic deformations occur in the direction of travel (in the traction mode, the bandage material is compressed and the rail is stretched), in the area «B», phenomena similar to boxing are observed. If the direction of movement of the wheel set does not coincide with the track axis, the deformation of the contacting fibers of the wheel and the rail occurs in the direction of movement of the center of each wheel.

Due to the difference in deformation of the wheel and the rail in the contact area, the path traversed by the geometric center of the wheel is less than the path calculated by the angular speed of the wheel rotation under the assumption of rolling without sliding. This phenomenon is considered from the kinematic point of view as slippage and, given the cause of its occurrence, is called elastic, pseudo-slip, or creep.

Studying the movement of a wheel set or a bogie, the response of the wheel and rail is modeled according to the theory of creep, assuming it is proportional to the relative rate of deformation of the contacting particles of the wheel and the rail in the direction of wheel movement.

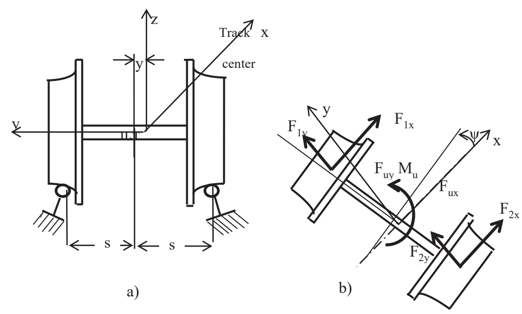
The reaction forces arising in the contact spot due to elastic deformations of the surface fibers of the contacting bodies are usually called creep forces [2, 3]. In the future, in presenting the material, we will adhere to the notations and concepts given there. In particular, we introduce the notation (Pic. 2): x – longitudinal displacement of the center of mass of the wheel set; y – transverse displacement of the center of mass of the wheel set; ψ – rotation angle around the vertical axis z; θ – angular variation of the nominal value of the angular velocity ω of rotation of the wheel set about the y axis, where $\omega = v/r_0$; r_0 – radius of rolling circle of wheel of the wheel set in central installation (nominal rolling radius); F_i (F_{ix}, F_{iy}) – creep force and its components in the wheel contacts, respectively along the track center x and across – y, i = 1,2.

Creep forces F_i (i = 1,2) are creep functions, which are defined as the relative linear displacements between the wheel and the rail [3, p. 153], directed along the wheel's motion. In the same source it is stated that «the longitudinal creep ε_{ix} is given as the ratio of the difference between the longitudinal velocity of the contact point and the longitudinal velocity of the rail contact point to the nominal speed;



Pic. 1.

Pic. 2.



the transverse creep ε_{iy} is given as the ratio of the difference between the transverse velocity of the contact point and the transverse velocity of the rail contact point to the nominal speed» [3] of the movement of the wheel set in the central installation.

Such calculation of creep components occurs in practically all early and late studies devoted to the problem of the motion of rail vehicles [4–8]. Thus, in [2] longitudinal and transverse creep are calculated by the formulas:

$$\varepsilon_{1x} = \frac{1}{v}(r_1 \cdot \dot{\omega} - \dot{x} + s \cdot \dot{\psi});$$

$$\varepsilon_{2x} = \frac{1}{v}(r_2 \cdot \omega - \dot{x} - s \cdot \dot{\psi}); \varepsilon_{1,2y} = \frac{1}{v}(\dot{y} - \dot{x}\psi), \qquad (1)$$

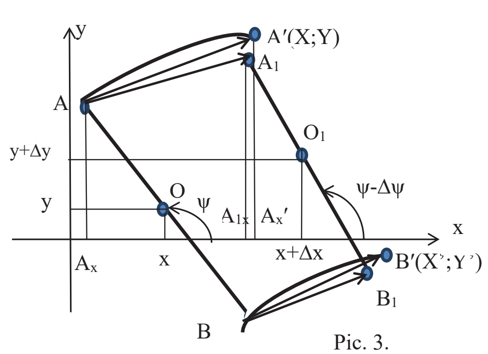
and in [3, p. 155] – by the formulas:

$$\begin{cases} \varepsilon_{1x} = \frac{1}{v}\{v[\frac{r_1}{r_0} - 1] + s \cdot \dot{\psi}\} = \frac{1}{v}(r_1 \cdot \dot{\omega} - \dot{x} + s \cdot \dot{\psi}); \\ \varepsilon_{2x} = \frac{1}{v}\{v[\frac{r_2}{r_0} - 1] - s \cdot \dot{\psi}\} = \\ = \frac{1}{v}(r_2 \cdot \omega - \dot{x} - s \cdot \dot{\psi}); \\ \varepsilon_{1y} = \frac{1}{v}(\dot{y} + r_1 \cdot \dot{\phi} - \dot{x}\psi) \cong; \frac{1}{v}(\dot{y} - \dot{x}\psi); \\ \varepsilon_{1y} = \frac{1}{v}(\dot{y} + r_2 \cdot \dot{\phi} - \dot{x}\psi) \cong \frac{1}{v}(\dot{y} - \dot{x}\psi). \end{cases} \qquad (2)$$

If we compare the formulas (1) and (2), then it is easy to see that the formulas for the longitudinal creep are identical, while for the transverse creep they differ in the term $r_i \cdot d\phi/dt$, which, as compared to other terms, is usually neglected and, therefore, they also coincide.

II.

Let us consider in more detail the derivation of formulas for calculating the longitudinal and transverse creep as the difference of the path traversed by the geometric center of the <u>wheel</u> and the path calculated from the angular speed of rotation of the wheel under the assumption of rolling without sliding.



Pic. 3.

Let the coordinates of the center of mass of the wheel set O and the points of contact between the wheels A and B be at the moment of time: $O(x; y)$, $A(x - s \cdot \psi; y + s)$, $B(x + s \cdot \psi; y - s)$. We give the increment of time Δt. Then in the time Δt the points O_1, A_1, B_1 (Pic. 3), the coordinates of which will be $O_1(x + \Delta x; y + \Delta y)$, $A_1[x + \Delta x - s \cdot (\psi - \Delta\psi); y + \Delta y + s]$, $B_1[x + \Delta x + s \cdot (\psi - \Delta\psi); y + \Delta y - s]$, and the axis of the wheel set turns in the horizontal plane by an angle $\Delta\psi > 0$.

From Pic. 3 it is clear that the points of the contact spots in time Δt have moved the left and right wheels from A to A_1 and from B to B_1, respectively. In view of the smallness of Δt, the trajectory of motion is assumed to be rectilinear and then the projections of the vectors of the contact points

$$\overrightarrow{BB_1}(\Delta x - s \cdot \Delta\psi; \Delta y), \quad \overrightarrow{AA_1}(\Delta x + s \cdot \Delta\psi; \Delta y).$$

As a result of the kinematic displacement, as the displacement of absolutely rigid bodies, the points of the contact spots of the wheels with the rails will move from B and A to B′ and A′ respectively (Pic. 3). We will assume, as above, the trajectory of the motion of contact spots to be straight lines

$$\overrightarrow{BB_1}(\Delta X'; \Delta Y'), \quad \overrightarrow{AA_1}(\Delta X; \Delta Y).$$

It is clear (Pic. 3), that

$$\begin{cases} \left|\overrightarrow{AA_1}\right| < \left|\overrightarrow{AA'}\right| \\ \left|\overrightarrow{BB_1}\right| < \left|\overrightarrow{BB'}\right| \end{cases} \qquad (3)$$

and, consequently, these inequalities obey the projections of the vectors [9]:

$\Delta x - s \cdot \Delta\psi < \Delta X; \Delta y < \Delta Y.$ (4)

$\Delta x + s \cdot \Delta\psi < \Delta X'; \Delta y < \Delta Y'.$ (5)

Let's consider the difference in the coordinates of the vectors of the real displacement of the points of the contact and kinematic spots. As in [2, 3], the length of the kinematic displacement of the contact spots will be assumed to be equal to:

$\Delta l_i = r_i \cdot \Delta\theta, (i = 1,2),$ (6)

where r_1, r_2 are radii of the left and right wheels respectively; $\Delta\theta$ –angle of rotation of the wheel set in time Δt.

Obviously, in this case, the projections of the kinematic displacement vector are:

$$\begin{cases} \Delta X = \Delta l_1 \cdot Cos\psi \cong r_1 \cdot \Delta\theta; \Delta Y = \Delta l_1 \cdot Sin\psi \cong r_1 \cdot \Delta\theta \cdot \psi; \\ \Delta X' = \Delta l_2 \cdot Cos\psi \cong r_2 \cdot \Delta\theta; \Delta Y' = \Delta l_2 \cdot Sin\psi \cong r_2 \cdot \Delta\theta \cdot \psi. \end{cases} \qquad (7)$$

Taking into account formulas (7), we obtain the differences of the projections of the vectors $\overrightarrow{AA'}, \overrightarrow{AA_1}$ and $\overrightarrow{BB'}, \overrightarrow{BB_1}$, i.e. the projections of vectors $\overrightarrow{A_1A'}, \overrightarrow{B_1B'}$ that are the differences of linear displacements between wheels and wheel set rails.

Thus, we have:

$$\begin{cases}\overrightarrow{A_1A'}(r_1\cdot\Delta\theta-\Delta x+s\cdot\Delta\psi;\ r_1\cdot\Delta\theta\cdot\psi-\Delta y)\\ \overrightarrow{B_1B'}(r_2\cdot\Delta\theta-\Delta x-s\cdot\Delta\psi;\ r_2\cdot\Delta\theta\cdot\psi-\Delta y).\end{cases} \qquad (8)$$

Analysis of the formulas (8) shows that physically projecting displacements on the Oy axis express deformations of the surface layers of the wheel and rail in contact spots along and across the track. And, note, the transverse deformations for the left and right wheels are different by an amount equal to the difference in the wheels' rolling circles.

We compare the deformations (8) with those given in formulas (1), (2). To this end, we integrate on the interval Δt the right-hand sides of the expressions of the formulas (1), (2), without taking into account the denominator equal to the speed of translational motion.

Omitting the intermediate calculations, we obtain for the left wheel

$$\begin{cases}\int\limits_{\Delta t}(r_1\cdot\dot{\omega}-\dot{x}+s\cdot\dot{\psi})dt=r_1\cdot\Delta\theta-\Delta x+s\cdot\Delta\psi\\ \int\limits_{\Delta t}(\dot{y}-\dot{x}\psi)dt=\Delta y-\Delta x\cdot\psi\end{cases} \qquad (9)$$

and for the right wheel

$$\begin{cases}\int\limits_{\Delta t}(r_2\cdot\dot{\omega}-\dot{x}-s\cdot\dot{\psi})dt=r_2\cdot\Delta\theta-\Delta x-s\cdot\Delta\psi\\ \int\limits_{\Delta t}(\dot{y}-\dot{x}\psi)dt=\Delta y-\Delta x\cdot\psi.\end{cases} \qquad (10)$$

Analyzing the right-hand sides of the first formulas (9) and (10), we note that they express the difference of longitudinal displacements and coincide with the first coordinates of the vectors (8). Analysis of the second formulas (9) and (10) shows that they are equal. But this can not be, since with the translational-rotational movement of the wheel set, the path traversed by the wheels is not the same. At the same time, the physical essence of the terms Δy and $\Delta x\cdot\psi$ differs. The first term represents the actual displacement of the center of mass of the wheel set across the track, and the second is the actual displacement of the center of mass along the track, multiplied by the angle of rotation of the wheel set. These displacements are neither geometrically nor physically related to the contact spots of the wheels of the wheel set.

But this contradicts the definition of creep.

And this is not a difference in the kinematic and actual movements of the contact spots, which is the foundation of the creep, and cannot cause deformation of the surface layers of the wheel and rail in the direction along the Oy axis.

It is known that the use of creep, calculated from the formulas (2), for mathematical modeling of the motion of a wheel set or a bogie with two degrees of freedom [2] leads to two linear differential equations of the second order. The study of the trajectory of the center of mass, obtained from this model, indicates its instability. Perhaps this is a consequence of incorrect creep modeling in the transverse direction.

The use of formulas (8) for the creep modeling is physically justified, although the differential equations can be nonlinear. However, with the development of computer technology and methods for investigating the stability of solutions of differential equations, nonlinearity will not cause significant difficulties.

Conclusion. *We return in conclusion to the formulas for calculating the creep (9), (10). The longitudinal creep of the left and right wheels are projections of the real creep, the direction of which does not coincide with the direction of the longitudinal axis of coordinates. Therefore, it would be advisable from the physical and geometric points of view to calculate the transverse creep by multiplying the longitudinal creep by the angle of rotation of the wheel set. If we denote the longitudinal creep of the wheels as Δ_{xi}, the longitudinal – Δ_{yi} (i = 1,2), then it is more natural to calculate the longitudinal creep as:*

$$\begin{cases}\Delta_{y1}=\Delta_{x1}\cdot\psi=(r_1\cdot\Delta\theta-\Delta x+s\cdot\Delta\psi)\cdot\psi\\ \Delta_{y2}=\Delta_{x2}\cdot\psi=(r_2\cdot\Delta\theta-\Delta x-s\cdot\Delta\psi)\cdot\psi.\end{cases} \qquad (11)$$

Formulas (11) are obtained under the assumption that the directions of the real $(\overrightarrow{AA_1}, \overrightarrow{BB_1})$ and kinematic $(\overrightarrow{AA'}, \overrightarrow{BB'})$ displacements coincide. Nevertheless, formulas (11) reflect the physical nature of creep, in contrast to the formulas for transverse creep (9, 10).

Thus, to calculate the interaction forces between wheels and rails, the application of the creep (8) and (11) formulas is physically more justified.

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