SITING OF RESCUE SERVICES NEAR ONE-SIDED TRANSPORT HUB

Gusev, Sergey A., *LLC Stroitorgi, Moscow, Russia.*

ABSTRACT

When designing social facilities, it is important to choose a right location for them, to make it easier for them to perform their functions in the allotted area. In the works of the author [1–5], the problems of the optimal allocation of resources associated with traffic safety programs in transport are being studied. In works [7–8] an optimal location of rescue services with a normal and uniform distribution of transport hubs is found. The article deals with a one-sided transport hub as an exponential distribution law with certain intensity. The problem of the optimal location of several rescue services on the main line is studied.

Keywords: *safety, exponential distribution law, one-sided transport hub, extremum of variable functions, rescue services, hub intensity.*

Background. *To maintain the safety and rhythmic operation of the railway, it is necessary to have repair and rescue services. The placement of repair and rescue services influences the cost of maintenance, service time, convenience in work. Different parts of the main line have different intensity of transport, so areas with greater intensity require more frequent maintenance. The article considers a one-sided transport hub with intensity in the form of an exponential distribution law and for it the problem of the optimal location of several rescue services is solved.*

Objective. *The objective of the author is to consider location of rescue services near one-sided transport hub.*

Methods. *The author uses general scientific methods, comparative analysis, mathematical apparatus, evaluation approach.*

Results.

Indicative law for a one-sided node

Let's suppose that on the main line we have a transport hub T(0), and it is the final and the starting point of departure (i. e. the start and end of the main line). Some trains go from the transport hub along a dedicated main line, and some leave it in some places. Let us denote by G the intensity of the transport hub – the number of cars that have passed through it in one year. Suppose that the number of cars on the main line near the transport hub is distributed according to the exponential law [8] with the parameter λ and the intensity G (see Pic. 1). Thus, we get that the number of cars that were on the section AB for the year:

$$N = G\int_A^B \lambda e^{-\lambda x}dx.$$

It is necessary to place several rescue services on the main line so that the total maintenance costs are minimal.

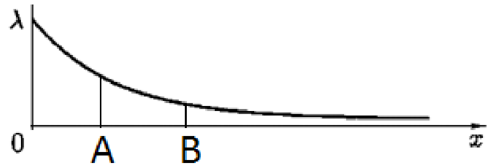
Optimal location of one rescue service

Let the rescue service C be located at point c (Pic. 2).

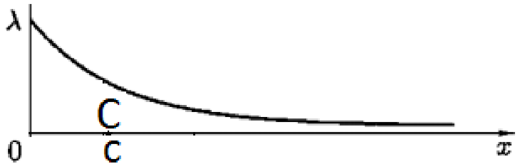
The total costs of the rescue service C will be proportional to:

$$\mathrm{A} = \mathrm{G}\int_0^c (c-x)\lambda e^{-\lambda x}dx + \mathrm{G}\int_c^\infty (x-c)\lambda e^{-\lambda x}dx.$$

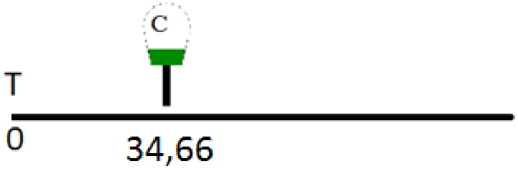
It is necessary to find c, for which A is minimal. We denote the first integral by A_1, and the second by A_2 and compute them separately.



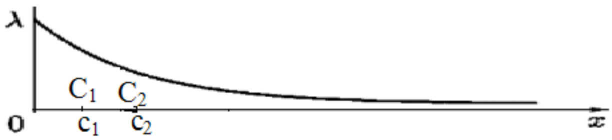
Pic. 1. Indicative location of transport near the transport hub.



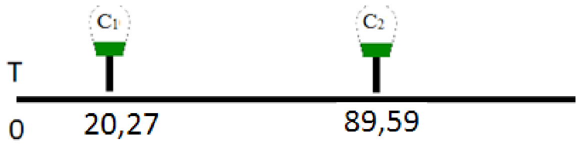
Pic. 2. Location of the rescue service on the main line.



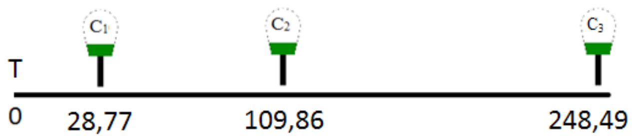
Pic. 3. Optimal location of one rescue service.



Pic. 4. Location of rescue services.



Pic. 5. Optimal location of two rescue services.



Pic. 6. Optimal location of three rescue services.

$$A_1 = -G\int_0^c (c-x)de^{-\lambda x} =$$

$$= -G(c-x)e^{-\lambda x}\Big|_0^c - G\int_0^c e^{-\lambda x}dx = Gc + \frac{Ge^{-\lambda x}}{\lambda}\Big|_0^c =$$

$$= Gc + \frac{G}{\lambda}(e^{-\lambda c}-1).$$

$$A_2 = -G\int_c^\infty (x-c)de^{-\lambda x} = -G(x-c)e^{-\lambda x}\Big|_c^\infty +$$

$$+G\int_c^\infty e^{-\lambda x}dx = \frac{Ge^{-\lambda c}}{\lambda}.$$

$$A = Gc + \frac{G}{\lambda}(2e^{-\lambda c}-1).$$

$\frac{dA}{d} = G - 2Ge^{-\lambda c} = 0$ or $e^{-\lambda c} = \frac{1}{2}$, hence $c = \frac{ln2}{\lambda}$.

When $c = \frac{ln2}{\lambda}$ the derivative $\frac{dA}{d} = 0$, when $c > \frac{ln2}{\lambda}$ the derivative > 0, and when $c < \frac{ln2}{\lambda}$ the derivative < 0.

Hence it follows that $c = \frac{ln2}{\lambda}$ – minimum point.

Let's consider a numerical example of applying the obtained result.

<u>Example 1.</u> Let a one-sided transport hub have $1/\lambda = 50$ (km). It is required to place one rescue service in the optimal way for it.

<u>Solution.</u> $50 \cdot ln2 \approx 34{,}66$, so the rescue service should be at point C(34,66(km)). Pic. 3. schematically depicts the location of the rescue service.

<u>Note</u>. The intensity G does not affect the results, so from now we set G = 1.

Optimal location of two rescue services

Suppose that we need to optimally arrange two rescue services $C_1(c_1)$ and $C_2(c_2)$ to service the transport hub (for the sake of definiteness, assume that $c_1 < c_2$). Pic. 4 shows the schematic placement of rescue services.

Then the total costs of rescue services $C_1(c_1)$ and $C_2(c_2)$ will be proportional:

$$A = \int_0^{c_1}(c_1-x)\lambda e^{-\lambda x}dx + \int_{c_1}^{\frac{c_1+c_2}{2}}(x-c_1)\lambda e^{-\lambda x}dx +$$

$$+ \int_{\frac{c_1+c_2}{2}}^{c_2}(c_2-x)\lambda e^{-\lambda x}dx + \int_{c_2}^{\infty}(x-c_2)\lambda e^{-\lambda x}dx.$$

It is necessary to find c_1 and c_2, while A is minimal. We divide the integrals into three groups:

$$A_1 = \int_0^{c_1}(c_1-x)\lambda e^{-\lambda x}dx = c_1 + \frac{1}{\lambda}(e^{-\lambda c_1}-1).$$

$$A_3 = \int_{c_2}^{\infty}(x-c_2)\lambda e^{-\lambda x}dx = \frac{1}{\lambda}e^{-\lambda c_2}.$$

$$A_2 = \int_{c_1}^{\frac{c_1+c_2}{2}}(x-c_1)\lambda e^{-\lambda x}dx + \int_{\frac{c_1+c_2}{2}}^{c_2}(c_2-x)\lambda e^{-\lambda x}dx =$$

$$= -\int_{c_1}^{\frac{c_1+c_2}{2}}(x-c_1)de^{-\lambda x} - \int_{\frac{c_1+c_2}{2}}^{c_2}(c_2-x)de^{-\lambda x} =$$

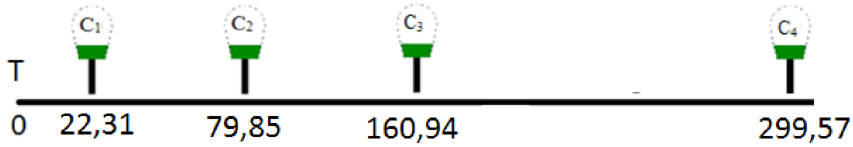
$$= -(x-c_1)e^{-\lambda x}\Big|_{c_1}^{\frac{c_1+c_2}{2}} + \int_{c_1}^{\frac{c_1+c_2}{2}}e^{-\lambda x}dx - (c_2-x)e^{-\lambda x}\Big|_{\frac{c_1+c_2}{2}}^{c_2} -$$

$$- \int_{\frac{c_1+c_2}{2}}^{c_2}e^{-\lambda x}dx =$$

$$= -\frac{c_2-c_1}{2}e^{-\lambda\frac{c_1+c_2}{2}} -$$

$$-\frac{e^{-\lambda x}}{\lambda}\Big|_{c_1}^{\frac{c_1+c_2}{2}} + \frac{c_2-c_1}{2}e^{-\lambda\frac{c_1+c_2}{2}} + \frac{e^{-\lambda x}}{\lambda}\Big|_{\frac{c_1+c_2}{2}}^{c_2} =$$

$$= \frac{1}{\lambda}(e^{-\lambda c_1} + e^{-\lambda c_2} - 2e^{-\lambda\frac{c_1+c_2}{2}}).$$



Pic. 7. Optimal location of four rescue services.

Thus, for A we have:

$$A = c_1 + \frac{2}{\lambda}\left(e^{-\lambda c_1} + e^{-\lambda c_2} - e^{-\lambda\frac{c_1+c_2}{2}}\right) - \frac{1}{\lambda}.$$

We write down the necessary condition for the extremum of a function of two variables A(c_1; c_2):

$$\frac{\partial A}{\partial c_1} = 1 - 2e^{-\lambda c_1} + e^{-\lambda\frac{c_1+c_2}{2}} = 0;$$

$$\frac{\partial A}{\partial c_2} = -2e^{-\lambda c_2} + e^{-\lambda\frac{c_1+c_2}{2}} = 0.$$

From the second equation we have:

$$-\lambda\frac{c_1+c_2}{2} = -\lambda c_2 + ln2, \text{ or } \frac{c_2-c_1}{2} = \frac{ln2}{\lambda},$$

or $c_2 = c_1 + \frac{2 \bullet ln2}{\lambda}.$

Substituting in the first equation, we get:

$$1 - 2e^{-\lambda c_1} + e^{-\lambda(c_1+\frac{ln2}{\lambda})} = 0, \text{ or } 1 - \frac{3}{2}e^{-\lambda c_1} = 0,$$

hence $c_1 = \frac{ln\frac{3}{2}}{\lambda}.$

Substituting in the expression for c_2, we get:

$$c_2 = \frac{ln6}{\lambda}.$$

Let us verify the resulting stationary point in terms of sufficient conditions:

$$\frac{\partial^2 A}{\partial c_1^2} = 2\lambda e^{-\lambda c_1} - \frac{\lambda}{2}e^{-\lambda\frac{c_1+c_2}{2}};$$

At the stationary point $\frac{\partial^2 A}{\partial c_1^2} = \frac{7}{6}\lambda.$

$$\frac{\partial^2 A}{\partial c_2^2} = 2\lambda e^{-\lambda c_2} - \frac{\lambda}{2}e^{-\lambda\frac{c_1+c_2}{2}};$$

At the stationary point $\frac{\partial^2 A}{\partial c_2^2} = \frac{1}{6}\lambda.$

$$\frac{\partial^2 A}{\partial c_1 \partial c_2} = -\frac{\lambda}{2}e^{-\lambda\frac{c_1+c_2}{2}};$$

At the stationary point $\frac{\partial^2 A}{\partial c_1 \partial c_2} = -\frac{1}{6}\lambda.$

We compute the Jacobian:

$$J = \lambda^2 \begin{vmatrix} \frac{7}{6} & -\frac{1}{6} \\ -\frac{1}{6} & \frac{1}{6} \end{vmatrix} = \frac{1}{6}\lambda^2 > 0.$$

Here the stationary point is extreme, since $\frac{\partial^2 A}{\partial c_1^2} > 0,$ *it follows that this is the minimum point.*

Let's consider a numerical example of applying the obtained result.

<u>Example 2</u>. Let a one-sided transport hub have 1/λ= 50 (km). It requires an optimal placement of two rescue services.

<u>Solution</u>. $50 \bullet ln\frac{3}{2} \approx 20,27;\ 50 \bullet ln6 \approx 89,59,$

therefore, rescue services should be placed at points C_1(20,27 (km)), C_2(89,59 (km)). Pic. 5 schematically shows the location of rescue services:

Optimal location of three rescue services

Suppose that we need to optimally arrange three rescue services $C_1(c_1)$, $C_2(c_2)$ and $C_3(c_3)$ to serve a one-sided transport hub (for the sake of definiteness assume that $c_1 < c_2 < c_3$). Performing similar operations of the previous section, we write down the necessary condition for the extremum of a function of three variables A(c_1; c_2; c_3):

$$\frac{\partial A}{\partial c_1} = 1 - 2e^{-\lambda c_1} + e^{-\lambda\frac{c_1+c_2}{2}} = 0;$$

$$\frac{\partial A}{\partial c_2} = -2e^{-\lambda c_2} + e^{-\lambda\frac{c_1+c_2}{2}} + e^{-\lambda\frac{c_2+c_3}{2}} = 0;$$

$$\frac{\partial A}{\partial c_3} = -2e^{-\lambda c_3} + e^{-\lambda\frac{c_2+c_3}{2}} = 0.$$

From the third equation we have:

$$-\lambda\frac{c_2+c_3}{2} = -\lambda c_3 + ln2, \text{ or } \frac{c_3-c_2}{2} = \frac{ln2}{\lambda},$$

or $c_3 = c_2 + \frac{2 \bullet ln2}{\lambda}.$

Substituting the obtained expression for c_3 into the second equation, we obtain:

$$-2e^{-\lambda c_2} + e^{-\lambda\frac{c_1+c_2}{2}} + e^{-\lambda(c_2+\frac{\ln 2}{\lambda})} = 0; \text{ or}$$

$$-2e^{-\lambda c_2} + e^{-\lambda\frac{c_1+c_2}{2}} + \frac{1}{2}e^{-\lambda c_2} = 0,$$

Hence we have:

$$\frac{c_2 - c_1}{2} = \frac{\ln(\frac{3}{2})}{\lambda}, \text{ or } c_2 = c_1 + \frac{2\cdot\ln(\frac{3}{2})}{\lambda}.$$

Substituting the obtained expression for c_2 in the first equation and expressing c_1, we obtain:

$$c_1 = \frac{ln\frac{4}{3}}{\lambda}, \text{ hence we find } c_2 = \frac{ln3}{\lambda},\ c_3 = \frac{ln12}{\lambda}.$$

Let us verify the resulting stationary point for sufficient conditions:

$$\frac{\partial^2 A}{\partial c_1^2} = 2\lambda e^{-\lambda c_1} - \frac{\lambda}{2} e^{-\lambda \frac{c_1 + c_2}{2}};$$

$$\frac{\partial^2 A}{\partial c_2^2} = 2\lambda e^{-\lambda c_2} - \frac{\lambda}{2} e^{-\lambda \frac{c_1 + c_2}{2}} - \frac{\lambda}{2} e^{-\lambda \frac{c_2 + c_3}{2}};$$

$$\frac{\partial^2 A}{\partial c_3^2} = 2\lambda e^{-\lambda c_3} - \frac{\lambda}{2} e^{-\lambda \frac{c_2 + c_3}{2}};$$

$$\frac{\partial^2 A}{\partial c_1 \partial c_2} = -\frac{\lambda}{2} e^{-\lambda \frac{c_1 + c_2}{2}};$$

$$\frac{\partial^2 A}{\partial c_1 \partial c_3} = 0;$$

$$\frac{\partial^2 A}{\partial c_2 \partial c_3} = -\frac{\lambda}{2} e^{-\lambda \frac{c_2 + c_3}{2}}.$$

We compose the Hessian matrix (we will not write the general factor λ):

$$\begin{pmatrix} \frac{5}{4} & -\frac{1}{4} & 0 \\ -\frac{1}{4} & \frac{1}{3} & -\frac{1}{12} \\ 0 & -\frac{1}{12} & \frac{1}{12} \end{pmatrix}.$$

We apply the Sylvester's criterion $\Delta_1 = 5/4 > 0$;

$$\Delta_2 = \begin{vmatrix} 5/4 & -1/4 \\ -1/4 & 1/3 \end{vmatrix} > 0;$$

$$\Delta_3 = \begin{vmatrix} 5/4 & -1/4 & 0 \\ -1/4 & 1/3 & -1/12 \\ 0 & -1/12 & 1/12 \end{vmatrix} > 0.$$

It follows that the stationary point is the minimum point.

So, we have got the final answer: the optimal coordinates of three rescue services

$$c_1 = \frac{ln\frac{4}{3}}{\lambda},\ c_2 = \frac{ln3}{\lambda},\ c_3 = \frac{ln12}{\lambda}.$$

Let's consider a numerical example of applying the obtained result.

<u>Example 3.</u> Let a one-sided transport hub have $1/\lambda = 100$ (km). It requires an optimal placement of three rescue services.

<u>Solution</u>.

$$100\cdot ln\frac{4}{2} \approx 28{,}77;\ 100\cdot ln3 \approx 109{,}86;\ 100\cdot ln12 \approx 248{,}49,$$

therefore, rescue services should be located at points C_1(28,77 (km)), C_2(109,86 (km)), C_3(248,49 (km)). Pic. 6 schematically depicts the location of rescue services.

Optimal location of several rescue services

In this section, we will analyze the general case, i. e. suppose that we need to optimally arrange κ rescue services $C_1(c_1)$, $C_2(c_2)$, ..., $C_\kappa(c_\kappa)$ to serve a one-sided transport hub (for definiteness we assume that $c_1 < c_2 < ... < c_\kappa$). Proceeding analogous to the previous calculations of the action, we write the necessary condition for the extremum of the function of κ variables $A(c_1; c_2; ..., c_\kappa)$:

$$\frac{\partial A}{\partial c_1} = 1 - 2e^{-\lambda c_1} + e^{-\lambda \frac{c_1 + c_2}{2}} = 0;$$

$$\frac{\partial A}{\partial c_2} = -2e^{-\lambda c_2} + e^{-\lambda \frac{c_1 + c_2}{2}} + e^{-\lambda \frac{c_2 + c_3}{2}} = 0;$$

....

$$\frac{\partial A}{\partial c_{\kappa-2}} = -2e^{-\lambda c_{\kappa-2}} + e^{-\lambda \frac{c_{\kappa-3} + c_{\kappa-2}}{2}} +$$
$$+ e^{-\lambda \frac{c_{\kappa-2} + c_{\kappa-1}}{2}} = 0;$$

$$\frac{\partial A}{\partial c_{\kappa-1}} = -2e^{-\lambda c_{\kappa-1}} + e^{-\lambda \frac{c_{\kappa-2} + c_{\kappa-1}}{2}} +$$
$$+ e^{-\lambda \frac{c_{\kappa-1} + c_{\kappa}}{2}} = 0;$$

$$\frac{\partial A}{\partial c_{\kappa}} = -2e^{-\lambda c_{\kappa}} + e^{-\lambda \frac{c_{\kappa-1} + c_{\kappa}}{2}} = 0.$$

From the last equation we have:

$$-\lambda \frac{c_{\kappa-1} + c_\kappa}{2} = -\lambda c_\kappa + ln2, \text{ or } \frac{c_\kappa - c_{\kappa-1}}{2} = \frac{ln2}{\lambda},$$

$$\text{or } c_\kappa = c_{\kappa-1} + \frac{2\cdot ln2}{\lambda}.$$

Substituting the obtained expression for c_κ into the preceding equation, we obtain:

$$-2e^{-\lambda c_{\kappa-1}} + e^{-\lambda \frac{c_{\kappa-2} + c_{\kappa-1}}{2}} +$$
$$+ e^{-\lambda (c_{\kappa-1} + \frac{\ln 2}{\lambda})} = 0; \text{ or}$$

$$-2e^{-\lambda c_{\kappa-1}} + e^{-\lambda \frac{c_{\kappa-2} + c_{\kappa-1}}{2}} +$$
$$+ \frac{1}{2} e^{-\lambda c_{\kappa-1}} = 0,$$

Hence we have:

$$\frac{c_{\kappa-1} - c_{\kappa-2}}{2} = \frac{\ln(\frac{3}{2})}{\lambda}, \text{ or } c_{\kappa-1} = c_{\kappa-2} + \frac{2\cdot\ln(\frac{3}{2})}{\lambda}.$$

Substituting this expression into the equation for $\frac{\partial A}{\partial c_{\kappa-2}}$ and expressing $c_{\kappa-2}$, we obtain:

$$c_{\kappa-2} = c_{\kappa-3} + \frac{2\bullet\ln(\frac{4}{3})}{\lambda}.$$

Then we get:

$$c_{\kappa-\text{м}+1} = c_{\kappa-\text{м}} + \frac{2\bullet\ln(\frac{\text{м}+1}{\text{м}})}{\lambda},\ \text{м} = 1,\ 2,\ \ldots,\ \kappa-1.$$

When м = κ – 1 we have:

$$c_2 = c_1 + \frac{2\bullet\ln(\frac{\kappa}{\kappa-1})}{\lambda}.$$

Substituting in the first equation, we get:

$$c_1 = \frac{\ln(\frac{\kappa+1}{\kappa})}{\lambda} = \frac{\ln(\frac{(\kappa+1)\kappa}{\kappa^2})}{\lambda}.$$

Substituting in the expression for c_2, we get:

$$c_2 = \frac{\ln(\frac{(\kappa+1)\kappa}{\kappa^2})}{\lambda} + \frac{2\bullet\ln(\frac{\kappa}{\kappa-1})}{\lambda} = \frac{\ln(\frac{(\kappa+1)\kappa}{(\kappa-1)^2})}{\lambda}.$$

Then we get:

$$c_\text{м} = \frac{\ln(\frac{(\kappa+1)\kappa}{(\kappa-\text{м}+1)^2})}{\lambda},\ \text{м} = 1,\ 2,\ \ldots,\ \kappa.$$

Let us check the derived formulas for the variants already studied.

For κ = 1 we had $c = \frac{ln2}{\lambda}$.

In the general case $c_1 = \frac{\ln(\frac{(\kappa+1)\kappa}{\kappa^2})}{\lambda}$, *that when κ = 1 coincides with the available result.*

For κ = 2 we had $c_1 = \frac{ln\frac{3}{2}}{\lambda},\ c_2 = \frac{ln6}{\lambda}$.

In the general case

$c_1 = \frac{\ln(\frac{(\kappa+1)\kappa}{\kappa^2})}{\lambda},\ c_2 = \frac{\ln(\frac{(\kappa+1)\kappa}{(\kappa-1)^2})}{\lambda}$, *that when κ = 2 coincides with the result obtained.*

For κ = 3 we had $c_1 = \frac{ln\frac{4}{3}}{\lambda},\ c_2 = \frac{ln3}{\lambda},\ c_3 = \frac{ln12}{\lambda}$.

In the general case

$$c_1 = \frac{\ln(\frac{(\kappa+1)\kappa}{\kappa^2})}{\lambda},\ c_2 = \frac{\ln(\frac{(\kappa+1)\kappa}{(\kappa-1)^2})}{\lambda},\ c_3 = \frac{\ln(\frac{(\kappa+1)\kappa}{(\kappa-2)^2})}{\lambda},$$

that when κ = 3 coincides with the result obtained.

To apply the resulting general result, let's look at a numerical example.

<u>Example 4.</u> Let a one-sided transport hub have 1/λ = 100 (km). It requires optimal placement of four rescue services.

<u>Solution</u>. In the general case, we write out the optimal points:

$$c_1 = \frac{\ln(\frac{(\kappa+1)\kappa}{\kappa^2})}{\lambda},\ c_2 = \frac{\ln(\frac{(\kappa+1)\kappa}{(\kappa-1)^2})}{\lambda},$$

$$c_3 = \frac{\ln(\frac{(\kappa+1)\kappa}{(\kappa-2)^2})}{\lambda},\ c_4 = \frac{\ln(\frac{(\kappa+1)\kappa}{(\kappa-3)^2})}{\lambda}.$$

When κ = 4 we have

$$c_1 = \frac{ln\frac{5}{4}}{\lambda},\ c_2 = \frac{ln\frac{20}{9}}{\lambda},\ c_3 = \frac{ln5}{\lambda},\ c_4 = \frac{ln20}{\lambda}.$$

$$100\bullet ln\frac{5}{4} \approx 22,31;\ 100\bullet ln\frac{20}{9} \approx 79,85;$$

$$100\bullet ln5 \approx 160,94;\ 100\bullet ln20 \approx 299,57,$$

therefore, rescue services should be located at the points C_1(22,31 (km)), C_2(79,85 (km)), C_3(160,94 (km)), C_4(299,57 (km)).

Pic. 7 schematically depicts the location of rescue services.

Note that the test for sufficient conditions cannot be carried out, since it can be seen from the statement of the problem that the minimum is attained, and since it has been shown that there is only one stationary point, hence it is a minimum point.

Conclusion. *The article deals with the problem of optimal placement of several rescue services when servicing one-sided transport hub. Analytical formulas for the coordinates of rescue services are found. A few examples are given. The developed approaches and schemes can be used for the optimal placement of various objects (medical stations, transport stops, etc.) near the terminal points.*

REFERENCES

1. Gusev, S. A. Theorems of Resource Allocation. *World of Transport and Transportation*, Vol. 8, 2010, Iss. 3, pp. 30–35.
2. Gusev, S. A. Income Functions and Risk Prevention. *World of Transport and Transportation*, Vol. 9, 2011, Iss. 3, pp. 88–91.
3. Gusev, S. A. Economic methods of safety management in railway transport [*Ekonomicheskie metody upravleniya bezopasnost'yu na zheleznodorozhnom transporte*]. *Transportnoe delo Rossii*, 2011, Iss. 2, pp. 110–114.
4. Gusev, S. A. Effectiveness of investing in programs to improve safety [*Effektivnost' vlozheniya sredstv v programmy povysheniya bezopasnosti*]. *Transportnoe delo Rossii*, 2011, Iss. 3, pp. 125–129.
5. Gusev, S. A. Linear and discrete functions of the safety. *World of Transport and Transportation*, Vol. 10, 2012, Iss. 2, pp. 182–185.
6. Gnedenko, B. V. Course of the theory of probability [*Kurs teorii veroyatnostei*]. 6th ed., rev. and enl. Moscow, Nauka publ., 1988, 448 p.
7. Gusev, A. I., Gusev, S. A. Optimal location of rescue service. *World of Transport and Transportation*, Vol. 15, 2017, Iss. 4, pp. 194–201.
8. Gusev, A. I., Gusev, S. A., Milevsky, A. S. Provision of optimal transit service under even distribution of facilities. *World of Transport and Transportation*, Vol. 15, 2017, Iss. 6, pp. 32–46. ●

Information about the author:
Gusev, Sergey A. – manager of the financial department of LLC Stroitorgi, Moscow, Russia, 7781011@gmail.com.

Article received 21.02.2017, revised 15.04.2018, accepted 23.05.2018.