CONFIDENCE LIMITS OF MEASUREMENT RESULTS

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ABSTRACT

Problems relating to density distribution of estimated probability are solved on the basis of existing theoretical and methodological approaches to the calculation of confidence limits of measurement error. Attention is paid to the diversity of actual occurring distributions, including normal, trapezoidal, exponential Laplace distribution, the t-distribution, and so on. Particular emphasis of the authors is placed on the use of a method based on the simulation of arrays of numbers with a given distribution in the mathematical apparatus for calculating the confidence level. The simulation results with comments are given in seven tables that allow comparing values and justifying advantages and disadvantages of each type of distribution using the criterion of confidence in end values of measurements.

ENGLISH SUMMARY

Background. *To calculate the confidence limits of measurement errors [1–3 et al.] it is necessary to know the probability density distribution w(x), which in most cases is supposed to be an even function. Given this assumption, the confidence limits are symmetric* $\pm\Delta_{дов}$ *and if given confidence level is* $P_{до}$*, then* $\Delta_{дов}$ *is found from the condition*

$$\int_{-\Delta_{дов}}^{\Delta_{дов}} w(x)dx = P_{дов}. \qquad (1)$$

Most often the probability density of an error is assumed to be normal:

$$w(x) = e^{-x^2/2\sigma^2} \big/ \sqrt{2\pi}\sigma\,, \qquad (2)$$

where σ is a mean-square deviation (hereinafter-MSD). This notation will be used for MSD of other distributions.

Despite widespread use of normal distribution, it does not cover the whole variety of really occurring distributions. In particular, in [4] for calculation of uncertainty it is offered to use a trapezoidal distribution, extreme special cases of which are rectangular (uniform) and triangular distributions.

Rectangular distribution is given by

$$w(x) = \begin{cases} 1/2b & |x| \le b; \\ 0 & |x| > b. \end{cases} \qquad (3)$$

For this distribution $\sigma = b/\sqrt{3} \approx 0{,}58b.$

Triangular distribution is given by

$$w(x) = \begin{cases} \big(1-|x|/b\big)\big/b & |x| \le b; \\ 0 & |x| > b. \end{cases} \qquad (4)$$

In this case $\sigma = b/\sqrt{6} \approx 0{,}41b.$

The normal distribution is different from zero on the whole number axis. Trapezoidal, rectangular and triangular distributions, in contrast to normal one are finite, they differ from zero on a finite interval. In this case, near the maximum, which is achievable with x = 0, the normal distribution is more flat than triangular, and less flat than rectangular.

Non-finite distribution, which is less flat and decreases with increasing modulus of the argument slower than normal, will be two-sided exponential distribution (Laplace distribution)

$$w(x) = e^{-|x|/m} \big/ 2m\,, \qquad (5)$$

for which $\sigma = m\sqrt{2} \approx 1{,}41m.$

An example of nonfinite distribution with a flat top, which decreases with increasing modulus of the argument faster than normal:

$$w(x) = 2e^{-x^4/m^4} \big/ m\Gamma(0{,}25) \approx 0{,}552 e^{-x^4/m^4} \big/ m\,. \qquad (6)$$

In this case $\sigma = m\sqrt{\Gamma(0{,}75)/\Gamma(0{,}25)} \approx 0{,}58\,m,$ *where* $\Gamma(.)$ *is a complete gamma function. Below this distribution will be called a rapidly decreasing distribution.*

Objective. *The objective of the authors is to investigate particular aspects of confidence limits of measurements, approaches to their measurement and measurement error distributions.*

Methods. *The authors use mathematical calculations, simulation and analysis.*

Results. *Substituting (2) into (1) leads to the known rules of two sigma and two and a half sigma at confidence levels P = 0,95 and 0,99, respectively. In [5] it is noted that the use of statistical methods without assumptions under which they are derived, can lead to significant errors. This, in particular, applies to the said rules, which under other probability density distributions are unfair.*

Table 1 lists MSD relating to all enlisted distributions. It shows clearly that for more slowly decreasing Laplace distribution $\Delta_{дов}/\sigma$ *is higher and with increasing confidence level is growing more significantly than for a normal distribution. For a rapidly decreasing distribution the situation is reversed. For finite distributions* $\Delta_{дов}/\sigma$ *MSD is smaller than for normal, and with an increase in the confidence level it grows to a lesser extent. For this reason, in case of finite distributions* $\Delta_{дов}$*=b can be set for almost all confidence levels.*

If the law of error distribution and its parameters are known, the calculation of confidence limits for a single measurement in accordance with (1) is quite simple. The situation is more complex for multiple measurements. Thus it is necessary to consider two options:

– MSD of errors of single observations is known a priori;

– MSD of an error of a single observation is determined by the results of multiple measurements.

Let us consider the first option. As a result N of multiple equal measurements arithmetic mean is taken. The error of this result is equal to arithmetic mean of the errors of single observations

$$\Delta_{ср.\,ар.} = \frac{1}{N}\sum_{j=1}^{N}\Delta_j. \qquad (7)$$

In the assumption of independence of single observation errors $\sigma_{ср.ар} = \sigma/\sqrt{N}$*. in case of a normal distribution the error of arithmetic mean will also be normally distributed. Therefore values*

Table 1

Distribution	Confidence level $P_{дов}$			
	0,95	0,98	0,99	0,997
Normal	1,96	2,32	2,57	3,00
Uniform	1,69	1,72	1,72	1,73
Triangular	1,90	2,10	2,20	2,32
Laplace	2,12	2,77	3,26	4,11
Rapidly decreasing	1,80	2,02	2,15	2,34

Table 2

$P_{дов}$	N						
	1	2	3	5	10	20	30
Uniform distribution							
0,95	1,65	1,90	1,93	1,93	1,96	1,95	1,93
0,98	1,70	2,08	2,21	2,27	2,27	2,27	2,26
0,99	1,71	2,18	2,35	2,49	2,52	2,53	2,48
0,997	1,72	0	2,572,3	2,78	2,87	2,79	2,76
Triangular distribution							
0,95	1,86	1,96	1,93	1,90	1,98	1,94	1,92
0,98	2,10	2,24	2,24	2,22	2,35	2,25	2,36
0,99	2,18	2,40	2,49	2,45	2,53	2,44	2,60
0,997	2,32	2,64	2,68	2,78	2,82	2,83	2,98
Laplace distribution							
0,95	2,13	2,02	1,98	1,94	1,94	1,92	1,90
0,98	2,75	2,50	2,48	2,42	2,39	2,31	2,28
0,99	3,23	2,79	2,74	2,76	2,57	2,51	2,47
0,997	3,85	3,32	3,27	3,16	3,07	3,03	2,96
A rapidly decreasing distribution							
0,95	1,85	1,91	1,95	1,94	1,97	1,94	1,93
0,98	2,10	2,31	2,42	2,54	2,53	2,55	2,51
0,95	2,03	2,17	2,24	2,28	2,27	2,29	2,25
0,997	2,15	2,56	2,69	2,80	2,90	2,86	2,86

of the ratio $\Delta_{дов}/\sigma_{ср.ар}=\Delta_{дов}\sqrt{N}/\sigma$ expectedly coincide with values, given in Table 1 for the normal distribution.

For other distributions with unbounded increase in the number of observations distribution of the arithmetic mean according to the central limit theorem will seek to normal. However, at finite N it will differ from the error distribution of a single observation, and from the normal distribution. This distribution can be found by rather cumbersome calculations.

We used a simpler method, based on the simulation of arrays of numbers with given distributions.

Many software packages (e. g., Excel, Mathcad) enable to create random numbers with uniform and some other distributions. Using nonlinear transformation of uniformly distributed random value, it is possible to obtain random variables with any desired distribution.

In nonlinear monotonic transformation of a random variable ν with a probability density $w_\nu(y)$ the result of the transformation $\xi=f(\nu)$ will have a probability density $w_\xi(x)$, defined by the relation

$$w_\xi(x)\left|\frac{df(y)}{dy}\right|=w_\nu(y). \qquad (8)$$

With a uniform distribution of the random variable ν in the interval [-0,5; 0,5] $w_\nu(y)=1$ and for generating a random variable with a triangular distribution (4) according to (8) it is necessary to use a nonlinear transformation

$$\xi=b\left(1-\sqrt{1-2\nu}\right).$$

For formation of a random variable with Laplace distribution (5), we can use the conversion

$\xi=m\ln(1-2\nu)$.

For the distribution (6), condition (8) leads to the equation

$$\int_{-\infty}^{\xi/m} e^{-z^4}dz=y.$$

The authors failed to obtain an exact solution of this equation. Therefore, the authors used an approximate power series expansion that provides acceptable accuracy:

$\xi=m(3{,}118\nu+6{,}734\nu^5+52{,}57\nu^9+489{,}5\nu^{13}+32770\nu^{17})$.

For each summand in (7) 1000 values were formed, and on the results of the processing of the arithmetic mean values of the ratio $\Delta_{дов}/\sigma_{ср.ар}=\Delta_{дов}\sqrt{N}/\sigma$. were obtained. Results of the simulation for distributions (3) – (6) are shown in Table 2.

Table 3

$P_{дов}$	Number of freedom degrees *N*					
	2	3	5	10	20	30
0,95	4,3	3,18	2,57	2,23	2,09	2,04
0,98	6,96	4,54	3,36	2,76	2,53	2,46
0,99	9,92	5,84	4,03	3,17	2,85	2,75

Table 4

$P_{дов}$	Number of freedom degrees *N*					
	2	3	5	10	20	30
Uniform distribution						
0,95	3,27	2,41	1,97	1,80	1,71	1,64
0,98	5,39	3,23	2,50	2,01	1,89	1,76
0,99	7,70	4,49	2,85	2,17	1,98	1,85
Triangular distribution						
0,95	4,45	3,11	2,38	2,18	2,03	1,88
0,98	7,05	4,60	3,06	2,54	2,33	2,12
0,99	9,79	5,86	3,60	2,76	2,50	2,22
Laplace distribution						
0,95	6,34	4,32	3,37	2,67	2,32	2,12
0,98	11,89	7,04	5,09	3,97	3,48	2,90
0,99	16,43	9,49	6,43	4,61	4,22	3,30
A rapidly decreasing distribution						
0,95	3,70	2,70	2,27	2,02	1,90	1,81
0,98	6,12	3,73	2,85	2,42	2,25	2,03
0,99	8,52	4,96	3,39	2,58	2,34	2,19

At N=1 confidence limits coincide with values given in Table 1, significantly different for different distributions. When N increases due to the normalization of the distribution of the arithmetic mean this difference reduces. At N=30 values practically coincide with the confidence limits for a normal distribution.

When MSD of an error in the calculation of confidence limits is unknown, the experimentally obtained estimate is used

$$S = \sqrt{\frac{1}{N}\sum_{j=1}^{N}\Delta_j^2}, \qquad (9)$$

whose square distribution coincides in form with the distribution of chi-square with N degrees of freedom.

Taking into account (9), we assume that in the error assessment the actual value of the measured value is known. Otherwise, if Δ_j is deviation of the arithmetic mean, N in the denominator of the formula (9) is replaced by N–1and the number of degrees of freedom reduces by one. Usually, there are two cases.

The value S can be used to calculate the confidence limits of an error of a single observation and the arithmetic mean. Under the assumption of normality and independence of errors of single observations the calculation is based on the t-distribution.

T-distribution with N degrees of freedom implies a random variable $t=\xi_0\big/\sqrt{\nu/N}$, where ξ_0 is normally distributed with zero mathematical expectation and unit variance, and the value ν, which is independent from it, has chi-square distribution with N degrees of freedom. Given the definition of a random variable with a chi-square distribution, we can write

$$t=\xi_0\Big/\sqrt{\sum_{i=1}^{N}\xi_i^2\Big/N}, \qquad (10)$$

where independent normal random variables $\xi_0, \xi_1, \dots, \xi_N$ have zero mathematical expectations and equal (optional unit) MSDs.

T-distribution implies a rather simple analytical expression, although for its practical application tables are commonly used. And it is important to note that this distribution as normal is defined on the whole number axis.

With account of (9) and (10) for a normally distributed error of a single observation, not used in the calculation of S, the ratio Δ/S will have a t-distribution. Accordingly, Table 3 shows values $\Delta_{дов}/S$ for some values of a confidence level and the number of degrees of freedom.

For distributions of errors, which differ from normal, according to Table 4 values $\Delta_{дов}/S$ become mentioned, obtained from the results of simulated options (3) – (6).

The table shows that the difference between the distribution of a single observation from a normal one rather significantly influences the confidence limits of a single observation. The most significant differences appear for the uniform distribution with a small number of degrees of freedom (due to distribution of the numerator in the formula (10)) and the Laplace distribution (due to a slower decrease of the probability density of the numerator in (10)). For large N, the confidence limits in

Table 5

$P_{дов}$	Number of freedom degrees *N*					
	2	3	5	10	20	30
Normal distribution						
0,95	1,411	1,63	1,81	1,86	1,91	1,93
0,98	1,414	1,70	1,97	2,05	2,23	2,25
0,99	1,414	1,72	2,08	2,15	2,38	2,44
Uniform distribution						
0,95	1,407	1,58	1,64	1,64	1,66	1,64
0,98	1,413	1,67	1,80	1,78	1,77	1,74
0,99	1,414	1,70	1,89	1,90	1,82	1,81
Triangular distribution						
0,95	1,410	1,64	1,77	1,82	1,84	1,84
0,98	1,413	1,69	1,91	2,04	2,04	2,06
0,99	1,414	1,72	2,01	2,21	2,22	2,16
Laplace distribution						
0,95	1,412	1,69	1,98	2,14	2,18	2,18
0,98	1,414	1,72	2,12	2,45	2,59	2,67
0,99	1,414	1,72	2,15	2,65	2,86	2,99
A rapidly decreasing distribution						
0,95	1,408	1,61	1,73	1,77	1,81	1,82
0,98	1,413	1,68	1,89	1,95	1,98	1,98
0,99	1,414	1,70	1,96	2,08	2,07	2,08

Table 6

$P_{дов}$	Number of freedom degrees *N*					
	2	3	5	10	20	30
Uniform distribution						
0,95	3,35	2,54	2,14	2,06	1,98	1,93
0,98	5,58	3,44	2,72	2,50	2,39	2,27
0,99	7,57	4,62	3,25	2,79	2,58	2,44
Triangular distribution						
0,95	4,00	3,04	2,34	2,21	2,01	1,93
0,98	7,12	4,14	2,94	2,60	2,42	2,35
0,99	10,51	5,50	3,35	2,84	2,79	2,74
Laplace distribution						
0,95	6,65	4,39	3,06	2,43	2,27	2,27
0,98	11,44	6,33	4,19	3,43	2,79	2,76
0,99	17,49	8,71	5,06	4,10	3,21	3,13
A rapidly decreasing distribution						
0,95	3,70	2,77	2,24	2,23	2,04	2,05
0,98	6,34	3,86	2,87	2,91	2,43	2,41
0,99	8,42	5,15	3,52	3,38	2,66	2,65

Table 4 are close to the values in Table 1. It occurs due to the fact that with increasing N the spread of S, which stands in the denominator (10), decreases, and the type of the distribution of the ratio t is determined mainly by the type of numerator distribution, i. e. by the type of an error distribution of a single observation.

Other material breach of the applicability of the t-distribution is the dependence of the random variable in the numerator (10) from random variables in the denominator. If as ξ_0 we take one of the values used in the calculation of S (for example, ξ_1), the value of t is limited in modulus

$$|t| = |\xi_1| \Big/ \sqrt{\sum_{i=1}^{N} \xi_i^2 \Big/ N} \le \sqrt{N}, \qquad (11)$$

and its distribution in contrast to t-distribution will be finite, specified in the interval $\pm\sqrt{N}$ for any error distributions of single observations. Therefore, even for a normal distribution of errors the use of confidence limits of t-distribution for calculation will not be justified.

Table 5 shows values $\Delta_{дов}/S$, obtained by the simulation results for the distributions (2) – (6), when we take ξ_1 as ξ_0.

Table 7

$P_{дов}$	The number of freedom degrees *N*					
	2	3	5	10	20	30
Normal distribution						
0,95	1,410	1,70	1,82	1,95	2,01	2,00
0,98	1,414	1,72	1,97	2,20	2,29	2,30
0,99	1,414	1,73	2,03	2,36	2,50	2,49
Uniform distribution						
0,95	1,413	1,69	1,90	1,91	1,92	1,90
0,98	1,414	1,72	2,08	2,24	2,22	2,26
0,99	1,414	1,73	2,13	2,38	2,47	2,51
Triangular distribution						
0,95	1,410	1,63	1,80	1,92	1,90	1,93
0,98	1,413	1,69	1,96	2,18	2,24	2,33
0,99	1,414	1,71	2,04	2,38	2,44	2,59
Laplace distribution						
0,95	1,408	1,62	1,76	1,84	1,90	1,89
0,98	1,413	1,68	1,92	2,09	2,15	2,29
0,99	1,414	1,71	2,00	2,24	2,32	2,46
A rapidly decreasing distribution						
0,95	1,408	1,62	1,76	1,84	1,90	1,89
0,98	1,413	1,68	1,92	2,09	2,15	2,29
0,99	1,414	1,71	2,00	2,24	2,32	2,46

In this case, there are significantly lower values of the confidence limits, especially for small N, compared with the data in Tables 3 and 4. Obviously, this understatement occurs not due to more accurate calculation of a confidence limit, but due to improper use of (10). Hence we talk about the validity of the well-known rule: the detection of gross errors during MSD estimation the verifiable observation, having a maximum deviation from the mean one, should not be taken into account. However, in assessing the confidence limits for errors of single observations, which are not considered as failures, improper use of the relation (10) sometimes takes place.

When calculating the confidence limits of the arithmetic mean to ensure the independence of the numerator and denominator of the expression (10) the arithmetic mean and MSD estimation should be calculated from different arrays, the number of single observations of which may be different. In practice, this condition is rarely taken into account, and the requirement for independence of numerator and denominator by using the t-distribution is violated.

Let the symbol N be the number of observations, on which S is calculated, then through $N_{cp.ap}$ *we denote the number of observations, on which the arithmetic mean is calculated.*

Since the variance of the arithmetic mean is $N_{cp.ap}$ *times smaller than the variance of averaged results, when it is substituted in (10) as* ξ_0*, a factor* $\sqrt{N_{cp..ap}}$ *is introduced. With account of this, in a normal distribution of errors of observation and calculation of the arithmetic mean and S for different arrays of values* $\Delta_{дов}\sqrt{N_{cp.\ ap}}\Big/S$ *at a given confidence level coincides with the values given in Table 3 and the number of degrees of freedom is still equal to N.*

In Table 6 for $N_{cp.ap}$*=N values of the ratio* $\Delta_{дов}\sqrt{N}\Big/S$ *are given for distributions (3) – (6) in calculating the arithmetic mean and MSD for different independent arrays.*

The results obtained are different from the values given in Table 3. As for the values in Tables 1, 2 and 4, the most significant differences relate to the Laplace distribution, because of its slow decrease with increasing modulus of the argument. Similarly to Table 2, with increasing N due to the normalization of the numerator, reduction in denominator spread in (10), and the normalization of its square, these differences decrease.

Similarly to the confidence limits of errors of single observations dependence of numerator and denominator in (10) will substantially understate estimates of the confidence limits of the arithmetic mean by changing the distribution of the variable t.

Without loss of generality, we can assume that the arithmetic mean is determined by the values of the first $N_{cp.ap}$ *values from N values, on which MSD is estimated. Then from (10) we obtain*

$$t=\sqrt{N_{cp.ap}}\sum_{i=1}^{N_{cp.ap}}\xi_i\Big/N_{cp.ap}\Bigg/\sqrt{\sum_{i=1}^{N}\xi_i^2\Big/N}\ .$$

Since

$$\left| \sum_{i=1}^{N_{ср.ар}} \xi_i \Big/ \sqrt{\sum_{i=1}^{N} \xi_i^2} \right| \le \sqrt{N_{ср.ар}},$$

we come to the condition $|t| \le \sqrt{N}$, *which coincides with the inequation (11).*

Thus, for the arithmetic mean, as well as for a single observation, we came to the finite distribution of t on the same interval $\pm\sqrt{N}$, *although the type of the distribution becomes different. This conclusion is fair for both finite and nonfinite distributions w(ξ), including normal.*

In Table 7 for $N_{ср.ар}$*=N values of the ratio* $\Delta_{дов}\sqrt{N}\big/S$ *are given with respect to the distributions under consideration in the calculation of the arithmetic mean and MSD for a single array.*

The values given in this table are significantly lower than the values in Tables 3 and 6. For a small number of observations, they differ by several times. The dependence of these results on the confidence level and the number of observations and their absolute values are close to those in Table 5. However, there with larger N value, the influence of the shape of the distribution of a single observation is more pronounced.

Once again, we note that if at the detection of gross errors the dependence of the numerator and denominator in (10) is usually eliminated, then in estimating the confidence limits for results of multiple measurements, the numerator and denominator in (10) are in most cases calculated by the same data. This leads to a significantly erroneous underestimation of absolute values of the confidence limits even when using the t-distribution for a normal distribution of errors of single observations.

Then it is necessary to briefly discuss the evaluation of the reliability of the simulation results. We will denote through Z values given in Tables 2, 4–7. In each case, they represent the confidence limits of a relationship, which allows obtaining the measurement error. Z value is a function ψ of the confidence level. The form of this function is determined by the form of the distribution of observational errors, the number of observations and the kind of ratio that is used in the calculation of the confidence limits. In principle, it can be determined, but its analytical expression requires rather complicated transformations.

When modeling Z^* *and* $P^*_{дов}$ *assessments are investigated and that is equivalent to the application of the ratio*

$$Z^* = \psi(P^*_{дов}). \qquad (12)$$

From (12) we obtain

$$\sigma_{Z^*} = \sigma_{P^*}\,|d\psi(P_{дов})\big/dP_{дов}| \approx \sigma_{P^*}\,|\delta Z^*\big/\delta P_{дов}|. \qquad (13)$$

*The approximate equation is obtained by replacing the derivative with difference quotient, found by modeling Z *, and the given values* $P_{дов}$*.*

Not MSD of the confidence limit estimation itself is of practical interest, but its relation to the confidence limit

$$\gamma = \sigma_{Z^*}\big/Z \approx \sigma_{P^*}\,|\delta Z^*\big/Z\delta P_{дов}|. \qquad (14)$$

MSD of the confidence level estimation on n results of modeling is given by

$$\sigma_{P^*} = \sqrt{(1-P_{дов})P_{дов}\big/n}. \approx \sqrt{(1-P_{дов})\big/n}.$$

In the simulation the authors used n=1000. Then, for a confidence level of 0,95, 0,98 and 0,99, σ_P *is respectively, equal to 0,0071, 0,0045 and 0,0032.*

Values of $\Delta Z^*/Z^*$ *and* $\Delta P_{дов}$ *can be determined by simulation results. For example, in the interval of confidence levels [0,98; 0,99]* $\Delta P_{дов}$*=0,01, and the ratio* $\Delta Z^*/Z^*$ *varies from a few hundredths to two-tenths (Tables 2, 4–7). Accordingly, we find that the value of γ does not exceed six hundredths, which is quite acceptable for estimating the confidence limits.*

Conclusions.

1. The results shown in Tables 1, 2, 4 and 6, with a priori information about the type of the law of an error distribution of a single observation enable to define the confidence limits. In the absence of this information, it is possible to get an upper estimation for the absolute value of end values. For the considered distributions the worst is the Laplace distribution.

2. The data in Tables 5 and 7 help to assess the degree of an erroneous underestimation of the absolute values of the confidence limits during the use of the same observational results in the evaluation of MSD and measurement errors.

Keywords: *probability theory, metrology, measurement errors, law of error distribution, confidence limits, Laplace distribution, t-distribution, modeling.*

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