PROVISION OF OPTIMAL TRANSIT SERVICE UNDER EVEN DISTRIBUTION OF FACILITIES

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ABSTRACT

The article continues the topic of optimal location of objects in the system of network structures (see World of Transport and Transportation, 2017, Iss. 4 [5]), in particular, develops and focuses on theoretical and methodological approaches to modeling the distribution of service points for urban residents within the boundaries of a residential microdistrict with piecewise- constant density of settlement. The goal of such a scientifically grounded distribution is to minimize the costs of moving between objects, while observing rational proportions including transport demand with transport offers.

Keywords: uniform distribution law, mathematical model, social object, transport, urban microdistrict, optimal location, extremum of functions.

Background. *We all use transport, shops, hospitals, and it is convenient when a social institution, a transport service point are located near us, those areas where our work, study, housing are located. The same applies to the locations of emergency, fire, rescue and other services. Here we will consider the economic-mathematical model of the optimal location of social facilities for servicing the territory in the event of their piece-wise permanent deployment.*

Objective. *The objective of the authors is to consider theoretical and methodological approaches to modeling the distribution of service points within a residential microdistrict.*

Methods. *The authors use general scientific and engineering methods, mathematical apparatus, evaluation approach, modeling.*

Results.

Mathematical model

Let's consider the following model. Suppose we have a residential zone, located along the section [c, d] with a nonnegative population density $f(x)$. We assume that the function $f(x)$ is piecewise continuous. It is required to place on this section several service points so that the total costs of moving to them are minimal. It is assumed that the potential visitor uses the nearest service point.

We denote the points by a_i, i = 1, 2, ..., n (see Pic. 1). Respectively

$$a_{1,2}=\frac{a_1+a_2}{2},\ a_{2,3}=\frac{a_2+a_3}{2},\dots$$

– middle of the sections.

Each point a_i has some «attraction zone» to the left and to the right, going to it from the middle of the section connecting with the next point, or (at the ends) from the extreme points c or d. The costs are proportional to the expression

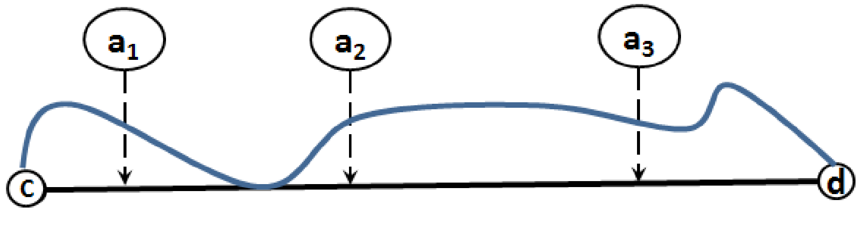
$$Q(a)=\int_c^{a_1}(a_1-x)f(x)dx+\int_{a_1}^{a_{1,2}}(x-a_1)f(x)dx+$$
$$+\int_{a_{1,2}}^{a_2}(a_2-x)f(x)dx+\int_{a_2}^{a_{2,3}}(x-a_2)f(x)dx+\dots+$$
$$+\int_{a_{n-1,n}}^{a_n}(a_n-x)f(x)dx+\int_{a_n}^{d}(x-a_n)f(x)dx.$$

We write down the extremum conditions for the objective function Q. Calculating the partial derivatives, we find:

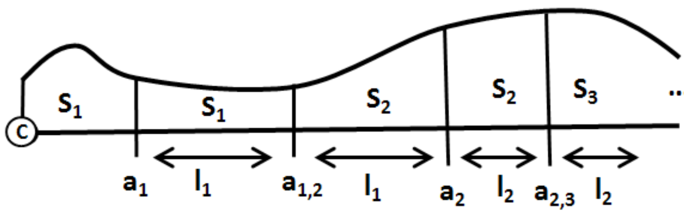
$$\frac{\partial Q}{\partial a_1}=\int_c^{a_1}f(x)dx-\int_{a_1}^{a_{1,2}}f(x)dx,$$
$$\frac{\partial Q}{\partial a_2}=\int_{a_{1,2}}^{a_2}f(x)dx-\int_{a_2}^{a_{2,3}}f(x)dx,\dots,$$
$$\frac{\partial Q}{\partial a_n}=\int_{a_{n-1,n}}^{a_n}f(x)dx-\int_{a_n}^{d}f(x)dx.$$

Equating the derivatives to zero, we obtain for each placed point the necessary conditions for the extremum in the form of the equality of the «population» in the left and right «attraction zones»:

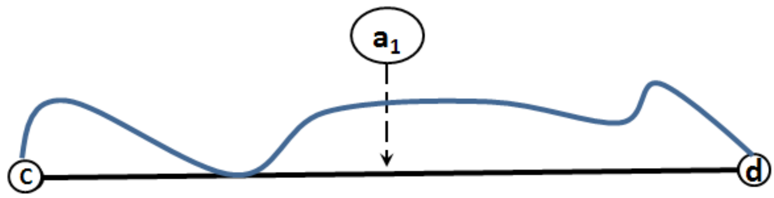
$$\begin{cases}\int_c^{a_1}f(x)dx=\int_{a_1}^{a_{1,2}}f(x)dx\\ \int_{a_{1,2}}^{a_2}f(x)dx=\int_{a_2}^{a_{2,3}}f(x)dx\\ \dots\\ \int_{a_{n-1,n}}^{a_n}f(x)dx=\int_{a_n}^{d}f(x)dx.\end{cases}\qquad(1)$$



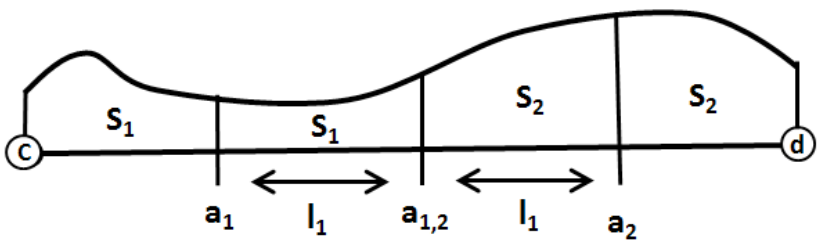
Pic. 1. Placement of objects.



Pic. 2. Necessary conditions for optimality



Pic. 3. One service point.



Pic. 4. Necessary optimality conditions for two points.

These conditions can easily be obtained by simple reasoning. Let there be no equality of population in the left and right attraction zones of a certain point. Then, for example, if the right area is «larger» than the left one, then a small shift of the item to the right will obviously reduce costs. This means that there is no extremum.

We rewrite conditions (1) in a more convenient form, using the notation

$$F(x)=\int_c^x f(t)dt.$$

Then we obtain a chain of equations

$$F(a_{1,2})=2F(a_1),\quad F(a_{2,3})=2F(a_2)-F(a_{1,2}),$$
$$F(a_{3,4})=2F(a_3)-F(a_{2,3}),\ldots,$$
$$F(d)=2F(a_n)-F(a_{n-1,n}).$$

In the assumption[1]
zeros of the function f(x) – isolated (2)
there is a chain of recurrence formulas:

$$a_{1,2}=G(2F(a_1)),\quad a_2=2a_{1,2}-a_1,$$
$$a_{2,3}=G(2F(a_2)-F(a_{1,2})),\quad a_3=2a_{2,3}-a_2,\ldots,$$
$$d=G(2F(a_n)-F(a_{n-1,n}))\Leftrightarrow \quad (3)$$
$$\Leftrightarrow F(d)=2F(a_n)-F(a_{n-1,n}),$$

[1] The case when this condition is violated will be discussed further.

where G(x) – function, inverse of F(x). Substituting each equality into the next one, we obtain an equation of the form

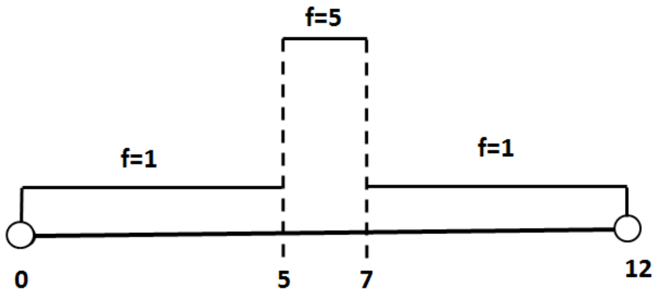
$H(a_1) = F(d)$ (4)

with some function H(x). The set of solutions of such an equation (which, as we shall see, may even be a continuum) determines the location of the first service station. Next points under condition (2) are uniquely determined by the recurrence formulas (3) (see Pic. 2).

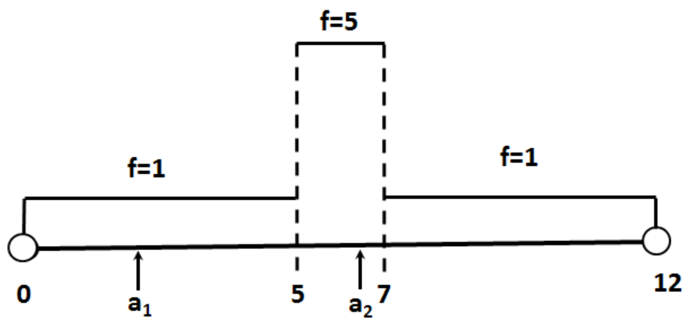
The variants found in this way satisfy the necessary extremum conditions, that is, they are stationary points of the function Q, but are not necessarily its local minima. The presence and form of the extremum, as a rule, can be determined by calculating partial derivatives of the second order:

$$\frac{\partial^2 F}{(\partial a_1)^2}=2f_1-0{,}5f_{1,2},$$
$$\frac{\partial^2 F}{\partial a_1\partial a_2}=-0{,}5f_{1,2}$$
$$\frac{\partial^2 F}{(\partial a_2)^2}=2f_2-0{,}5f_{1,2}-0{,}5f_{2,3},$$
$$\frac{\partial^2 F}{\partial a_2\partial a_3}=-0{,}5f_{2,3},\ldots,$$

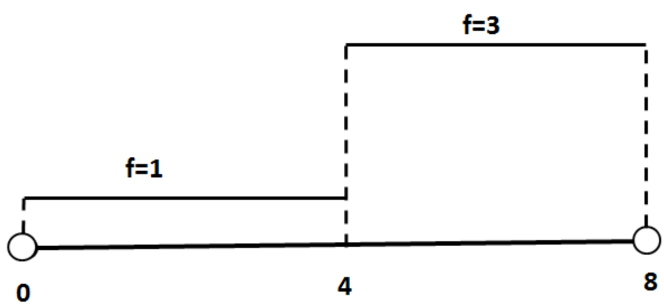
where the notations $f_i = f(a_i)$, $f_{i,j} = f(a_{i,j})$ *are used.*



Pic. 5. An example of a symmetric distribution.



Pic. 6. Optimal placement.



Pic. 7. An example of distribution.

As a result, we obtain the following (three-diagonal) Hessian matrix:

$$\begin{pmatrix} 2f_1-0,5f_{1,2} & -0,5f_{1,2} & 0 & \dots \\ -0,5f_{1,2} & 2f_2-0,5f_{1,2}-0,5f_{2,3} & -0,5f_{2,3} & \dots \\ 0 & -0,5f_{2,3} & 2f_2-0,5f_{1,2}-0,5f_{2,3} & \dots \\ \dots & \dots & \dots & \dots \end{pmatrix}.$$

To apply the Sylvester criterion, it is required that the function f(x) be continuous in the vicinity of the points a_i.

Let's consider simple special cases.

One service point

In this situation, the necessary conditions (1) become

$$\int_c^{a_1} f(x)dx = \int_{a_1}^{d} f(x)dx,$$

that is, the point a_1 should divide the «population» into two equal parts (see Pic. 3). It is clear that such a «median» point is

- *either one (in particular, this will happen when condition (2) is satisfied);*
- *or «median» points form a whole section [α; β] (where $\alpha \neq c$; $\beta \neq d$).*

In the latter case, $f(x) \equiv 0$ on the whole section (at the points α and β, we can redefine f by continuity). That is, all points of the section [α; β] give the same value of the objective function. On leaving the section [α; β], Q values increase, so that each median point is an acceptable response.

Two service points

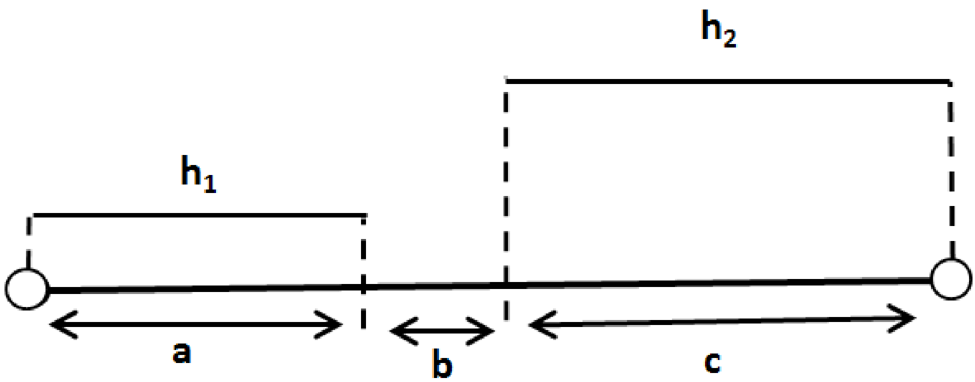
In this situation, the necessary conditions (1) become conditions for the «balance» of attraction zones:

$$\begin{cases} \int_c^{a_1} f(x)dx = \int_{a_1}^{a_{1,2}} f(x)dx \\ \int_{a_{1,2}}^{a_2} f(x)dx = \int_{a_2}^{d} f(x)dx. \end{cases}$$

The graphically necessary condition for the extremum looks as shown in Pic. 4.

In particular, $F(d) = 2S_1 + 2S_2$, $F(a_2) - F(a_1) = S_1 + S_2 = F(d)/2$.

The recurrence relations (3) in this case are written as follows:



Pic. 8. The presence of a «zero» zone.

$$a_{1,2}=G\left(2F\left(a_1\right)\right),\quad a_2=2a_{1,2}-a_1,$$
$$F\left(d\right)=2F\left(a_2\right)-F\left(a_{1,2}\right),$$

and equation (4) takes the form

$$2F\left(2G\left(2F\left(a_1\right)\right)-a_1\right)-2F\left(a_1\right)=F\left(d\right).$$

Accordingly, the Hessian matrix:

$$A=\begin{pmatrix}2f_1-0{,}5f_{1,2} & -0{,}5f_{1,2}\\ -0{,}5f_{1,2} & 2f_2-0{,}5f_{1,2}\end{pmatrix}.$$

Thus, under the assumption that the function f(x) is continuous in the vicinity of the points a_1 and a_2, the Sylvester criterion gives the following sufficient conditions for a local minimum:

$$4f_1>f_{1,2},$$
$$4f_1f_2>f_{1,2}\left(f_1+f_2\right). \qquad (5)$$

Example 1. *Let's take the following situation (see Pic. 5).*

Then

$$F\left(x\right)=\begin{cases}x, & 0\le x<5,\\ 5x-20, & 5\le x<7,\\ x+8, & 7\le x\le 12,\end{cases}$$

$$G\left(x\right)=F^{-1}\left(x\right)=\begin{cases}x, & 0\le x<5,\\ (x+20)/5, & 5\le x<15,\\ x-8, & 15\le x\le 20.\end{cases}$$

Let the point a_1 have the coordinate x. From the relations (3) we obtain successively

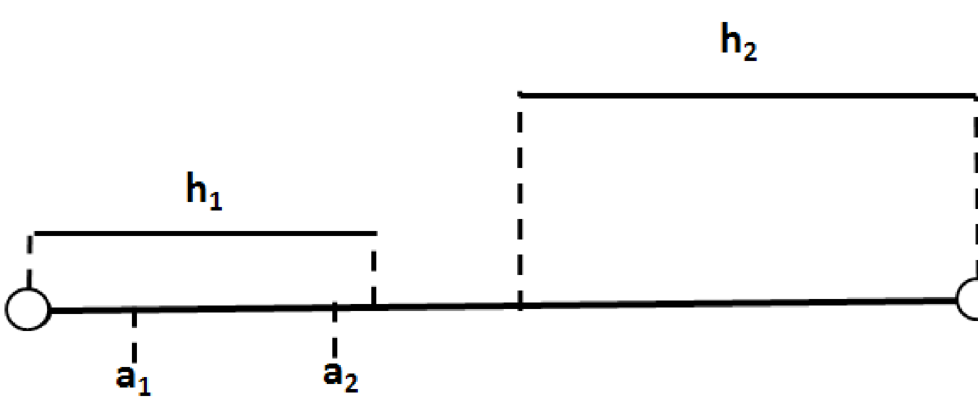
$$S_1=F\left(x\right),\quad a_{1,2}=G\left(2F\left(a_1\right)\right)=\begin{cases}2x, & 0\le x<2{,}5,\\ 0{,}4x+4, & 2{,}5\le x<5,\\ 2x-4, & 5\le x<5{,}5,\\ 10x-48, & 5{,}5\le x\le 6,\end{cases}$$

$$a_2=2a_{1,2}-a_1=\begin{cases}3x, & 0\le x<2{,}5,\\ -0{,}2x+8, & 2{,}5\le x<5,\\ 3x-8, & 5\le x<5{,}5,\\ 19x-96, & 5{,}5\le x\le 6,\end{cases}$$

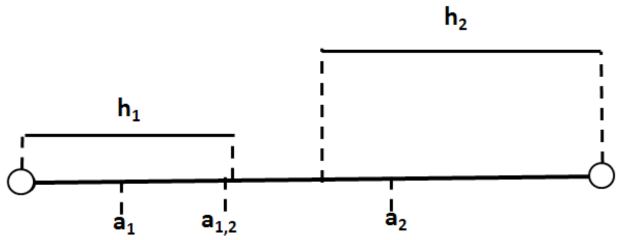
$$F\left(12\right)=20=2F\left(a_2\right)-F\left(a_{1,2}\right)=\begin{cases}4x, & 0\le x<5/3,\\ 28x-40, & 5/3\le x<7/3,\\ 4x+16, & 7/3\le x<2{,}5,\\ -2{,}4x+32, & 2{,}5\le x<5,\\ -4x+40, & 5\le x\le 5{,}5,\\ 28x-136, & 5{,}5\le x\le 6.\end{cases}$$

Hence we find three options:

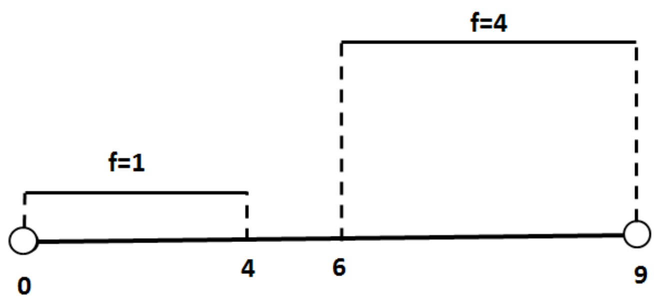
$$\begin{pmatrix}a_1=15/7, a_2=45/7,\\ Q=190/7\end{pmatrix},\quad \begin{pmatrix}a_1=5, a_2=7,\\ Q=30\end{pmatrix},$$
$$\begin{pmatrix}a_1=39/7, a_2=69/7,\\ Q=190/7\end{pmatrix}.$$



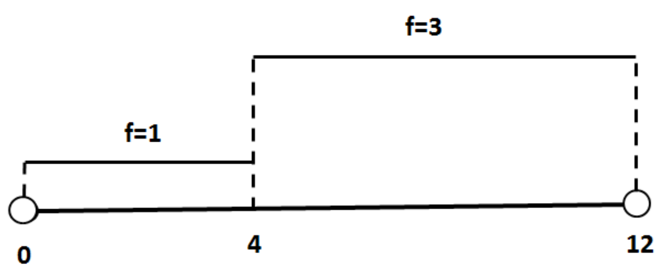
Pic. 9. Possible placement of points.



Pic. 10. Other possible placement of points.



Pic. 11. An example of distribution.



Pic. 12. An example of distribution.

The average, symmetric, is the local maximum of the objective function Q. The answers for the original task are the first variant (Pic. 6) and the third symmetric to it.

Example 2. *Let's consider the situation in Pic. 7.*

Analogous calculations lead to the equation

$$F(8)=16=2F(a_2)-F(a_{1,2})=\begin{cases}4x, & 0\le x<4/3,\\ 16x-16, & 4/3\le x<2,\\ 16, & 2\le x\le 4,\\ 12x-32, & 4<x\le 40/9.\end{cases}$$

The solution of this equation relative to x is the *whole section* *[2; 4]. As it is easy to verify, the value of the objective function is constant and equal to 16. The answer for the initial task is*

$$\begin{pmatrix} a_1=x, \quad 2\le x\le 4,\\ a_2=\dfrac{1}{3}x+\dfrac{16}{3},\\ Q=16 \end{pmatrix}.$$

Two points in violation of condition (2)

Now the situation when the condition (2) is violated and the function f turns to 0 on the inner section (Pic. 8).

Let it be necessary to place two service points.

1) First, the case where both points are placed on the left section (see Pic. 9).

From the condition of equality of areas we find

$$a_{1,2}=2a_1, \quad a_2=3a_1, \quad h_1\cdot a_1=h_1(a-a_2)+h_2\cdot c,$$

hence

$$a_1=\frac{1}{4}(a+Kc), \quad a_2=\frac{3}{4}(a+Kc), \quad K=\frac{h_2}{h_1}.$$

Both points really lie in the first section, if in addition

$$\frac{3}{4}(a+Kc)\le a \quad \Leftrightarrow \quad a\ge 3Kc,$$

or if we introduce the notation $a\cdot h_1=l_1$, $c\cdot h_2=l_2$ *– the number of «inhabitants»*

$l_1\ge 3\,l_2$. (6)

This arrangement, as is evident from condition (5), provides a local minimum of the objective function. The value of the function Q is:

$$Q=h_1\int_0^{2a_1}|a_1-x|dx+h_1\int_{2a_1}^{a}|3a_1-x|dx+h_2\int_{a+b}^{a+b+c}(x-3a_1)dx=$$

$$=h_1\frac{3a_1^2}{2}+h_1\frac{(a-3a_1)^2}{2}+h_2\frac{(a+b+c-3a_1)^2}{2}-h_2\frac{(a+b-3a_1)^2}{2}.$$

2) The case where both service points are located in the right zone is reduced to the previous via obvious re-designations:

$$a\leftrightarrow c, \quad h_1\leftrightarrow h_2, \quad x\leftrightarrow a+b+c-x.$$

It remains to consider the case where in the first and third zones there is one service point.

3) Immediately note that if the «central» point $a_{1,2}$ falls on the middle, «zero» section, then the only way to ensure equality of areas is to place points in the middle of the left and right sections. Then

$$\begin{pmatrix} a_1 = \dfrac{a}{2},\ a_2 = a+b+\dfrac{c}{2} \\ Q = \dfrac{h_1 \cdot a^2 + h_2 \cdot c^2}{4} \end{pmatrix}.$$

The point $a_{1,2}$ does indeed fall into the middle section under the condition

$$|c-a| \le 2b. \qquad (7)$$

4) Let's now take the case (Pic. 10), when the point $a_{1,2}$ falls on the left section (with the right one all the same).

Again from the equality of areas:

$$a_{1,2} = 2a_1, \quad a_2 = 3a_1,$$
$$h_1 \cdot (a - a_{1,2}) + h_2 (a_2 - a - b) = h_2 \cdot (a + b + c - a_2).$$

Hence when $K \ne 1/3$:

$$a_1 = \frac{K(a+b+c/2) - a/2}{3K-1}.$$

If $K < 1/3$, then the second of the inequalities (5) has the opposite sign, so there is no extremum.

If $K > 1/3$, then additional requirements arise:

$$0 \le a_{1,2} = \frac{2K(a+b+c/2) - a}{3K-1} \le a;$$
$$a+b \le a_2 = 3a_1 \le a+b+c.$$

If $K = 1/3$, then, depending on the value of the numerator, fractions for a_1 stationary points of the form in question are either not at all, or they can form a whole range of possible values. Indeed, a stationary arrangement of this kind is possible only if $a + b + c/2 = 3a/2$, that is, $a = 2b + c$. And it is easy to understand that then the solution is any pair (a_1; $3a_1$), where $(a + b)/3 \le a_1 \le a/2$.

Example 3. *Let's consider the situation in Pic. 11.*

In this case, $a = 4$, $b = 2$, $c = 3$, $K = 4$. For the «central» location of service points:

$$\begin{pmatrix} a_1 = 2,\ a_2 = 7{,}5, \\ Q = 13 \end{pmatrix}.$$

If $a_{1,2} \le 4$, then

$$a_1 = \frac{a_{1,2}}{2}, \quad a_2 = 2a_{1,2} - a_1 \le 6,$$

which is impossible due to violation of the condition of equality of areas.

Since $I_2 \ge 3I_1$ ($12 \ge 12$), there are two points on the right section. We find them using the formulas:

$$a_1 = \frac{1}{4}(a + Kc), \quad a_2 = \frac{3}{4}(a + Kc), \quad K = \frac{h_2}{h_1}.$$

But in the «opposite» direction:

$$\begin{pmatrix} a_1 = 6,\ a_2 = 8, \\ Q = 26. \end{pmatrix}.$$

This option is inferior to the «central» option.

Example 4. *Suppose that there are two residential units T_1 and T_2 in the service area with the corresponding sections of uniform distribution – [0, 400 m], [500 m, 700 m], and the «intensity» of residential units is 3000, 500. It is required to place two service points.*

We have:

$$a = 400, \quad I_1 = 3000, \quad I_2 = 500,$$
$$h_1 = \frac{3000}{400} = 7{,}5, \quad c = 700 - 500 = 200,$$
$$h_2 = \frac{500}{200} = 2{,}5.$$
$$b = 500 - 400 = 100, \quad K = \frac{2{,}5}{7{,}5} = \frac{1}{3}.$$

Condition (6) is satisfied, so that we first consider the case when both points are placed on the first section. We have:

$$a_1 = \frac{1}{4}(a + Kc) = \frac{400 + 200/3}{4} \approx 116{,}67, \quad a_2 = 3a_1 = 350.$$

Objective function value

$$Q = h_1 \frac{3a_1^2}{2} + h_1 \frac{(a - 3a_1)^2}{2} + h_2 \frac{(a+b+c-3a_1)^2}{2} - $$
$$-h_2 \frac{(a+b-3a_1)^2}{2} = 287500.$$

The condition (7) is also satisfied here, so we can also consider the «central» arrangement of service points:

$$\begin{pmatrix} a_1 = a/2 = 200, \\ a_2 = a+b+c/2 = 600, \\ Q = (h_1 \cdot a^2 + h_2 \cdot c^2)/4 = 300000 \end{pmatrix}.$$

This option is worse.

Finally, we take the case when the service points are located in both the first and the third zones, and the middle point is in the first zone.

Since $a + b + c/2 = 3a/2$, the solution is any pair (a_1; $3a_1$), where $(a + b)/3 \le a_1 \le a/2$, i.e. points

$$\begin{pmatrix} \dfrac{500}{3} \le a_1 \le 200, \\ a_2 = 3a_1, \\ Q = 325500 \end{pmatrix}$$

are stationary. But this option is also inferior to the first.

So, the optimal location of service points is:

$$\begin{pmatrix} a_1 \approx 116{,}67,\ a_2 = 350, \\ Q \approx 287500. \end{pmatrix}.$$

Example 5. *In the previous example, we change the third section from [500 m, 700 m] to [450 m, 650 m]. Then, as before*

$$a = 400, \quad h_1 = 7{,}5, \quad c = 200, \quad h_2 = 2{,}5, \quad K = \frac{2{,}5}{7{,}5} = \frac{1}{3}.$$

only b will change. A variant with a continuous arrangement of stationary points becomes impossible, and from the other two optimal as before remains

$$(a_1 \approx 116{,}67,\ a_2 = 350).$$

Example 6. *Let there be two residential units T_1 and T_2 in the service area with the corresponding sections of the uniform distribution – [0, 400 m], [500 m, 600 m], and the «intensities» of the residential units are respectively equal to 3000, 1000. It is required to place two service points.*

$$a=400,\quad I_1=3000,\ I_2=500,$$
$$h_1=\frac{3000}{400}=7{,}5,\quad c=600-500=100,$$
$$h_2=\frac{1000}{100}=10.\ b=500-400=100,$$
$$K=\frac{10}{7{,}5}=\frac{4}{3}.$$

Condition (6) $I_1 \ge 3I_2$ is satisfied, therefore we consider first the case when both service points are located in the first section. We have:

$$a_1=\frac{1}{4}\left(a+Kc\right)=\frac{400+400/3}{4}\approx 133{,}33,\quad a_2=3a_1=400.$$

Objective function value

$$Q=h_1\frac{3a_1{}^2}{2}+h_1\frac{\left(a-3a_1\right)^2}{2}+h_2\frac{\left(a+b+c-3a_1\right)^2}{2}-$$
$$-h_2\frac{\left(a+b-3a_1\right)^2}{2}=350000.$$

The condition (7) is not satisfied, so that the «central» arrangement of the points is not considered.

The option remains, when the service points are located in the extreme zones and the middle point does not fall into the zero section.

$$a_1=\frac{K\left(a+b+c/2\right)-a/2}{3K-1}\approx 177{,}78,\quad a_2=3a_1\approx 533{,}33.$$

In this case, the midpoint hit the first section, and the value of the objective function Q = 316666,67, which is better.

So, the optimal location of service points: $\left(a_1\approx 177{,}78,\ a_2\approx 533{,}33\right)$.

Arbitrary number of points

If condition (2) is satisfied for a piecewise constant function f, the chain of equalities (3) makes it possible to construct a solution explicitly for a large number of service points.

Example 7. *Let the function f be constant throughout the section, which corresponds to the presence of only one residential unit. Then the equality condition for the areas can obviously be satisfied only if the points a_1, $a_{1,2}$, a_2, $a_{2,3}$, ... divide the section into equal parts. The segment is divided into 2k equal parts, and points with odd numbers are chosen to place the points.*

Example 8. *Let's find the optimal arrangement of three, four and five service points for the density function:*

Let the point a_1 have the coordinate x. For three points, recurrent calculations lead to the equation

$$F\left(12\right)=28=2F\left(a_3\right)-F\left(a_{2,3}\right)=\begin{cases}6x, & 0\le x<0{,}8,\\ 26x-16, & 0{,}8\le x<1,\\ -6x+16, & 1\le x\le 4/3,\\ 30x-32, & 4/3<x\le 2,\\ -2x+32, & 2<x\le 4,\\ 18x-48, & 4<x\le 68/15.\end{cases}$$

Hence there are two options:

$$\begin{pmatrix}a_1=2,\ a_2=6,\ a_3=10,\\ Q=28\end{pmatrix},$$
$$\begin{pmatrix}a_1=38/9,\ a_2=22/3,\ a_3=94/9,\\ Q=244/9\end{pmatrix}.$$

The global minimum is provided by the second of these options.

Now four points. We have:

$$F\left(12\right)=28=2F\left(a_4\right)-F\left(a_{3,4}\right)=$$
$$=\begin{cases}8x, & 0\le x<4/7,\\ 36x-16, & 4/7\le x<2/3,\\ -12x+16, & 2/3\le x\le 0{,}8,\\ 48x-32, & 0{,}8<x\le 1,\\ -16x+32, & 1<x\le 4/3,\\ 44x-48, & 4/3<x\le 68/37.\end{cases}$$

Hence we find the best option:

$$\begin{pmatrix}a_1=19/11, a_2=57/11, a_3=87/11, a_4=117/11,\\ Q=218/11\end{pmatrix}.$$

For five service points, the optimal location is as follows:

$$\begin{pmatrix}a_1=46/29,\ a_2=138/29,\ a_3=198/29,\\ a_4=258/29,\ a_5=318/29,\\ Q=452/29\end{pmatrix}.$$

Conclusion. *The developed mathematical model allows selecting options of siting of everyday demanded facilities within an urban district (under the condition of approximately even distribution of their density), which are less expensive for transit of inhabitants.*

The model can be used in the planning of residential areas and the street-road network, while finding the optimal location of rescue services in transport, bus stops and other social facilities.

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