FORMULA FOR CALCULATING THE RESTORING FORCE FOR WHEELS WITH A CURVILINEAR PROFILE

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ABSTRACT

A method is proposed for calculating the restoring force of a wheel set using wheels with a curvilinear profile having a parabolic shape. The corresponding mathematical calculations are made. The results of the calculations showed that the application of the curvilinear profile increases the restoring force compared to the wheels having a conical surface. Simultaneously it was established that the restoring force plays the role of elastic force.

Keywords: wheel set, restoring force, calculation method, mathematical model, wheel profile, curvature, cone.

Background. *First, let's define the main thing: the restoring force will be the sum of the projections on the horizontal plane of the normal reactions of the rails to the wheels of the wheel set when it is transversely moved from the axis of the track.*

In the presentation of the material, the coordinate system (Pic. 1) is used, moving with velocity v along the axis O_x, whose origin is located on the axis of the track and coincides with the center of gravity of the wheel set (WS) in the central installation, let's call it so. The basis vectors form the right-hand triple, so that the axis O_y is directed transversely to the track, to the left with respect to its axis.

Objective. *The objective of the authors is to consider a method for calculating the restoring force for wheels with a curvilinear profile.*

Methods. *The authors use general scientific and engineering methods, comparative analysis, mathematical calculations.*

Results. *In the presented system, the following coordinate notations are accepted: x – movement along the track; y – movement across the track; z – vertical displacement of the center of gravity of WS. The designations of the WS elements and the track to the left of the origin along O_y axis are indexed by 1, and to the right by the number 2.*

The local coordinate systems $O_i\xi\Delta r$ and $O_i\eta z$, in which the equations of the rolling profiles of wheels and rails are given, are associated with the contact points of the wheels with rails. For the uniformity of the equations of the profiles of the rolling surface of wheels (PRSW), the horizontal axes $O_i\xi$ and $O_i\eta$ (i = 1,2) are turned inside the track, the axes $O_i\Delta r$, O_iz are vertically upward.

In these systems, the PRSW equation is presented in the form

$$\Delta r = \beta\xi^2 + b\xi, \tag{1}$$

and the equation of the rail head – in the form

$$(z-z_0)^2 + (\eta-\eta_0)^2 = \rho^2, \tag{2}$$

where ρ is radius of the head profile, (z_0, η_0) are coordinates of the center of circle $b = -\eta_0/z_0$.

Each of the local coordinate systems (Pic. 2) is rigidly connected to the corresponding contact points in the central installation, so that the $O_i\xi\Delta r$ systems move together with the wheel set along O_y and O_x axes, and the $O_i\eta z$ systems along O_x axis.

With the movement of WS along O_y axis, the radii of rolling of wheels, whose values in the central installation r_0 are the same, also change for $y > 0 r_1 > r_2$, and their difference $\Delta r = |r_1 - r_2|$. Therefore, $r_1 = r_0 + \Delta r$, and $r_2 = r_0 - \Delta r$.

It is known [2, 3] that with the conical rolling surface of wheels (linear profile), the radii of rolling are easily computed and for $y > 0 r_1 = r_0 + by$, $r_2 = r_0 - by$.

With the wear of the rolling surface, the determination of the radii of the rolling circles, the position of the normal reaction (Pic. 3) is significantly complicated in connection with the unknown equation of the profile of the rolling surface of the wheel.

In this paper, to take into account the effect of the curvilinearity of the profile on the horizontal components of the normal reaction, the equations of the profiles having the form of a parabola [4], in particular convex (1) and curvilinear-rectilinear are used:

$$\begin{cases} b\xi + \beta\xi^2, & \xi \ge 0, \\ b\xi, & \xi < 0. \end{cases} \tag{3}$$

In the given coordinate system (Pic. 3), equations (1), (3) take the convex form:

$$\Delta r = -b\xi - \beta\xi^2, \tag{4}$$

and curvilinear-rectilinear:

$$\begin{cases} -b\xi - \beta\xi^2, & \xi \ge 0, \\ -b\xi, & \xi < 0. \end{cases} \tag{5}$$

In these equations, b is the tangent of the tangent angle at the point O_1 of the rolling surface profile of the wheel; 2β is the curvature of the rolling profile at the point O_1, satisfying the inequality $0 < 2\beta < 1/\rho = \beta_0$, where ρ is the radius of the rail head.

In [4], formulas were obtained for calculating the abscissas of the contact points of wheels with rails when the wheel set is transversely displaced by the value y. For a convex profile, they can be written in the form:

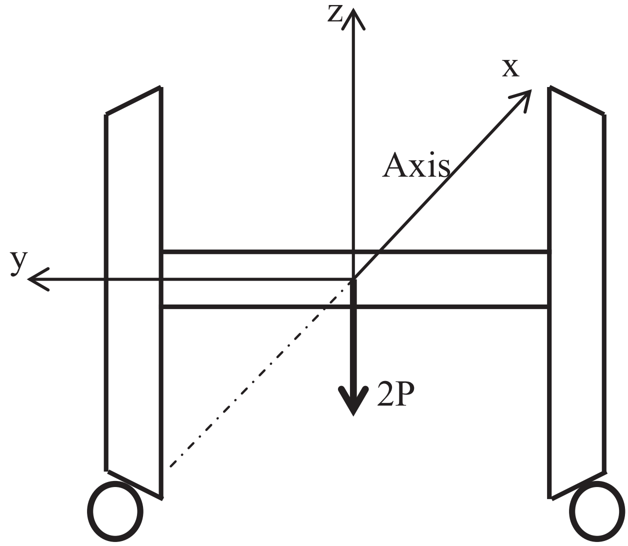
$$\begin{cases} \xi = (-1)^{n+1}\dfrac{1}{1-k}y, \\ \eta = (-1)^{n+1}\dfrac{k}{1-k}y, \end{cases} \tag{6}$$

where $k = 2\beta/\beta_0 < 1$; n = 1, 2.

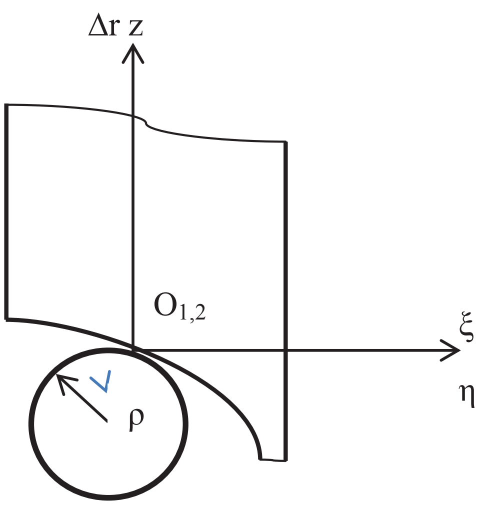
In the case of a linear profile, the abscissas of the contact points take the form:

$$\begin{cases} \xi = (-1)^{n+1}y, \\ \eta = 0. \end{cases} \tag{7}$$

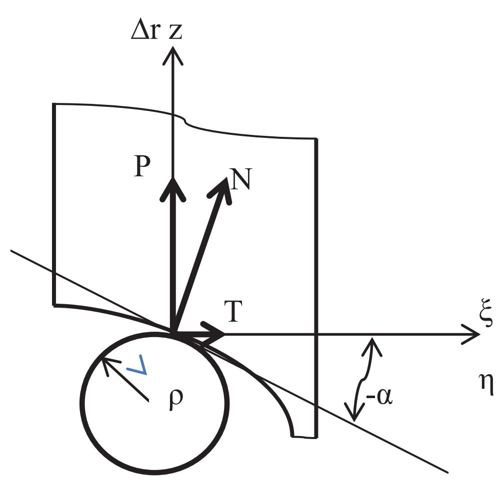
Comparing formulas (6) and (7), it can be established that contact points move along the profiles of the wheel and rail in the case of curvilinear rolling profile of the wheel, the contact zone being dependent on the curvature of the wheel profile. It can be seen that the contact area of the wheel in



Pic. 1.



Pic. 2.



Pic. 3.

$\frac{1}{1-\kappa}$ *is greater than in the rectilinear one, and this value is the larger, the larger is* β*:*
$\beta(0 \le 2\beta < 1)$.

If $\beta = 0$, *then* $-\Delta \le y \le \Delta$, *where* Δ *is a gap between the wheel flange and the rail head.*

Now let WS be shifted by the value «y». Then the coordinates of the contact point will move to the values ξ_1 *and* ξ_2 *on the left and right wheels respectively, assuming the surfaces are absolutely smooth, the normal reactions will be inclined to the horizontal axes* $0\,\xi$ *at angles* α_1 *and* α_2.

Consequently, the horizontal components T_1 *and* T_2 *of normal reactions directed into the inside of the track can be obtained as*

$$T_1 = P_1 \cdot tq\alpha_1 = P_1 \Delta r_1', \qquad (8)$$
$$T_2 = P_2 \cdot tq\alpha_2 = P_2 \Delta r_2',$$

where P_1, P_2 *– vertical reactions of rails, which we will consider the same* ($P_1 = P_2$); $\Delta r_i'$ *– derivatives of the equations of the wheel profiles.*

Considering the equation of the convex profile (3), we differentiate the function (3):

$$\frac{d(\Delta r)'}{d\xi} = -i - 2\beta\xi; \qquad (9)$$

$$\Delta r_1' = -i - 2\beta\frac{1}{1-k}y; \qquad (10)$$

$$\Delta r \;= -i + 2 \;\frac{\quad}{1\;\;k} y. \qquad (11)$$

We now calculate the sum of the components, taking into account their directions:

Table 1

2β	1/600	1/700	1/800	1/1000	1/1200	1/1800	1/2000	1/3000
$(1\text{-}\kappa)10$	1,66	2,85	3,75	5,0	5,83	6,67	7,5	8,33
$\frac{2\beta}{1\text{-}\kappa}10^3$	10,04	5,01	3,33	2,00	1,43	0,99	0,67	0,40
$\frac{2\beta S}{(1\text{-}\kappa)i}$	158	79	53	32	23	16	11	6,3

$$\Delta T_в = T_1 + T_2 = P\left(-i - 2\beta\frac{1}{1-k}y\right) - P\left(-i + 2\beta\frac{1}{1-k}y\right) = -\frac{4\beta}{1-k}yP. \qquad (12)$$

Thus, if y > 0, ΔT is directed to the side opposite to the movement of the wheel set, and this force is proportional to the displacement and vertical reaction of the rail.

If the profile is curvilinearly conical, then

$$\left.\begin{aligned} \Delta r_1^{'} &= -i - 2\beta\xi \\ \Delta r_2^{'} &= -i \end{aligned}\right\}$$

and $\Delta T = -\dfrac{2\beta}{1-K}yP.$ *(13)*

Analyzing formulas (12) and (13), one can see the advantage of using a convex profile in comparison with the curvilinear-rectilinear profile. Moreover, the magnitude of the restoring force is doubled. It should be noted that in deriving the calculation formulas ΔT, the rotation of WS around Ox axis was not taken into account, since the effect of the rotation is considered insignificant.

However, for a rectilinear profile Δr = –iξ, only rotation of WS around O_x axis causes the horizontal components of normal reactions to appear. By shifting WS by the value y > 0, the radii of the rolling of the wheels change, and the whole wheel set rotates by an angle $\varphi = \dfrac{iy}{S}$.

As a result, the sum of the horizontal components:

$$\Delta T_n = T_1 - T_2 = -\frac{2i}{S}yP. \qquad (14)$$

To compare the effect of the curvature of the profile on the horizontal component of the vertical forces, we take the radius of the rail head $\rho = 500\text{ mm}\left(\beta = \frac{1}{500}\right)$, *the tangent of the slope angle of the profile at the origin i = 1/20. The radius of curvature of the curvilinear sections of rolling will be changed. The calculation is made at the same pressure on the wheels and the same displacement y = 1 mm, i/S. Therefore, we can only compare the coefficients of the product yP. For the convex profile (4), the coefficients are given in Table 1. The coefficient for the rectilinear profile for the same product yP will be*

$$\frac{2i}{S} = 6.33 \cdot 10^{-5}.$$

Table 1 gives the ratio of the coefficients of the convex profile with different curvatures to the coefficient of the rectilinear profile.

It can be seen from the last line that with a decrease in 2β (with an increase in the radius of curvature of the profile of the rolling surface), the restoring force also decreases, and at 2β = 1/3000 it is only 6,3 times greater than the restoring force for a rectilinear profile. So, if we take y = 3 mm, $P_1 = P_2$ = 125000 n, then the restoring force for the rectilinear profile will be ΔT_n = 23,74 n, for the curvilinear-conic with 2β = 1/800 ΔT_{kk} = 1258 n, and for the convex profile – $\Delta T_в$ = 2516 n.

Conclusions.

1) The use of curvilinear profiles increases the restoring force by a factor of times compared to the wheels with the conical rolling surface.

2) The restoring force of the curvilinear profiles considered is proportional to the displacement and, consequently, plays the role of an elastic force.

3) As the curvature increases, the magnitude of the restoring force increases.

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