SOLUTION OF THE TRANSPORT PROBLEM BY THE METHOD OF SUCCESSIVELY DECREASING ITS DIMENSION

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ABSTRACT

The article considers the solution of the transport problem in two ways: by the method of the north-western angle and the method of the minimal element. As a result of the analysis, it is proved that the minimal element method allows to reduce the number of iterations by several times. In solving complex problems of large dimension, the choice of a rational method plays a decisive role, which is demonstrated by the method of successively reducing this dimension by means of the algorithms used to optimize the distribution of shipments of goods.

Keywords: transport problem, logistics, optimization, programming, decision methods, dimension, algorithms.

Background. *The transport problem is the search for the optimal distribution of supplies of a homogeneous product from suppliers to consumers at known costs of transportation (tariffs) between origin and destination points. It is a linear programming task of a special kind. Its solution begins with finding a basic plan.*

In the article published, two ways of solving this problem are compared:

1. The basic plan is formed by the method of the north-western angle.

2. The basic plan is found by the method of the minimal element.

The required result of solving the transport problem: all applications are satisfied, all stocks are exhausted, the total cost of all transportation is minimal. The solution methods are reduced to operations with the table, where in a certain order all the conditions of the problem are written down. Such a table is called a transport table. It records:

– points of departure and destination;

– stocks available at departure points (DepP);

– applications submitted by destination points (DesP);

– cost of transportation from each point of departure to each destination.

Objective. *The objective of the authors is to consider the solution of the transport problem in two ways: by the method of the north-western angle and the method of the minimal element.*

Methods. *The authors use general scientific and engineering methods, mathematical analysis, north-western angle method, minimal element method.*

Results.

Formulation of the problem

There are m departure points: $A_1,\dots, A_m$, in which the stocks of some homogeneous goods (cargo) are concentrated in the amount of $a_1,\dots, a_m$ units. In addition, n destination points are known: $B_1,\dots, B_n$, who applied for $b_{1,\dots,} b_n$ units of goods, respectively. It is assumed that the sum of all applications is equal to the sum of all stocks: $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$. The cost a_{ij} of transportation of a unit of cargo from each point of departure to each point of destination has been established. The table (matrix) of transportation costs a_{ij} of the unit of cargo is given in the following form:

$$\begin{Vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{Vmatrix}.$$

The dimension of this matrix m • n can also be used to determine the dimension of the transport problem itself.

It is required to make such a plan of transportation, in which all applications would have been fulfilled, and the total cost of all transportations would be minimal. With such a statement of the problem, the indicator of the efficiency of the transportation plan is the cost.

Let x_{ij} be the amount of cargo sent from the departure point A_i to the destination point B_j. Non-negative variables x_{ij} must satisfy the following conditions:

1. The total amount of cargo sent from each point of departure to all destinations should be equal to the cargo stock at this point. This gives m conditions for the equalities: $\sum_{j=1}^{n} x_{ij} = a_i, i = 1,\dots, m.$

2. The total amount of cargo delivered to each destination point from all points of departure must be equal to the application filed by the relevant paragraph. This gives n conditions for the equalities:

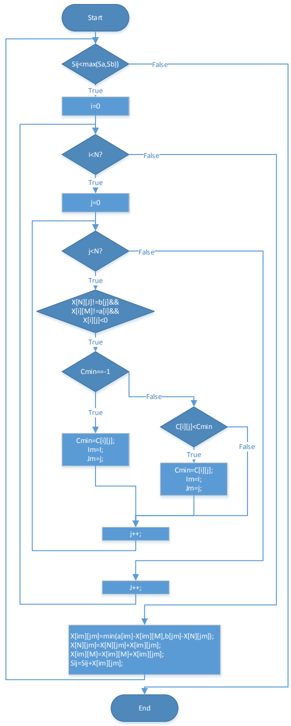
$$\sum_{i=1}^{m} x_{ij} = b_j, j = 1,\dots, n.$$

3. The total cost of all transportation should be minimal, i.e $\min_{x_{11},\dots,x_{mn}} \sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}x_{ij}$ *must be found.*

Table 1

Sample of a transport table

DepP / DesP	B_1	B_2	...	B_n	Stocks a_i
A_1	a_{11}	a_{12}	...	a_{1n}	a_1
A_2	a_{21}	a_{22}	...	a_{2n}	a_2
...	...	...	...	...	...
A_m	a_{m1}	a_{m2}	...	a_{mn}	a_m
Applications b_j	b_1	b_2	...	b_n	$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$



Pic. 1. Block diagram of the program operation algorithm.

```
//Makarenko A.A. UIS-411
float Sij = 0;
 do
 {
 int im;
 int jm;
 int Cmin = -1;
 for(int i = 0;i<N;i++)
 for(int j = 0;j<M;j++)
 if(X[N][j]! = b[j])
 if(X[i][M]! = a[i])
 if(X[i][j]<0)
 {
 if(Cmin =  = -1)
 {
 Cmin = C[i][j];
 im = i;
 jm = j;
 }
 else
 if(C[i][j]<Cmin)
 {
 Cmin = C[i][j];
 im = i;
 jm = j;
 }
 }

 X[im][jm] = min(a[im]-X[im][M],b[jm]-X[N][jm]);
 X[N][jm] = X[N][jm]+X[im][jm];
 X[im][M] = X[im][M]+X[im][jm];
 Sij = Sij+X[im][jm];

 }while(Sij<max(Sa,Sb));
```

Pic. 2. The code of the program. Construction of the initial basic plan by the minimal element method.

Costs of transportation are placed in the upper right corner of each cell of the table in order to place x_{ij} transportation in the cell itself when drawing a plan (see table 1).

The rank of the constraint-equation system is $r = m + n - 1$, where m is the number of rows, n is the number of columns in the transport table. In each basic plan, no more than $m + n = 1$ transportation will be different from zero. The cells of the table, in which we will write the nonzero transportations, are called basic, and the remaining (empty) are free.

The solution of the problem reduces to the following. It is necessary to find those values of transportation, which, if affixed to the basic cells of the transport table, would satisfy such conditions:

– the amount of transportation in each row of the table should be equal to the stock of this DepP;

– the amount of transportation in each column should be equal to the application of this DesP;

– the total cost of transportation is minimal.

Ways of solution

The north-western angle method is designed to obtain an accessible initial solution to the transport problem. It was proposed by J. Dantzig (1951) and later called «the rule of the north-western angle» (Charnes, Cooper). The essence of the method is the sequential search for the rows and columns of the transport table, starting with the left column and the top row, and writing out the maximum possible shipments to the appropriate table cells so that the stated resources of the supplier or the needs of the consignee are not exceeded. The method is not focused on delivery prices, since in the future the optimization of shipments is expected.

The method of successively reducing the dimension of the transport problem (the minimal element method) consists in using the following algorithm.

1. We find the cell of Table 1 with the minimum cost of transporting the unit of cargo, i.e. $\min_{1\le i\le m, 1\le j\le n} a_{ij}$ *, where i – number of the departure point (DepP), j – number of the destination point (DesPПН).*

2. In this cell there will be the cost $a_{i1,j1}$ and the cell number is (i_1, j_1), that is, it is at the intersection of the j_1-th column with the application b_{j1} and the i_1-th row with the stock a_{i1}.

3. Then we find $\min(a_{i1}, b_{j1})$.

4. If $\min(a_{i1}, b_{j1}) = a_{i1}$, then we put in the cell with the number (i_1, j_1) transportation $x_{i1,j1} = a_{i1}$, the stock a_{i1} is set equal to zero and the row A_{i1} is deleted. In this case the transport problem of the original dimension $m \cdot n$ turns into a transport problem of dimension $(m - 1) \cdot n$. The cost of serviced transportation will be equal to $a_{i1} \cdot a_{i1,j1}$.

5. If $\min(a_{i1}, b_{j1}) = b_{j1}$, we put in the cell with the number (i_1, j_1) transportation $x_{i1,j1} = b_{i1}$, the application b_{i1} is set equal to zero and the column B_{i1} is deleted. In this case the transport problem of initial dimension $m \cdot n$ turns into a transport problem of the dimension $m \cdot (n - 1)$. The cost of transportation will be $b_j \cdot a_{i1,j1}$.

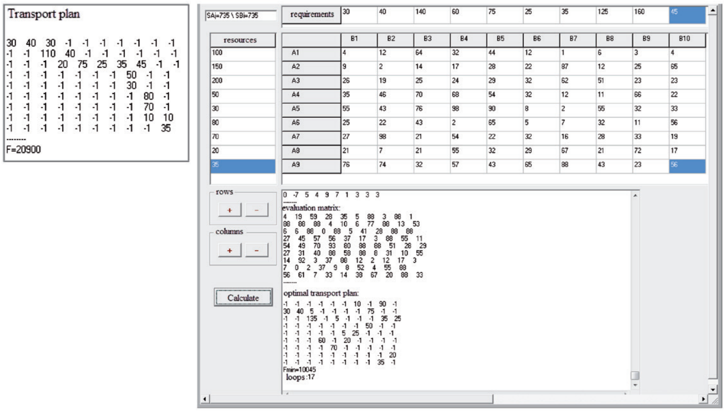
6. If $\min(a_{i1}, b_{j1}) = a_{i1} = b_{j1}$, then we put in the cell with the number (i_1, j_1) transportation $x_{i1,j1} = b_{i1} = a_{i1}$, the stock a_{i1} is set equal to zero and the row A_{i1} is deleted and the application b_{1i} is set equal to zero and the column B_{i1} is deleted. The cost of transportation $b_{j1} \cdot a_{i1,j1} = a_{i1} \cdot a_{i1,j1}$.

7. In this case the transport problem of initial dimension $m \cdot n$ turns into a transport problem of dimension $(m - 1) \cdot (n - 1)$.

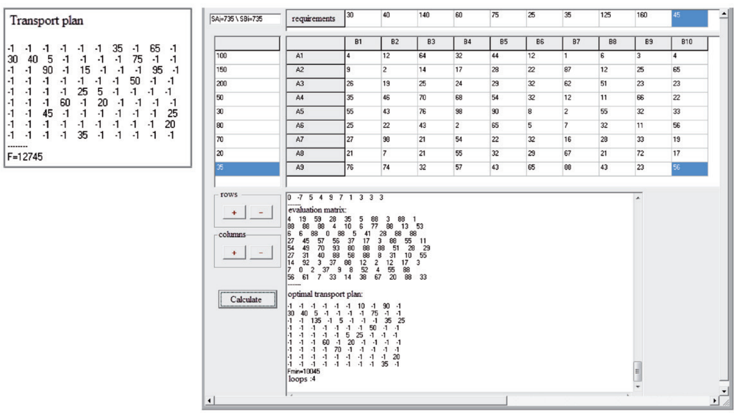
8. Further in the modified table 1 we find a cell with a minimum cost of transportation of a unit of cargo, i.e. $\min_{1\le i\le m, 1\le j\le n} a_{ij}$ *, where i – number of the point of departure, j – number of the point of destination.*

9. Then steps 2–7 of the algorithm are repeated until all the rows and columns of Table 1 are deleted.

The maximum number of steps of the proposed



Pic. 3. North-western angle method.



Pic. 4. Minimum element method.

E = 17/4 = 4.25

algorithm is equal to n + m – 1. The minimum number of steps is $]\frac{n+m-1}{2}[$*, i.e. the whole part from* $\frac{n+m-1}{2}$ *plus 1.*

We describe now the algorithm for searching for a cell in Table 1 with the minimum cost of transportation of a unit of cargo.

1. We allocate three cells for storing the current minimum of the cost of transportation of the unit of cargo and the coordinates of this cost: number of the row and number of the column. In these three cells, we record successively the value of the vector ($a_{1,1}$, *1,1).*

2. In the top row of Table 1 we take the vector ($a_{1,2}$, *1,2).*

3. We verify the fulfillment of the inequality $a_{1,1} > a_{1,2}$. *If it is fulfilled, then in the allocated three cells for storing the current minimum of the cost of transportation of the unit of cargo and the coordinates of this cost, we write sequentially the value of the vector* ($a_{1,2}$, *1,2). If it is not fulfilled, then nothing changes in the selected cells.*

4. Further, step 3 is satisfied for all remaining values of the first row.

5. Then, steps 3 and 4 are satisfied for all subsequent rows of Table 1.

Thus, we find the cell of Table 1 with the minimum transportation cost of the unit of cargo.

Conclusions. *Having carried out a series of experiments, one can be sure that the construction of the initial basic plan with the help of the minimal element method gives a great advantage in its optimization. Based on the results, it can be stated that due to the minimal element method, the number of optimization iterations is reduced by two or more times, depending on the complexity of the problem.*

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