FUZZY NUMERICAL PROBABILITY ANALYSIS OF RAIL FASTENINGS RELIABILITY

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ABSTRACT

As a probabilistic model for calculating reliability of rail fastenings, a triangular distribution with two parameters, the maximum and the most probable, is used. Fuzzy numerical probability analysis with the use of simulation modeling suggests that experts, using statistical and expert information, can numerically estimate the maximum value of the operating time, and for the second value (mode) may offer only the interval of its change. This is the most realistic situation in practical applications in conditions of uncertainty in the initial data. The algorithm of fuzzy analysis, which is approved on the basis of expert information, is proposed.

Keywords: rail fastenings, reliability indicators, numerical probability analysis, simulation modeling.

Background. *When creating a railway track, various components are used, including rails, sleepers, linings, ballast and roadbed. The connecting elements of this system are rail fastenings. In recent years, in the organization of track facilities, great attention is given to them, which is reflected in various scientific and normative works [1–5].*

The main task of such intermediate designs is to preserve the geometric parameters of the rail track during transmission and damping of forces from the dynamic impact of rolling stock. At the same time, properties of fastenings should take into account not only the standard operating conditions of the track, but also the negative effects of the transport load with an uneven distribution of forces on the rails while the rolling stock is curving in [4]. Another task of fastening is electrical insulation of rails from the under-rail base to minimize the loss of signals of rail electrical circuits.

The type of fastening significantly affects the cost characteristics of construction and operational complexes of the railway track. One of the promising types used on the Ulaanbaatar railway (UBZD) is ARS – elastic unlined threadless fastening with an inseparable anchor bolted to the sleeper. ARS fastenings are supported and constructively developed by MIIT specialists who conduct monitoring at 26 sections of eight railways under different operating conditions [1].

From the point of view of reliability theory, rail fastenings can be attributed to non-renewable objects, because in case of failure they cannot be restored (only the main connection nodes can be replaced). And since failures occur for a variety of reasons, the problem arises of determining reliability indicators for rail fastenings under conditions of uncertainty with a small amount of statistical data.

Objective. *The objective of the authors is to consider fuzzy numerical probability analysis of rail fastenings reliability.*

Methods. *The authors use general scientific and engineering methods, mathematical calculation, evaluation approach, graph construction.*

Results.

Models for calculating reliability indicators

In [6], as a probabilistic runtime model for ARS-type fastenings, a triangular distribution is offered

$$f(t)=\begin{cases}\dfrac{2\cdot t}{t_m\cdot t_0}, & 0<t\le t_0\\ \dfrac{2(t_m-t)}{t_m\cdot(t_m-t_0)}, & t_0<t<t_m\end{cases}. \quad (1)$$

Mathematical expectation for it is:

$$\bar{t}=\frac{1}{3}(t_m+t_0)\ . \quad (2)$$

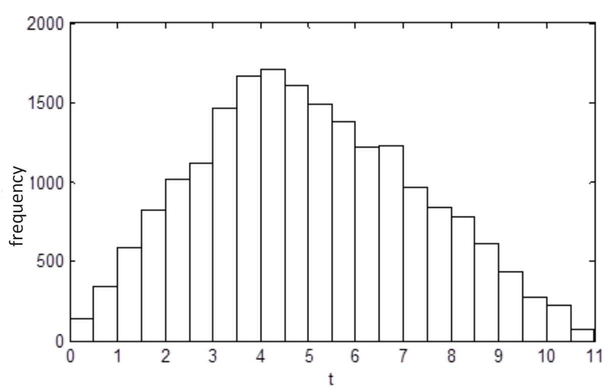
Experts, having some statistical information, estimate the highest value (t_m) and the most probable value (t_o), which is called the mode. These parameters are determined on the basis of statistical and expert information available in the literature and regulatory documents.

Thus, in [3, 5], the experimental sections of the track are described, which are located on the most complex and cargo-stressed sections. Measurements of the state parameters are carried out according to the method of comparative operational tests of the structures of intermediate rail fastenings. Similar measurements are made on the UBZD in areas with a small radius and heavy traffic.

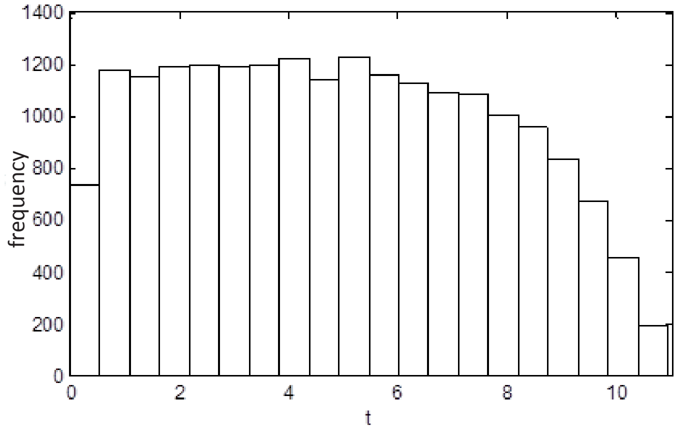
In our study, the maximum operating time of rail fastening is 550 mln tkm, and the most probable value is 200 mln tkm. As a unit of measurement, 50 mln tkm was taken, this determined the values of the initial data:

t_m = 11 (550 mln tkm), t_o = 4 (200 mln tkm). (3)

For parameters (3) mathematical expectation (2) is equal to 5.



Pic. 1. Frequency histogram for a triangular distribution.



Pic. 2. Frequency histogram for operating time, fuzzy probabilistic analysis.

It should be emphasized that the described study is methodical in nature and does not pretend to accurately estimate the parameters for operating time of rail fastenings.

In [6], analytical expressions for the following reliability indicators were obtained for this distribution:

1) probability of failure-free operation (probability that the object will not fail before time t);

2) intensity of failures;

3) average operating time before failure;

4) gamma-percentage resource – the total operating time, during which the object does not reach the limit state with probability γ;

5) average residual life to failure;

6) gamma-percentage residual resource for the probability γ.

The listed indicators are recommended in various sources, for example [7, 8].

Numerical algorithms for calculating reliability indicators based on the results of simulation and scientific recommendations [11–13] were obtained in [9, 10] for the triangular distribution of the operating time.

For distribution (1), the algorithm for modeling the operating time as a random variable has the form [9]:

$$t = \begin{cases} \sqrt{t_m \cdot t_0 \cdot r}, \ 0 < r \le \dfrac{t_0}{t_m}; \ 0 < t \le t_0 \\ t_m - \sqrt{t_m (t_m - t_0)(1-r)}, \ \dfrac{t_0}{t_m} < r < 1; \ t_0 < t < t_m. \end{cases} \quad (4)$$

Here r is a value of pseudo-random quantity, uniformly distributed on the interval (0, 1).

Pic. 1 shows the frequency histogram obtained by simulation using formula (4) for parameters (3). A good connection with the triangular distribution (1) is seen.

Probability analysis algorithm

Fuzzy numerical probability analysis with the use of simulation modeling assumes that experts can numerically estimate the maximum value of the operating time (t_m) based on the available statistical and expert information, and only the interval of its variation (a, b) for the second value (mode). This is the most realistic situation in practical applications in conditions of uncertainty in the initial data.

And we must proceed from the fact that the operating time mode is a random variable with a uniform distribution and known values of mathematical expectation ($\overline{t_o}$) and coefficient of variation (k_r)). The choice of the coefficient of variation is due to the fact that this is a normalized value, which can be changed during the research. The mathematical expectation of the mode is determined by the method of moments from the formula (2) according to the expectation of the operating time chosen by the experts, and the coefficient of variation is assigned expertly.

The mathematical expectation ($\overline{t_o}$) of a random variable having a uniform distribution, the coefficient of its variation (k_r), and the intervals of variation (a, b) are related by the dependence

$$a = \overline{t_o} \cdot (1 - \sqrt{3} \cdot k_r); \ b = \overline{t_o} \cdot (1 + \sqrt{3} \cdot k_r). \quad (5)$$

Algorithm of modeling of values of a random variable is:

$$t_{qo} = a + (b - a) \cdot r. \quad (6)$$

The following algorithm (MD) is proposed for fuzzy numerical probability analysis with the use of simulation modeling:

1. Knowing the mathematical expectation ($\overline{t}$) of the operating time by the method of moments from the formula (2), we obtain the value of the mathematical expectation of the mode

$$\overline{t_o} = 3 \cdot \overline{t} - t_m. \quad (7)$$

2. Knowing the value of the mathematical expectation of the mode (7) as a random variable and the coefficient of its variation (k_r), we determine the mode variation range (a, b) by the formula (5).

3. Using the formula (6), we determine the selective value of the operating time mode as a random variable (t_{qo}).

4. Using the formula (4), we determine the selective value of the operating time (t_q).

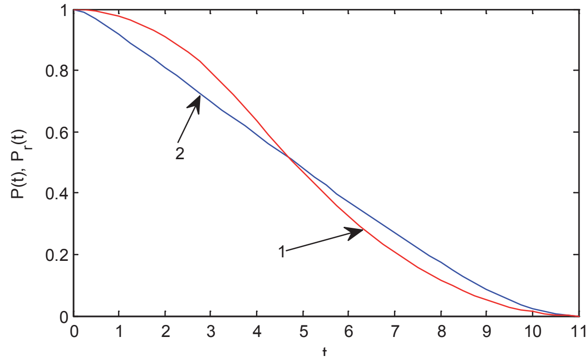
5. Points 3 and 4 are repeated n times, resulting in the creation of the required sample of operating time of the volume n

$$T_н = (t_1, \ldots, t_q, \ldots, t_n). \quad (8)$$

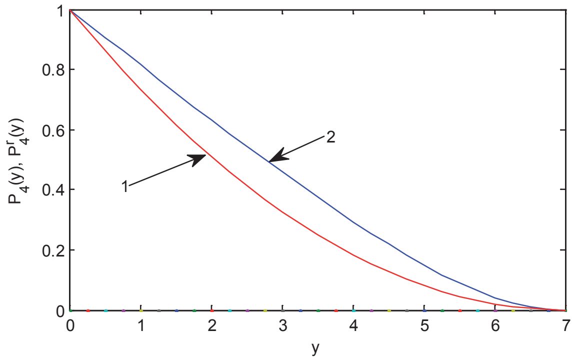
The sample (8) is processed by special numerical algorithms to obtain the reliability indicators described in [9, 10]. In addition, there is a point estimate of the mathematical expectation of the operating time ($\tilde{t}$) and a confidence interval for the mathematical expectation (t_1; t_2) [8]:

$$\tilde{t} = \frac{1}{n} \sum_{q=1}^{n} t_q; \ t_1 = \tilde{t} - \delta; \ t_2 = \tilde{t} + \delta; \quad (9)$$

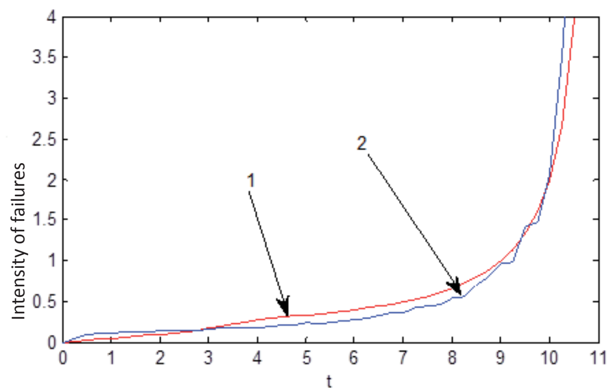
$$\delta = \frac{z_\gamma \cdot s}{\sqrt{n}}; \ s = \sqrt{\frac{\sum_{q=1}^{n} t_q^2 - n \cdot \tilde{t}^2}{n-1}}, \quad (10)$$



Pic. 3. Probability of failure-free operation.



Pic. 4. Probability of failure-free operation for the remaining resource.



Pic. 5. Failure intensity.

here t_q means selective values of operating time (8); z_γ is the quantile of the normalized normal distribution for the confidence probability γ, s is the root-mean-square deviation estimate.

Pic. 2 shows the frequency histogram obtained as a result of simulation based on the proposed MD algorithm (4–8): $\bar{t} = 5$, $t_m = 11$, sample size $n = 20000$, coefficient of variation $k_r = 0{,}50$, range of the mode change (5) – (0,536; 7,464).

This histogram differs significantly from the histogram in Pic. 1, obtained for a triangular distribution, but for a «clear» mode value ($t_o = 4$).

Spot (9) and interval (10) estimates of the operating time for the sample (8) are equal to

$$\tilde{t} = 4{,}922;\ (t_1;\ t_2) = (4{,}883;\ 4{,}961)\,. \qquad (11)$$

The mathematical expectation $\bar{t} = 5$ did not fall within the confidence interval (11). This allows us to conclude that the fuzziness of the mode with an increase in the value of its coefficient of variation significantly affects the mathematical expectation of the operating time. Moreover, the fuzziness of the mode has a significant effect on the estimation of the coefficient of variation of the operating time ($\tilde{k}_\nu$), it increases noticeably: for example, at $k_r = 0{,}15$ $\tilde{k}_\nu = 0{,}485$, and at $k_r = 0{,}50$ $\tilde{k}_\nu = 0{,}586$; the coefficient of variation for the distribution (1) with the parameters (3) is 0,454.

Approbation of the model range

Using fuzzy probability analysis, the reliability indicators of rail fastenings are calculated from the numerical algorithms described in [9, 10], taking into account the recommendations [11–13], but the sample (8) for them is determined by the MD algorithm.

Pic. 3 shows the probability of failure-free operation (FFOP): 1 – FFOP for the analytical model [6]; 2 – FFOP for the numerical model [9, 10], but for the fuzzy mode by the MD algorithm. The fuzziness of the mode affects this function, and with increasing coefficient of variation it affects it to a greater extent.

Numerical mean operating time to failure – $\bar{t}_r = 4{,}923$. This value falls within the confidence interval (11), which confirms the quality of the proposed algorithms.

The numerical gamma-percentage resource to failure for the fuzzy mode at $\gamma = 0,9$:

$t_r(0,9) = 1,855.$ (12)

For an analytical model, this value is 2,098. Taking into account (12), the fuzziness of the mode affected the gamma-percentage resource to failure.

Pic. 4 shows the probability of failure-free operation for the residual resource (FFOPRR): 1 – FFOPRR for the analytical model [6]; 2 – FFOPRR for the numerical model, but for the fuzzy mode by the MD algorithm. Unclear mathematical expectation affects this function – it is above the theoretical function, so there is a change in the average residual resource, and gamma-percentage residual resource.

The numerical average residual resource to failure for a fuzzy mode is $\tilde{y}_4 = 2,876$; *and the numerical gamma-percentage residual resource is* $\tilde{y}_4(0,9) = 0,539$. *When compared with the values obtained from analytical models (2,333 and 0,359, respectively), these values have increased significantly.*

Pic. 5 shows the intensity of failures (IF): 1 – IF for the analytical model; 2 – IF for a numerical model with fuzzy mode. Graphical representation of models does not quite match, i.e. fuzziness of the mathematical expectation affects this function.

Conclusions.

1. In case when it is not possible to unambiguously describe the operating time as a random variable in the conditions of uncertainty of the initial data, some of its parameters may become fuzzy. With regard to triangular distribution, it is suggested to consider this mode (t_o) as such a parameter.

2. The fuzziness of the mode is described by a random variable having a uniform distribution in the interval (a, b) with known values of the mathematical expectation and the coefficient of variation. These numerical characteristics are calculated expertly, taking into account the available recommendations.

3. The fuzzy description of the operating time as a random variable affects the frequency histogram (changes the law of its distribution), and hence the reliability of rail fastening.

4. Estimates of reliability indicators change significantly with increasing coefficient of mode variation. To obtain estimates of reliability indicators of rail fastenings with the required accuracy, it is necessary to use the method of simulation.

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