COMPLEX MATHEMATICAL MODEL OF THE MANIPULATION SYSTEM OF A MOBILE TRANSPORT-TECHNOLOGICAL MACHINE

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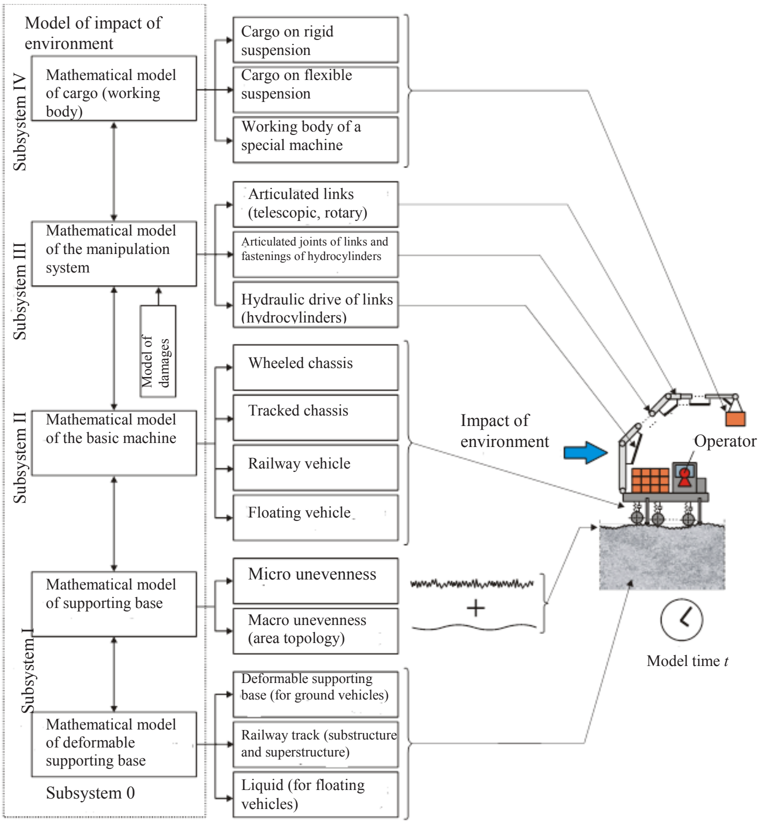
ABSTRACT

Manipulation systems are used as executive mechanisms of mobile transport-technological machines. The article offers approaches to modeling of dynamic processes in the operation of such systems. These approaches are based on the complex mathematical model developed by the author, which takes into account the interaction between the elements of the five-component system «Working body – Manipulation System – Basic Machine – Reference Base – Environment». By the method of statistical tests or simulation modeling, combinations of load factor values within each cycle of the system operation are determined. Then, for each combination of factors, a piecewise implementation of the process of changing dynamic forces or mechanical stresses in the elements of the structural steel that are of interest is constructed. For piecewise implementations, the loading of the manipulation system during the entire service life is estimated.

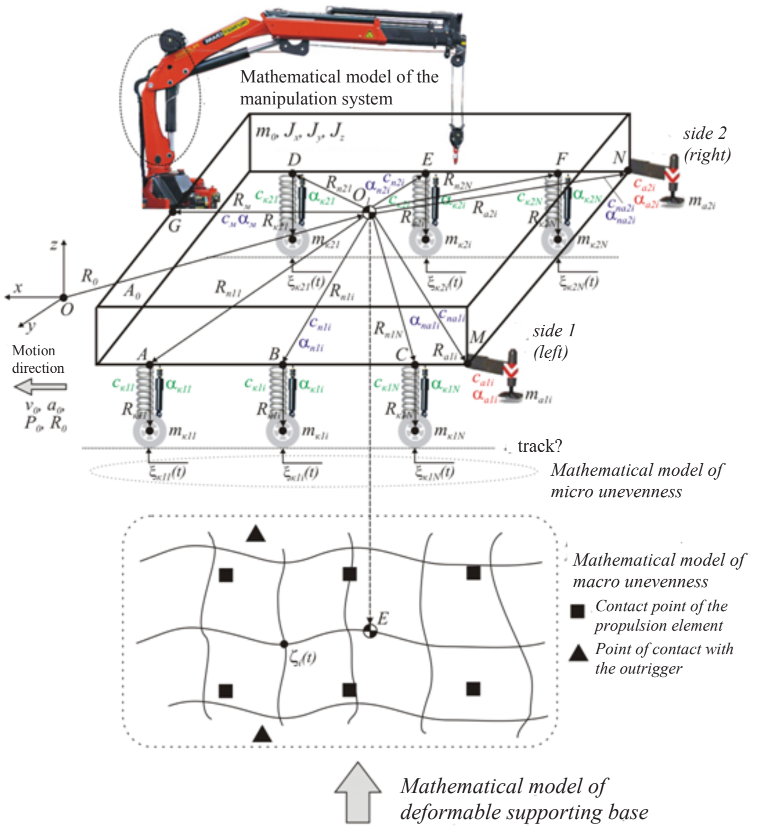
Keywords: manipulation system, transport-technological machine, bearing surface, mathematical model, dynamics, loading.

Background. *Manipulation systems (hereinafter – MS) are installed on mobile transport-technological machines (hereinafter – MTTM), used in various industries. They can also be used as actuators for the installation of the working bodies of special machines. The most common are hydraulic manipulators (general-purpose cranes), mainly for loading and unloading operations in confined spaces [1].*

Existing methods of modeling the operational loading of MS, as a rule, consider them as an isolated object installed on a stationary basis [1–5]. The links of the manipulation system can be modeled without taking into account [1–3] or taking into account [4, 5] elastic compliance. But in real operating conditions, MTTM is not guaranteed to fulfill this assumption. In particular, in work [6] the model of motion of MTTM with a load is considered with the non-working state of the manipulator crane, containing elements of the base chassis and manipulation system. However, for a number of MTTM, the MS operation can be accompanied by the movement of the base machine (forest machinery, excavators). In addition, even under the chassis mounted on the outriggers, the deformation of the supporting surface (compaction of the soil) is possible during the operation of the MS [7]. In this connection, further improvement of mathematical models is required in order to better take into account the mutual influence of MS, the basic chassis of MTTM and the deformable support base.



Pic. 1. Structure of complex mathematical model.



Pic. 2. Generalized model of the basic chassis of MTTM.

Objective. *The objective of the author is to consider a complex mathematical model of the manipulation system of a mobile transport-technological machine.*

Methods. *The author uses general scientific and engineering methods, mathematical methods, calculation methods.*

Results. *The complex mathematical model proposed for such purposes consists of a set of submodels that take into account the dynamics of each of the components of the system «Working body – Manipulation system – Base machine – Support base – Environment» (Pic. 1).*

Subsystem 0 *combines external environmental influences (wind load, temperature influence, impulse impact).*

Subsystem I *includes elements located under the MTTM, creating the reaction forces of the supports. The influence of the elements of the subsystem on the work of the MS is taken into account by: the mathematical model of the support surface (macro- and microroughness models) and the mathematical model of the deformable support base (when the base machine moves or the manipulation system works, the support surface is constantly changing). It is possible to distinguish the following types of deformable support base: for land vehicles (ground, road), railway track, liquid (for floating vehicles).*

Subsystem II *includes elements of MTTM (without MS elements) – elements of the base machine. The main ones are the hull, the suspension components, the propulsor, the engine with the transmission, the outriggers. When the MS is working, the influence of outriggers is taken into account, and when moving MTTM – is not taken into account. We can distinguish the following types of basic machines: wheeled or tracked chassis, railway carriage, floating vehicle.*

Subsystem III *includes MS elements: links of articulated jib, hydraulic cylinders, hinges. Also in this subsystem there are damages received by the MS during operation (destruction, backlash).*

Subsystem IV *includes a cargo fixed to a rigid or flexible suspension with a load-holding body, or a specialized working body MTTM (clamp hook, delimbing head, milling cutter, ram engine).*

If the links of the MS are considered absolutely rigid, then their dynamics is described by an equation of the form:

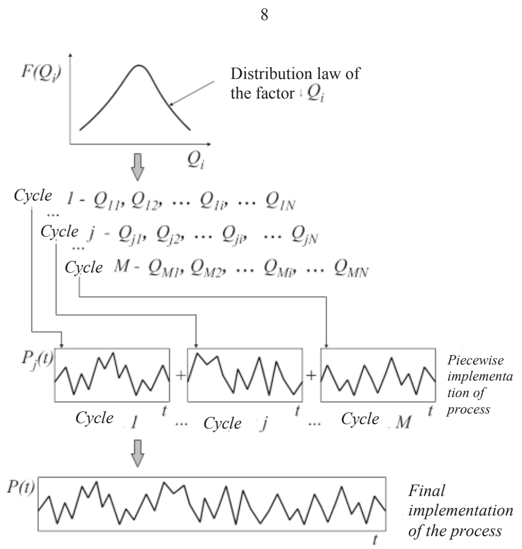
$$[H(q)]\{q''\} + \{C(q, q', F)\} = \{\tau\},$$

where $\{\tau\}$ is a vector of generalized forces; $[H(q)]$ is a matrix of inertia, the components of which depend on the generalized coordinates; $\{q''\}$ is a vector of generalized accelerations; $\{C(q, q', F)\}$ is a vector, depending on the generalized coordinates, generalized velocities and external influences F [2].

The equations of motion of the links of the MS with account of elastic compliance are constructed using the method of Lagrange multipliers, which allows taking into account the structural limitations:

$$M_{3i}\ddot{q}_i + C_q^T\lambda = Q_{esi} - Q_{v3i} - Q_{fsi},$$

where C_q is a matrix of coupling equations; λ are Lagrange multipliers, Q_{3i} is a vector of external forces;



Pic. 3. The method of dynamic loading simulation MS MTTM.

Q_{v3i} is a vector of forces that depend on the squares of the velocities; Q_{f3i} is a vector of elastic forces [4, 5].

A mathematical model of the power hydraulic drive is designed, oriented to its inclusion in the structure of the complex mathematical model of the multipurpose MTTM. The model makes it possible to determine the forces acting on the links of the MS, and is a system of differential and algebraic equations with respect to unknowns: pressure (p_i)) and flow rate (Q_j).

There is a large number of basic chassis MTTM, designed for installation of manipulation systems. The greatest influence on the dynamics of the base machine is provided by the propulsion type. A generalized model of the base machine is shown in Pic. 2.

The equations of motion of the system in general form are given below. For the sprung mass of the base machine, the first three equations of motion are the most important.

$$m_0\ddot{z}_{\text{м}} - \sum_i\sum_j\sum_k\left[F_{kij} + F_{aji}\right] = P_z;$$

$$J_x\ddot{\phi}_x - \sum_i\sum_j\sum_k\left[F_{kij}s_\phi R_{xkij} + F_{aij}s_\phi R_{xaij}\right] = M_x + P_y B;$$

$$J_y\ddot{\phi}_y - \sum_i\sum_j\sum_k\left[F_{kij}s_\phi R_{ykij} + F_{aij}s_\phi R_{yaij}\right] = M_y - P_z O_1 Z + P_x B;$$

$$m_0\ddot{x}_{\text{м}} - \sum_i\sum_j\sum_k\left[F_{xkij} + F_{xaji}\right] = P_x;$$

$$m_0\ddot{y}_{\text{м}} - \sum_i\sum_j\sum_k\left[F_{ykij} + F_{yaji}\right] = P_y;$$

$$J_z\ddot{\phi}_z - \sum_i\sum_j\sum_k\left[F_{xkij}y_{R_{kij}} + F_{xaji}y_{R_{akij}} + F_{ykij}x_{R_{kij}} + F_{yaji}x_{R_{akij}}\right] =$$
$$= M_z + P_y O_1 Z;$$

…

$$m_{kij}\ddot{x}_{kij} + \left(c_{nji} + c_{kij}\right)\left(x_{kij} - z_{\text{м}} - s_\phi\phi_y R_{ykij} - s_\phi\phi_x R_{xkij} + \xi_{kij}(t)\right) +$$
$$+\left(\alpha_{nji} + \alpha_{kij}\right)\left(\dot{x}_{kij} - \dot{z}_{\text{м}} - s_\phi\dot{\phi}_y R_{ykij} - s_\phi\dot{\phi}_x R_{xkij}\right) = 0;$$

…

$$m_{aij}\ddot{x}_{aji} + \left(c_{aji} + c_{naij}\right)\left(x_{aji} - z_{\text{м}} - s_\phi\phi_y R_{yaji} - s_\phi\phi_x R_{xaji} + \xi_{aij}(t)\right)$$
$$+\left(\alpha_{aji} + \alpha_{naij}\right)\left(\dot{x}_{aji} - \dot{z}_{\text{м}} - s_\phi\dot{\phi}_y R_{yaji} - s_\phi\dot{\phi}_x R_{xaji}\right) = 0,$$

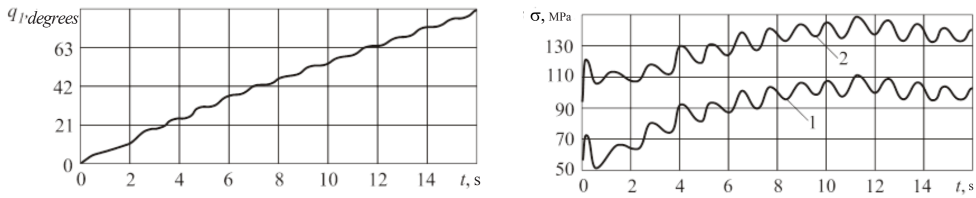
where c_{k1i}, α_{k1i} is coefficient of elasticity and viscosity of the mass suspension element m_{k1i}; c_{k2i}, α_{k2i} is coefficient of elasticity and viscosity of the mass suspension element m_{k2i}; c_{n1i}, α_{n1i} is coefficient of elasticity and viscosity of the frame of the base machine for the action of the mass suspension element m_{k1i}; c_{n2i}, α_{n2i} is coefficient of elasticity and viscosity of the frame of the base machine for the action of the mass suspension element m_{k2i}; c_{aij}, c_{naij}, α_{aij}, α_{naij} are coefficients of elasticity and viscosity of the outriggers and the frame of the base machine for impact from the outriggers; $c_{\text{м}}$, $\alpha_{\text{м}}$ is coefficient of elasticity and viscosity of the frame of the base machine for manipulator action; s_ϕ is the multiplier for selecting the sign (if the force on the side of the reference element leads to positive rotation $s_\phi = 1$, to negative rotation – $s_\phi = -1$); R_{ykij}, R_{yakij} is distance from the center of gravity to the point of attachment of the support element along the y axis; R_{xkij}, R_{xakij} is distance from the center of gravity to the point of attachment of the support element along the x axis; F_{xkij}, F_{xaji}, F_{ykij}, F_{yaji} are elastic forces, with shifts along the x and y axes, calculated similarly to the forces F_{kij} and F_{aji}, but with the corresponding rigidity and dissipation coefficients; x_R and y_R are components of vectors R; B is height of the point G above the gravity center. The parameters are determined either analytically or by the finite element method [8]. The manipulation system acts on the base chassis by means of six force factors: the forces P_x, P_y, P_z and moments M_x, M_y, M_z.

The deformation of the support base ε_z under the support element is studied using the nonlinear theory of elastic-visco-plastic materials. For this purpose, the dependence is constructed for one of the chassis types: tracked or wheeled with weak or strong pumping of the wheels. Such a relationship is as follows:

$\varepsilon_z = \varepsilon_z(t, E, K_z, \sigma_z, t_\kappa)$,

where t is time; E is instantaneous modulus of deformation; K_z is a function of creep speed; σ_z is stresses (pressure) at the point of contact of the support element and the support base; t_κ is the end time of deformation process of the soil under the support element [7].

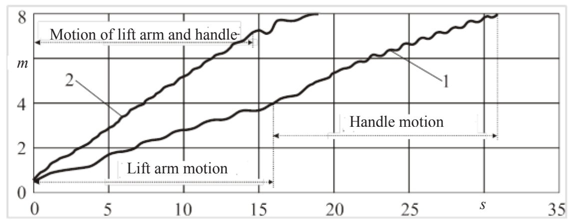
To calculate the metal structure of the manipulation system for strength, it is necessary to determine the loads acting directly on it and the values of the corresponding stresses at hazardous points, sections, and elements. The evaluation of the dynamic loading is carried out after the construction of a probabilistic



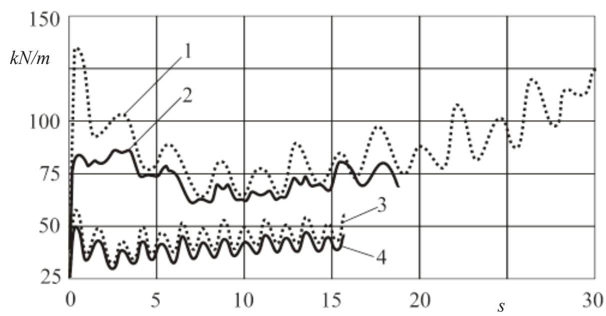
a) b)

Pic. 4. Results of modeling dynamics when moving the handle:
***a* – graph of the change in the general coordinate of the handle (angle of rotation);**
***b* – graph of the change in the maximum equivalent stresses in the links;**
1 – stresses in the handle; 2 – stresses in the lift arm.

Pic. 5. The graph of the change in the vertical coordinate of the suspension point of the cargo MS MTTM AST-4-A: 1 – with separate movement (first the lift arm moves, then the handle); 2 – with joint movement of links.



Pic. 6. The graph of the change in the forces in the hydraulic cylinders MC MTTM AST-4-A: 1, 2 – hydraulic cylinder of the lift arm drive; 3, 4 – the hydraulic cylinder of the handle drive; 1, 3 – with separate movement (first the lift arm moves, then – the handle); 2, 4 – when moving together.



family of realizations of the process of changing stresses in hazardous locations. The general algorithm for estimating dynamic loading consists of the steps shown in Pic. 3.

1. During the simulation, the exact values of the random load factors Q_i are determined for each cycle of operation of the manipulation system of the mobile transport-technological machine during the period of work under study.

2. Dynamic processes in the MS are simulated using a deterministic complex dynamic model. The simulated values of loading factors serve as the initial parameters of these models. The result of the simulation is a piecewise realization of the process $P_j(t)$ of changing the load characteristics of the MS.

3. As a result, from the piecewise implementations corresponding to different combinations of loading factors, the final realization P(t) of the process of changing the loading characteristic (dynamic forces or stresses in the metal structure) is added.

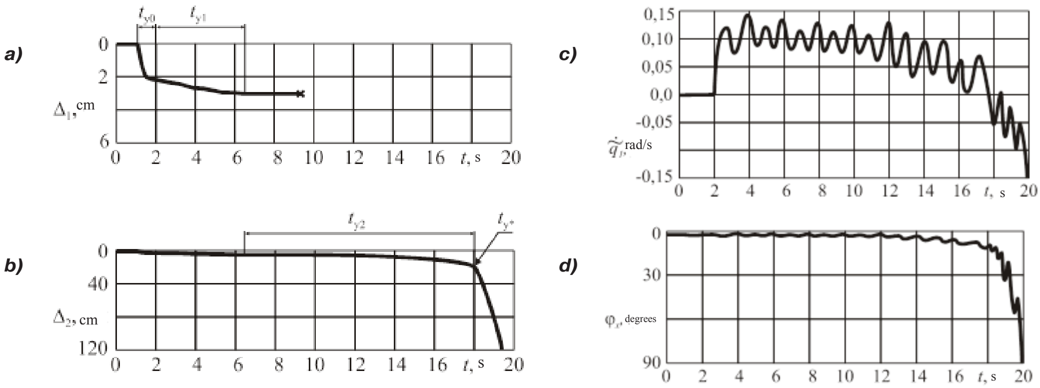
4. This process is repeated many times, after which a probabilistic family of realizations is formed. If necessary, the simulation results of load modeling are performed (blocks are constructed or distribution laws are determined).

Using the developed model, the dynamics of the three-link MS MTTM AST-4-A was simulated on the basis of the crawler tractor TT-4M [5, 6, 8]. The manipulation system of this machine consists of three links: a rotating column, an arrow and a handle, on which a hook suspension is fixed. The MS links can move both separately and together. The results of modeling the dynamics when moving the MS handle are shown in Pic. 4. Its position is determined by the generalized coordinate q_1.

The elasticity of the MS links causes the oscillatory character of the graphs (Pic. 4), even at the stage of steady motion, when the solution for the model with rigid links gives a zero value. The range of oscillation of the magnitude of acceleration thus reaches 15–20 MPa. The duration of the transient process at the beginning of the movement of the link increases from 0,1–0,2 to 2,0–2,5 s.

Hydraulic drive and kinematic scheme MS MTTM AST-4-A allows simultaneous movement of the handle and lift arm. In this case, the pump feed is distributed between the hydraulic cylinders through the consumption divider. The graph of the vertical coordinate change of the suspension point of the load when the lift arm and the handle are moving together is shown in Pic. 5. After 15–16 seconds, the handle reaches its end position and its drive switches off. This causes a temporary increase in the amplitude of the oscillations (transient process). The rest of the time only the lift arm drive works. As soon as the lift arm reaches its extreme position, the hydraulic actuator MS is completely shut off. Combination of movements leads to a reduction in cycle time by 40–50 %.

The graph of the change in the forces in the hydraulic cylinders MS MTTM AST-4-A with the joint movement of the lift arm and the handle is shown in Pic. 6. It can be seen that the combination



Pic. 7. Results of simulation of subsidence under MTTM AST-4-A:
a – deformation of soil under the track No. 1; b – deformation of soil under the track No. 2; c – generalized speed of the MS handle; d – angle of the side heel of the base chassis of the machine.

of movements leads to a decrease in effort by 25–30 %.

One of the goals of creating a complex model was the possibility of assessing the stability of MTTM, taking into account the influence of the base machine and the deformation of the support surface. Let's consider the case of a sharp subsidence of the ground under one of the tracks MTTM AST-4-A. After installing the machine on the support base, the soil (loam) is compacted under both tracks (Pic. 7a). This process lasts for t_{y0}. At the time t = 2 s, the load begins to rise. For some time, t_{y1} dynamic loads caused by the operation of the MS, lead to additional compaction of the soil under the tracks. The total compaction is 3,1 cm.

After 9,5 seconds after the beginning of the lifting of the load, subsidence starts under one of the tracks, caused by external influences (erosion or collapse of the underground cavity). This leads to the disarmament of a single track (Pic. 7a) and the appearance of a strong roll of MTTM in the installation plane of the MS (Pic. 7d). At time t = 18, subsidence under the loaded track bearing the main load begins to increase rapidly (Pic. 7b). With a delay of 0,3–0,5 s, the machine begins to abruptly lean (Pic. 7d). The stability angle for MTTM AST-4-A is 62.6˚. Its exceeding will lead to loss of stability of the machine. From Pic. 7d it can be seen that, taking into account the deformations of the ground, a sharp increase in the angle of inclination of the machine begins at $\phi_x = 55^o$, which then leads to its overturning (loss of overall stability).

Conclusions. *The developed complex model can be used to simulate the dynamics and estimation of the dynamic loading of MS MTMM under various operating conditions: movement of individual lift arm links, joint movement of links, movement of the base machine with the load on the stationary MS, subsidence of the ground under the MTTM. When the movements of MS MTMM links are combined, the cycle time is reduced by 40–50 %, and the forces overcome by hydraulic cylinders are reduced by 25–30 %. The sharp subsidence of the ground under MTTM can lead to a loss of overall stability. The risk of such a development of events can be estimated using the developed complex mathematical model of MS MTTM.*

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