EVALUATION OF TRACK'S VIBRATION AT HIGH SPEED

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ABSTRACT

This article proposes an analytical method of assessment of track's vibration level, based on the spectral decomposition of functions of sleeper deflections under the influence of passing axles of rolling stock. Train is considered to be formed from vehicles of the same type and of unlimited length. Vibrations are studied within a stable system of coordinated, related to the ground, for one sleeper during the passage of a train of unlimited length through track's cross-section. The computational scheme takes into account vibration damping of a track, considered as an infinitely long beam on a viscoelastic foundation. The algorithm and the results of numerical calculation of track's vibration are given at speeds in the area of conventionally critical speed (the speed at which the model that does not take into account energy dissipation on the way predicts vibrations of unrestricted amplitude).

It is shown that when the train's speed is equal to conditionally critical (of about 500–600 km / h), the track's vibrations reach their peak in the whole frequency range. Results are obtained for a wide frequency range from 16 to 32000 Hz in 12 octave bands in decibel scale. Methods, outlined in this article, make it possible to solve problems of accumulation of residual track's deformations, as well as give direction for solving problems associated with the vibrations of the roadbed.

ENGLISH SUMMARY

Background. *Speed of high-speed transport is continuously increasing. Relatively recently speed of 300–350 km / h has been considered as very high, but now the question is how to achieve the level of 400–450 km / h on Russian railways. In this regard, the task arises to evaluate track's vibrations at such or even speeds.*

Objective. *The objective of the authors is to propose a new evaluation method of track's vibrations, which occur in high-speed traffic.*

Methods. *The authors use mathematical method, analysis and scientific description.*

Results. *Article [1] is devoted to problems of analytical evaluation of track's vibration level under passing trains, formed from vehicles of the same type. To determine the shape of the curve of a rail's deflection a model was used in which, to simplify the solution, energy dissipation was not taken into consideration. Solutions obtained in this case are sufficiently accurate at speeds of up to 300–350 km / h. At speeds of over 500 km / h and disregarding damping a system may become unstable, and solutions become unbounded. In fact, the energy in the system always dissipates. In the vicinity of the speed, which is critical in the absence of damping (conditionally critical), we should expect increased dynamic effects.*

It is known that in France for high-speed line speed of 574 km / h was implemented, which, according to the authors, is higher than conventionally critical.

In general, with account for damping the deflection of a fixed track's point when singular vertical force Q = 1 passes through the cross section of a track with speed V is described by the differential equation [2]:

$$EI_y^o \frac{d^4 Z_p^o}{du^4} + \left(N^o + m_z^o V^2\right)\frac{d^2 Z_p^o}{du^2} - f_z^o V \frac{dZ_p^o}{du} + U_z^o Z_p^o = 0, \quad (1)$$

which is valid everywhere except at the point of application of force Q = 1, where there is discontinuity of the third derivative resulting from the vertical deflection of a rail.

In formula (1) there are following notations:

E – elasticity modulus of rail steel;

I_y^o *– inertia moment of a rail with regard to the main cross horizontal axis y;*

Z_p^o *– vertical deflection of a rail;*

u – distance of driving force from considered fixed cross- section of a rail;

N^0 – axial force in a rail;

m_z^o *– distributed along the length reduced mass of a rail and a foundation at vertical vibrations of a track;*

f_z^o *– distributed along the length vibration damping of a track at vertical vibrations;*

U_z^o *– elasticity modulus of a rail base in the vertical plane.*

The characteristic equation of the differential equation (1) takes a form:

$$r^4 + \frac{N^o + m_z^o V^2}{EI_y^o} r^2 - \frac{f_z^o V}{EI_y^o} + \frac{U_z^o}{EI_y^o} = 0. \quad (2)$$

Since all coefficients of the equation (2) are valid, it has two pairs of complex conjugate roots $r_1 = \bar{r}_3,\ r_2 = \bar{r}_4$. *Applying Hurwitz criterion to the equation (2), it can be proved that its two roots represent negative real part, while the other two represent positive real part.*

We assume for definiteness

$$Re\,r_1 < 0;\ Re\,r_3 < 0;\ Re\,r_2 > 0;\ Re\,r_4 > 0.$$

In the cross-section under the driving force rail's deflection, angular deflection and bending moment are continuous functions of the abscissa u *, and the shear force is discontinuous with a jump equal to the external unit force Q = 1. Given these boundary conditions, the solution of equation (1) takes the form*

$$Z_p^o(u) = \begin{cases} A_{11}e^{-\alpha_1 u}\sin\beta_1 u + A_{12}e^{-\alpha_1 u}\cos\beta_1 u & when\ u \ge 0; \\ A_{21}e^{\alpha_2 u}\sin\beta_2 u + A_{22}e^{\alpha_2 u}\cos\beta_2 u & when\ u \le 0, \end{cases} \quad (3)$$

where $\alpha_1 = |Re\,r_1|;\ \alpha_2 = |Re\,r_2|;\ \beta_1 = |Im\,r_1|;\ \beta_1 = |Im\,r_2|.$

Coefficients A_{ij} (i j=1,2) are determined as follows:

$$A_{11} = \frac{1}{EI_y^o \beta_1}\bullet\frac{C}{C^2 + D^2}; \qquad A_{12} = \frac{1}{EI_y^o \beta_1}\bullet\frac{D}{C^2 + D^2};$$

$$A_{21} = -\frac{1}{EI_y^o \beta_2}\bullet\frac{F}{F^2 + G^2}; \qquad A_{22} = \frac{1}{EI_y^o \beta_2}\bullet\frac{G}{F^2 + G^2};$$

$$C = (\alpha_1 + \alpha_2)^2 - \beta_1^2 + \beta_2^2; \qquad D = -2\beta_1(\alpha_1 + \alpha_2);$$

$$F=(\alpha_1+\alpha_2)^2-\beta_2^2+\beta_1^2; \quad G=2\beta_2(\alpha_1+\alpha_2).$$

If at time t = 0 driving unit force Q = 1 is located in cross-section over the considered cross-sleeper, rail's deflection in this cross-section at time t is determined by following expression:

$$Z_p^o(t)=\begin{cases}-A_{11}e^{\alpha_1 Vt}sin\beta_1 Vt+A_{12}e^{\alpha_1 Vt}cos\beta_1 Vt \;\; when\, t\le 0;\\ -A_{21}e^{-\alpha_2 Vt}sin\beta_2 Vt+A_{22}e^{-\alpha_2 Vt}cos\beta_2 Vt \;\; when\, t\ge 0.\end{cases} \quad (4)$$

In this case evident relation u =–Vt was used. In a particular case, when there is no energy dissipation in the system ($f_z^o=0$) characteristic equation (2) degenerates into a biquadratic. Then deflection of a fixed rail point under the passage of a unit vertical force Q = 1 is determined by the expression:

$$Z_p^o(t)=\frac{exp\{-\alpha V|t|\}}{4EI_y^o(\alpha^2+\beta^2)\alpha\beta}(\alpha sin\beta V|t|+\beta cos\beta Vt), \quad (5)$$

where

$$\alpha=\sqrt{\sqrt{\frac{U_z^o}{4EI_y^o}}-\frac{N^o+m_z^oV^2}{4EI_y^o}};$$

$$\beta=\sqrt{\sqrt{\frac{U_z^o}{4EI_y^o}}+\frac{N^o+m_z^oV^2}{4EI_y^o}}.$$

Track's vibrations at rail's deflection under the influence of unit force Q = 1 are considered in article [1]. As already mentioned, solutions obtained using the formula (5) are sufficiently accurate at speeds of trains of 300–350 km / h.

At high speeds it is necessary to consider that in the absence of vibration damping in the system ($f_z^o=0$) and speed

$$V=\sqrt{\frac{1}{m_z^o}\left(2\sqrt{EI_y^oU_z^o}-N^o\right)}$$

system becomes unstable.

Indeed, in this case, we have α=0, and solution (5) becomes infinite. We define the Fourier spectrum of the function (4). In contrast to the function (5) it is asymmetric with respect to the point t = 0. This circumstance provides complexity to the Fourier spectrum; it includes not only the cosine transform, but the sine-Fourier transform of function (4).

We first calculate the cosine Fourier transform of function (4). Obviously, we have

$$\Phi_{zc}(\omega)=-A_{11}\int_{-\infty}^{0}e^{\alpha_1 Vt}sin\beta_1 Vtcos\omega tdt+A_{12}\int_{-\infty}^{0}e^{\alpha_1 Vt}cos\beta_1 Vtcos\omega tdt-$$

$$-A_{21}\int_{0}^{\infty}e^{-\alpha_2 Vt}sin\beta_2 Vtcos\omega tdt+A_{22}\int_{0}^{\infty}e^{-\alpha_2 Vt}cos\beta_2 Vtcos\omega tdt.$$

Changing in the first and second integrals a variable t into –t and performing integration, we obtain the following result:

$$\Phi_{zc}(\omega)=\left\{\begin{matrix}\frac{\beta_1V+\omega}{2}\left[\alpha_1^2V^2+(\beta_1V+\omega)^2\right]^{-1}+\\ +\frac{\beta_1V-\omega}{2}\left[\alpha_1^2V^2+(\beta_1V-\omega)^2\right]^{-1}\end{matrix}\right\}A_{11}+$$

$$+\frac{\alpha_1V}{2}\left\{\left[\alpha_1^2V^2+(\beta_1V+\omega)^2\right]^{-1}+\left[\alpha_1^2V^2+(\beta_1V+\omega)^2\right]^{-1}\right\}A_{12}-$$

$$-\left\{\begin{matrix}\frac{\beta_2V+\omega}{2}\left[\alpha_2^2V^2+(\beta_2V+\omega)^2\right]^{-1}+\\ +\frac{\beta_2V-\omega}{2}\left[\alpha_2^2V^2+(\beta_2V-\omega)^2\right]^{-1}\end{matrix}\right\}A_{21}+$$

$$+\frac{\alpha_2V}{2}\left\{\begin{matrix}\left[\alpha_2^2V^2+(\beta_2V-\omega)^2\right]^{-1}+\\ +\left[\alpha_2^2V^2+(\beta_2V+\omega)^2\right]^{-1}\end{matrix}\right\}A_{22}. \quad (6)$$

We now calculate the sine-Fourier transform of function (4):

$$\Phi_{zs}(\omega)=-A_{11}\int_{-\infty}^{0}e^{\alpha_1 Vt}sin\beta_1 Vtsin\omega tdt+A_{12}\int_{-\infty}^{0}e^{\alpha_1 Vt}cos\beta_1 Vtsin\omega tdt-$$

$$-A_{21}\int_{0}^{\infty}e^{-\alpha_2 Vt}sin\beta_2 Vtsin\omega tdt+A_{22}\int_{0}^{\infty}e^{-\alpha_2 Vt}cos\beta_2 Vtsin\omega tdt.$$

Changing in the first and second integrals t into –t and integrating, we obtain:

$$\Phi_{zs}(\omega)=-\frac{\alpha_1V}{2}\left\{\begin{matrix}\left[\alpha_1^2V^2+(\beta_1V-\omega)^2\right]^{-1}-\\ \left[\alpha_1^2V^2+(\beta_1V+\omega)^2\right]^{-1}\end{matrix}\right\}A_{11}-$$

$$-\left\{\begin{matrix}\frac{\beta_1V+\omega}{2}\left[\alpha_1^2V^2+(\beta_1V+\omega)^2\right]^{-1}-\\ -\frac{\beta_1V-\omega}{2}\left[\alpha_1^2V^2-(\beta_1V-\omega)^2\right]^{-1}\end{matrix}\right\}A_{12}-$$

$$-\frac{\alpha_2V}{2}\left\{\begin{matrix}\left[\alpha_2^2V^2+(\beta_2V-\omega)^2\right]^{-1}-\\ -\left[\alpha_2^2V^2+(\beta_2V+\omega)^2\right]^{-1}\end{matrix}\right\}A_{21}+$$

$$+\left\{\begin{matrix}\frac{\beta_2V+\omega}{2}\left[\alpha_2^2V^2+(\beta_2V+\omega)^2\right]^{-1}-\\ -\frac{\beta_2V-\omega}{2}\left[\alpha_2^2V^2+(\beta_2V-\omega)^2\right]^{-1}\end{matrix}\right\}A_{22}. \quad (7)$$

Complex Fourier spectrum of the function (4) has the form

$$\Phi_z(\omega)=\Phi_{zc}(\omega)+i\Phi_{zs}(\omega) \quad (8)$$

where $i=\sqrt{-1}$ *– imaginary unit.*

We define the density of spectrum of process square $Z_p^o(t)$*, appearing on the way under the passage of a random force Q (t), which mean square is*

$$Q^2=Q^2+\sigma_Q^2, \quad (9)$$

where Q *– mean value of force Q (t);* σ_Q *– RMS deviation of force's value from the mean value* Q.

Parameters Q *and* σ_Q *can be determined by considering processes in the mobile, related to the vehicle's coordinate system [1].*

With account for formulas (8) and (9) the spectral density of a square of rail's deflection reduces to the expression

$$S_z^o(w)=\langle Q^2\rangle\Phi_z(w)\bar{\Phi}_z(w)=Q^2\left(\Phi_{zc}^2(w)+\Phi_{zs}^2(w)\right), \quad (10)$$

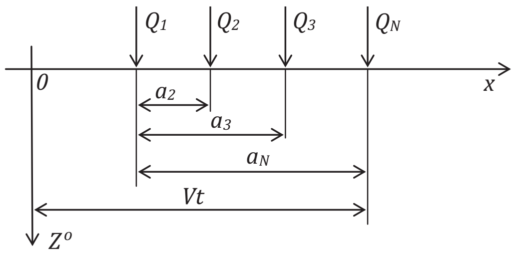
where $\bar{\Phi}_z(\omega)$ *– function, complex conjugate to function* $\Phi_z(\omega)$.

In expression (10) value Q^2 *is determined by formula (9), and functions* $\Phi_{zc}(\omega)$ *and* $\Phi_{zs}(\omega)$ *– respectively by formulas (6) and (7).*

Assume that when vehicle passes through a fixed track's cross-section all its axles transfer vertical forces to the rails with the same statistical characteristics:

$$Q=\frac{1}{N}\sum_{j=1}^{N}Q_j; \quad \sigma_Q^2=\frac{1}{N}\sum_{j=1}^{N}\sigma_Q^2, \quad (11)$$

where j is an axis number of a vehicle; N – total number of axles in the vehicle.



Pic. 1. Movement scheme of axles' group of a vehicle along the rail.

In this case, rail's deflection is determined by the sum of deflections resulting from individual forces, phase-shifted relative to wheel pair, which is the first in the direction of travel (Pic. 1).

In Pic.1 variable x is an abscissa of the current cross-section of a rail, calculated from the start of the fixed coordinate system.

In the frequency area shift is performed by the operator:

$$W(i\omega) = \sum_{j=1}^{N} exp\left\{i\omega \frac{a_j - a_N}{V}\right\},\ a_1 = 0. \qquad (12)$$

With account for (12) density of spectrum of process square $Z_p^o(t)$ when a single vehicle passes through the cross- section of a rail is:

$$S_z^*(\omega) = W(i\omega)\bar{W}(i\omega)S_y^o(\omega), \qquad (13)$$

where $\bar{W}(i\omega)$ is a frequency characteristics, complex conjugate to the frequency characteristics $W(i\omega)$.

If a chain of vehicles passes through rail's cross-section, spectrum density of process square $Z_p^o(t)$ of rail's deflection under the train can be determined as a superposition of the spectra of rail's deflections, caused by the passage of one vehicle with shifts jT_0, where T_0 – period, during which a vehicle passes a way, equal to its length l:

$$T_0 = \frac{l}{V}. \qquad (14)$$

Number j changes from – M to + M when $M \to \infty$.

After appropriate transforms [1], we obtain the following expression for the spectral density of the process of rail's deflection under a train formed from vehicles of the same type:

$$S_z(\omega) = S_z^*(\omega)\omega_0 \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0), \qquad (15)$$

where $\delta(\omega)$ – Dirac delta function;

$$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi V}{l}. \qquad (16)$$

With the expression (15), we can find the mean square of the process $Z_p^o(t)$:

$$\langle Z_p^{o2}(t)\rangle = \int_0^\infty S_z(\omega)d\omega. \qquad (17)$$

Substituting in (17), expression (15) and taking into account the properties of the delta function, we obtain:

$$\langle Z_p^{o2}(t)\rangle = \sum_{n=1}^{\infty} d_n^2 \qquad (18)$$

where d_n^2 – dispersion of process $Z_p^o(t)$, attributable to harmonic n.

Coefficients d_n^2 are determined as follows:

$$d_n^2 = 2\omega_0 S_z^*(n\omega_0). \qquad (19)$$

These coefficients fully reflect the dynamics of rail base when a train, formed from wagons of the same type, passes along the track.

Dispersion of vibration accelerations of a sleeper attributable to harmonic n, is written in the form [1]:

$$d^2_{\ddot{z}_{шn}} = \omega_0^4 n^4 d_n^2 \left|W^{z_p}_{z_ш}(in\omega_0)\right|^2, \qquad (20)$$

where $W^{z_p}_{z_ш}(i\omega)$ is frequency characteristics of the system with the input on the rail's deflection $Z_p^o(t)$ and output on the sleeper deflection $Z_ш^o(t)$.

Frequency characteristics $W^{z_p}_{z_ш}(i\omega)$ is determined with the help of expression [1]

$$W^{z_p}_{z_ш}(i\omega) = \frac{C_c}{-\omega^2 M_ш + i\omega f_ш + C_ш + C_c}, \qquad (21)$$

where C_c – stiffness of a clamping unit;
$C_ш$ – stiffness of sleeper base;
$M_ш$ – half mass of a sleeper;
$f_ш$ – vibration damping of a sleeper base.

Level of vibration acceleration of a sleeper is determined in octave bands and frequency range
$16 < n\omega_o < 32000$. (22)

Choice of octaves depends on the speed of a vehicle, since frequency of basic tone of vibrations ω_o is characterized by expression (16).

Harmonic number corresponding to the lower of the frequencies:

$$n_1 = \rceil \frac{16}{\omega_0} + 1, \qquad (23)$$

here $\rceil$ is a sign of the integer part of the fraction behind this sign.

It can be seen that the value n_1 is simultaneously a number $n^{(1)}$ of the first harmonic of the first frequency octave bands $16 < \omega < 32$, and value $n^{(1)}$ – the first member of geometric progression
$n^{(m)} = n^{(1)} \cdot 2^{m-1}$, (24)
where $n^{(m)}$ – number of the first harmonic of an octave band m.

In this task there are 11 octave bands.

Vibration accelerations of sleepers in frequency octave band shall be expressed in logarithmic units relative to the gravitational acceleration $g = 9{,}81\ m/s^2$:

$$L\left[\sum_{n=n^{(m)}}^{n=n^{(m+1)}-1} d^2_{\ddot{z}_H n}\right] = 10lg\left[\frac{1}{g^2}\sum_{n=n^{(m)}}^{n=n^{(m+1)}-1} d^2_{\ddot{z}_H n}\right]. \qquad (25)$$

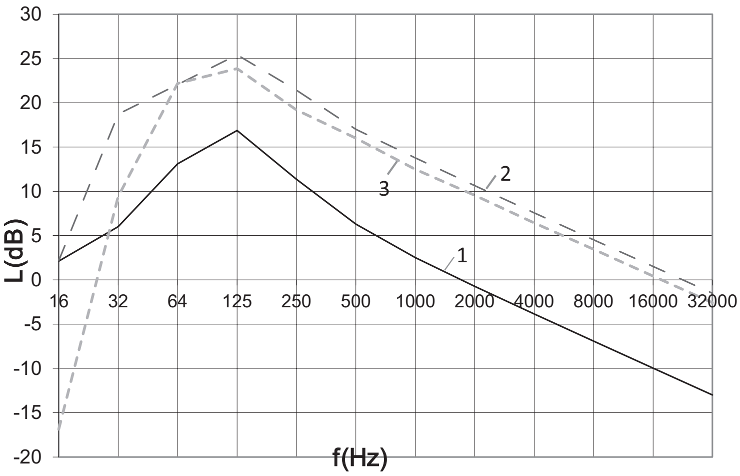
In expression (25) value $d^2_{\ddot{z}_ш n}$ is determined by formula (20). Value $\left[\sum_{n=n^{(m)}}^{n=n^{(m+1)}-1} d^2_{\ddot{z}_ш n}\right]$ is a sum of dispersions of all harmonics, which are located in frequency octave band m.

Below there are some results of calculations of track's vibration under the influence of high-speed trains. Due to the fact that we do not know its parameters at a speed of 400 km / h or more, and the design parameters of track superstructure when such a train passes, calculations are made for the parameters of the train «Sapsan» and track design with ballast and concrete sleepers.

In the calculation scheme it is adopted, that train is formed from vehicles, the number of which is unlimited.

Initial parameters of a vehicle are:

- *base of a bogie $b_т$ = 2,6 m;*
- *base of a car $b_в$ = 17,375 m;*
- *length of the car by an automatic coupler (average in a train) l = 24,175 m.*



Pic. 2. Levels of vibration accelerations of a sleeper when axial force is No=106N and vibration damping is $f_z^o = 4,5 \cdot 10^4\, Pa \cdot s$ (1 – speed 300 km/h; 2 – speed 554 km/h; 3 – speed 700 km/h).

Parameters required for the calculation of the dispersion $d_п^2$ (Pic. 1):

$a_1 = 0$; $a_2 = b_т = 2,6$ m; $a_3 = b_в = 17,375$ m; $a_4 = b_в + b_т = 19,975$ m.

The mean value of the force transmitted from the wheel on the rail:

$Q = 90\, kN$, j = 1, 2, 3, 4.

RMS deviation of a force's value from the mean value σ_{Qj}=14 kN.

Initial parameters of a track:

• elasticity modulus of a rail base U_z^o = 100 MPa;

• moment of inertia of the rail relative to the main cross horizontal axis I_y^o = $3,54 \cdot 10^{-5}$ m^4;

• elasticity modulus of rail steel E = $2,1 \cdot 10^5$ MPa;

• reduced mass of a rail and a rail base distributed along the length m_z^o = 1013 kg/m;

• vibration damping of a track at vertical vibrations distributed along the length $f_z^o = 4,5 \cdot 10^4\, Pa \cdot s$;

• weight of a sleeper $2M_ш$= 275 kg;

• stiffness of a clamping unit $C_c = 9 \cdot 10^7$ N/m;

• stiffness of a rail base $C_ш = 3 \cdot 10^7$ N/m;

• vibration damping of a sleeper base $f_ш = 10^5$ Ns/m.

Level of vibration accelerations of sleepers in decibel scale in frequency octave bands was calculated using formulas, given in this article. Calculations were performed at speeds of 300, 554 and 700 km/h. Speed 554 km/h is conditionally critical in the absence of vibration damping of the track ($f_ш = 0$).

As it can be seen from Pic. 2, when a train moves at a speed, which is equal to conditionally critical (V = 554 km / h), vibration accelerations of a sleeper reach a maximum in the whole frequency range of track's vibrations. In the frequency range of 64–250 Hz level of vibration accelerations of a sleeper comes to 22–25 dB.

With further increase in train's speed level of vibration accelerations of a track decreases at all frequencies, to the greatest extent – in the low frequency range (less than 64 Hz).

Conclusions.

1. The article presents the algorithm and results of numerical calculation of track's vibrations at speeds in the area of conditionally critical speed (when the model, not considering energy dissipation on the way, predicts emergence of vibrations of indefinitely large amplitude).

2. When a train moves at a speed equal to conventionally critical, track's vibrations reach their maximum in the entire frequency range. Further increase in speed implies reducing vibration acceleration at all frequencies.

Keywords: railway, sleeper's vibration, high-speed trains, critical speed, frequency octaves, spectral density, decibels, analytical method, numerical calculation, Hurwitz criterion, Fourier transform.

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Статья поступила в редакцию / article received 01.04.2014
Принята к публикации / article accepted 02.06.2014